Symmetry breaking in the opinion dynamics of a multi-group project organization*

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A bounded confidence model of opinion dynamics in multi-group projects is presented in which each group's opinion evolution is driven by two types of forces: (i) the group's cohesive force which tends to restore the opinion back towards the initial status because of its company culture; and (ii) nonlinear coupling forces with other groups which attempt to bring opinions closer due to collaboration willingness. Bifurcation analysis for the case of a two-group project shows a cusp catastrophe phenomenon and three distinctive evolutionary regimes, i.e., a deadlock regime, a convergence regime, and a bifurcation regime in opinion dynamics. The critical value of initial discord between the two groups is derived to discriminate which regime the opinion evolution belongs to. In the case of a three-group project with a symmetric social network, both bifurcation analysis and simulation results demonstrate that if each pair has a high initial discord, instead of symmetrically converging to consensus with the increase of coupling scale as expected by Gabbay's result (*Physica* A **378** (2007) p. 125 Fig. 5), project organization (PO) may be split into two distinct clusters because of the symmetry breaking phenomenon caused by pitchfork bifurcations, which urges that apart from divergence in participants' interests, nonlinear interaction can also make conflict inevitable in the PO. The effects of two asymmetric level parameters are tested in order to explore the ways of inducing dominant opinion in the whole PO. It is found that the strong influence imposed by a leader group with firm faith on the flexible and open minded follower groups can promote the formation of a positive dominant opinion in the PO.

Keywords: opinion dynamics, pitchfork bifurcation, symmetry breaking, project management

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1. Introduction

Despite the fact that megaprojects delivered through various partnerships are increasingly indispensable in improving people's living standards in modern society, recent studies show that the majority of megaprojects overrun on costs, fall behind schedule, and fail to be delivered at the desired quality.[1] Since numerous engineering practices suggest that a positive project culture can facilitate the successful completion of projects, project culture is increasingly attracting the attention of experts and scholars.[2] Project culture refers to the shared values, basic assumptions and beliefs that the participants involved in a project hold and are used to determine the way they process the project and the relationship with each other in the project environment. Since the culture of a project organization (PO) will affect all the participant's attitudes towards work within the project, it is extremely important to establish a unified, strong project culture in a megaproject. In spite of the strong cooperation willingness the participants may have, creating an excellent project culture is never an easy task due to the inherent divergence in their interests and company cultures. Discord among the contracting parties inevitably appears, which may cause conflicts and create an adversarial and mistrusting environment, thereby having detrimental effects on the performance of the project. The focus of this paper is to explore how the company culture of each participant affects the formation of a unified strong project culture.

In the past decade, physicists and sociologists have worked actively in opinion evolution and culture dynamics,[3−22] and many models have been presented. For example, the voter model, majority rule model, Sznajd model, and bounded confidence mod-

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els, just to quote a few examples.[5,6,22] To characterize the elementary process that determines opinion dynamics, some of social mechanisms that are considered to be reasonable have been defined and turned into mathematical rules. One of these mechanisms is bounded confidence (BC), which argues that, in practice, there is a really effective discussion only if the opinions of the people involved are sufficiently close to each other. BC is corroborated by the findings of social judgment theory that the attitude change in pair-wise interaction will wane past a critical discrepancy. However, classic BC models such as the Deffuant model have their limitations when applied to a project context. They ignore the existence of an individual's bias. Since a PO is a temporary organization with a limited life cycle, the influence of member groups' company culture, as a self-bias force, cannot be eliminated. Recently Gabbay[16] established a model of opinion dynamics driven by a self-bias force and a coupling force to study the effects of nonlinear interactions in small group opinion dynamics. He found that for high bias difference, group discord is characterized by discontinuous transitions as the coupling is varied, and a broker network can reduce group discord more effectively than a clique network. As for the special issue of project culture, most of the published papers are about empirical studies on evolution of project culture of several megaprojects while only a few papers deal with the dynamics of conflict behaviours in a construction project using catastrophe models.[23] However, papers are rarely seen on the formations and evolutions of the opinions of various participant groups coming from different companies in a mega-project, especially on the emergence of the macroscopic opinion of the PO via the interaction and communication between different group participants with disparate opinions.

The rest of this paper is organized as follows. A population dynamical model incorporating the actions of a linear restoring force and nonlinear coupling forces is presented in Section 2 to explore the emergence of dominant opinion and consensus in the course of opinion coupling between cooperative groups in a project. The assumption and the general form of the model and its relationship with other existing relevant models are discussed in Section 2. Section 3 is describes the case of two-group project, where the nonlinear dynamical behaviours are analysed using the bifurcation theory method, and a critical value of initial discord is derived to help identify the regime where these bifurcations may take place. In Section 4 we discuss the case of a three-group project, where diverse bifurcations are studied and the effects of several parameters on the dynamical behaviours are discussed. The paper concludes with offering some insights into future research directions in Section 5.

2. Assumption and formulation of the model

The opinion dynamics of a PO consisting of n member groups with different mainstream opinions is considered. For the sake of simplifying analyses, suppose that all member groups have the same scales: each with N/n individuals, where N is the total number of individuals in the project. Furthermore, the individuals' attitudes (opinions) towards the project, such as paying more/less attention to project quality, schedule and cost, are coarse-grained into 1/0. The ratio, individuals with opinion 1 over the total number (N/n) in group j ($j = 1, \ldots, n$) at time t, is denoted by $x_j(t)$; in the sense of statistics, the collective opinion of group $j$can be measured by x_j, with range [0,1].[24]

The opinion dynamics system (S1) for a PO with n groups is given by the following equations

$$D_t x_i = F_{ii}(x_i, \theta_i) + \sum_{i \neq j} F_{ij}(x_i, x_j), \quad i, j = 1, \ldots, n, \tag{1}$$

$$F_{ii}(x_i, \theta_i) = -h_i(x_i - \theta_i), \tag{2}$$

$$F_{ij}(x_i, x_j) = \mu_{ij} B(x_j - x_i), \tag{3}$$

$$B(u) = u \exp\left(-(u/\lambda)^{\beta}/\beta\right), \tag{4}$$

where $\theta_i, h_i, \mu_{ij} \in (0,1), i, j = 1, \ldots, n$, and D_t is the derivative with respect to t.

It is assumed that group i's opinion x_i gives rise to action by two types of forces: its cohesive force $F_{ii}(x_i, \theta_i)$ wherein the cohesive strength h_i is the commitment of group i to its natural bias, and the coupling forces of groups j and i $F_{ij}(x_i, x_j)$, $j = 1, \ldots, n$ and $j \neq i$, wherein the coupling strength μ_{ij} describes the influential strength of group j on group i through interaction. $F_{ii}(x_i, \theta_i)$ is a linear force which tends to restore x_i back towards its natural bias θ_i, depicting that company culture has a power to guide its members' work attitude and behaviour. $F_{ij}(x_i, x_j)$ is a nonlinear function of the opinion's discrepancy between groups j and i, and will bring x_i closer to

x_j. It is also assumed that the interactions between groups are costly and, hence, there exists an upper bound value of coupling strength for each group, beyond which they will not put more efforts into communication due to its expensive cost. λ is the latitude of acceptance, a term that is used in social judgment theory to describe that attitude change will wane past a critical discrepancy, and we specify the whole PO as being in consensus if the steady difference between two most disparate group opinions equals one latitude of acceptance or less. β is the attitude gradations index, an even number whose value is not less than 2. Our model can be viewed as a generalized BC model that includes the Gabbay model,[16] the social influence network model,[25] and the Deffuant model[26,27] as its special cases in the population dynamics context. When $\beta = 2$, our model is reduced into the Gabbay model. Let $\lambda \to \infty$ and $h_i = 0$, our model is transformed into a social influence network model driven by linear coupling forces; let $\beta \to \infty$, $h_i = 0$, and

$$F_{ij}(x_i, x_j) \to \begin{cases} \mu_{ij}(x_j - x_i), & |x_j - x_i| \leq \lambda \\ 0, & \text{otherwise,} \end{cases}$$

our model is just reduced into the Deffuant dynamics model frequently discussed in the literature about opinion dynamics.

Let k_{ij} be the relative coupling strength of group j to group i, $k_{ij} = \mu_{ij}/h_i$. Since h_i is positive, system S1 has the same equilibrium solutions to the following dynamics system S2

$$D_t x_i = -(x_i - \theta_i) + \sum_{j=1, j\neq i}^{n} k_{ij} B(x_j - x_i), \quad x = 1, \ldots, n. \tag{5}$$

For simplicity, we investigate system S2 in the remainder of the paper instead. Unlike most of existing investigations about opinion dynamics which focus only on consensus formation, project performance is not only related to the consensus between different groups, but also to the proportion of proponents of this positive attitude in the whole PO. In this paper, the global average opinion (GAO) of PO, z, and the maximum discord between different groups, δ, will be discussed later, and are defined as $z = \sum_{i=1}^{n} x_i/n$, and $\delta = \max_{i\neq j} |x_j - x_i|$, respectively.

3. Opinion evolution between two groups

In the case of a two-group project, the system is

$$D_t x_1 = -(x_1 - \theta_1) + k_{12} B(x_2 - x_1), \tag{6}$$
$$D_t x_2 = -(x_2 - \theta_2) + k_{21} B(x_1 - x_2). \tag{7}$$

Perform the linear transformation as

$$\begin{bmatrix} z \\ u \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Since $B(u)$ is an odd function, let (Eq. (6)+Eq. (7))/2, then we will have

$$\begin{aligned} D_t z &= -(z - z_0) - (k_{12} - k_{21}) B(u)/2 \\ &= -(z - z_0) - k\rho B(u), \end{aligned} \tag{8}$$

where $z_0 = (\theta_1 + \theta_2)/2$ is the initial GAO of PO; $k = (k_{12} + k_{21})/2$ is the coupling scale, equivalent to the average of coupling strengths of the two groups; $\rho = (k_{12} - k_{21})/(k_{12} + k_{21})$ is a parameter that accounts for asymmetric level of pair-wise coupling strength with range $[-1, 1]$. If ρ is negative, then group 1 has more influence on group 2, and *vice versa*. Assuming that the initial discord between the two groups $\theta = \theta_1 - \theta_2 > 0$, it is obvious that $u > 0$, $B(u) > 0$.

From Eqs. (6) and (7), we have

$$D_t u = -(u - \theta) - 2kB(u). \tag{9}$$

Equation (9) indicates that the evolution of discord u is directly related to neither z_0 nor ρ.

Now, we study the discord dynamics described by Eq. (9) (see Fig. 1(a)). In terms of the necessary condition of limit point (LP), also known as saddle-node point, $f(u) = D_u f(u) = 0$, we have

$$f(u) = (\theta - u) - 2kB(u) = 0, \tag{10}$$
$$D_u f(u) = -1 - 2kB(u)\left(1 - (u/\lambda)^{\beta}\right)/u = 0. \tag{11}$$

Equation (10) means that the steady discord is determined by k and θ. Substituting k in Eq. (10) into Eq. (11), we have

$$\begin{aligned} D_u f(u) &= -1 + (u - \theta)(1 - (u/\lambda)^{\beta})/u \\ &= -\frac{1}{u\lambda^{\beta}} G_1(u, \theta) = 0, \end{aligned}$$

where $G_1(u, \theta) = u^{\beta+1} - \theta u^{\beta} + \theta\lambda^{\beta}$. We can figure out the bifurcation characteristic by investigating the

roots of the polynomial $G_1(u,\theta)$ when u lies in the interval [0,1] (see Fig. 1(b)). The derivative of $G_1(u,\theta)$ with respect to u is

$$D_u G_1(u,\theta) = [(\beta+1)u - \beta\theta]u^{\beta-1}.$$

Since $u \in (0,1)$ and $\theta \in (0,1)$, when $u^*(\theta) = \beta\theta/(\beta+1)$, the polynomial in this interval has the only minimum value $\theta\lambda^\beta[1-(\theta/\theta_c)^\beta]$, where $\theta_c = \lambda(\beta+1)^{(\beta+1)/\beta}/\beta$ is the critical value of θ which can be used to identify the bifurcation regime. For $\beta \to \infty$, $\theta_c \to \lambda$. For $u \in (0,u^*(\theta))$, $G_1(u,\theta)$ monotonically decreases as u increases. For $u \in (u^*(\theta),1)$, $G_1(u,\theta)$ monotonically increases with u increasing, the critical phenomenon takes place when $G_1(u^*(\theta),\theta) = 0$. Since $G_1(0,\theta)$ and $G_1(1,\theta)$ are both positive, only when $G_1(u^*(\theta),\theta)$ becomes negative can this polynomial have two different real roots in the interval. So it is clear that the LP bifurcation can occur only if $\theta \geq \theta_c$. In fact, when $\theta > \theta_c$, the polynomial may have two different real roots, $u_a(\theta) < u^*(\theta) < u_b(\theta)$. $(ku_a(\theta), k(u_a(\theta)))$ and $(ku_b(\theta), k(u_b(\theta)))$ are two LPs, where $k(u) = (\theta-u)/(2B(u))$ is derived from Eq. (10). In the (k,u) space, the former/latter is the point at which the lower/upper stable branch collides with the middle unstable branch (see Fig. 1(a)). Figure 1(c) shows the relationship between k at bifurcation point and θ. In the dyad case, our model is a generalized cusp catastrophe model in which θ is the splitting factor and k is the normal factor related to the dependent variables u and z in a consistent pattern (Fig. 1(a)). The splitting factor θ is the key variable as it specifies the conditions under which k will affect u in a continuous fashion, and the other conditions under which k will produce discontinuous changes in dependent variables (Figs. 1(a) and 1(d)). In a catastrophe model, when the intensities of the normal factor and splitting factor each reach a critical point, the dependent variable will change suddenly and radically. From Fig. 1(c), we can learn that there exist three distinctive phase regimes, i.e., a deadlock regime (area 1), a convergence regime (area 3), and a bifurcation regime (area 2). In the deadlock regime, two groups are unable to reach consensus due to a too high communication cost; in the convergence regime, the discord of two groups reduces smoothly like a linear dynamical system; in the bifurcation regime, the abrupt convergence may happen if coupling scale k rises above the upper threshold $k_b(\theta)$, while the abrupt divergence occurs if k falls below the lower threshold $k_a(\theta)$.

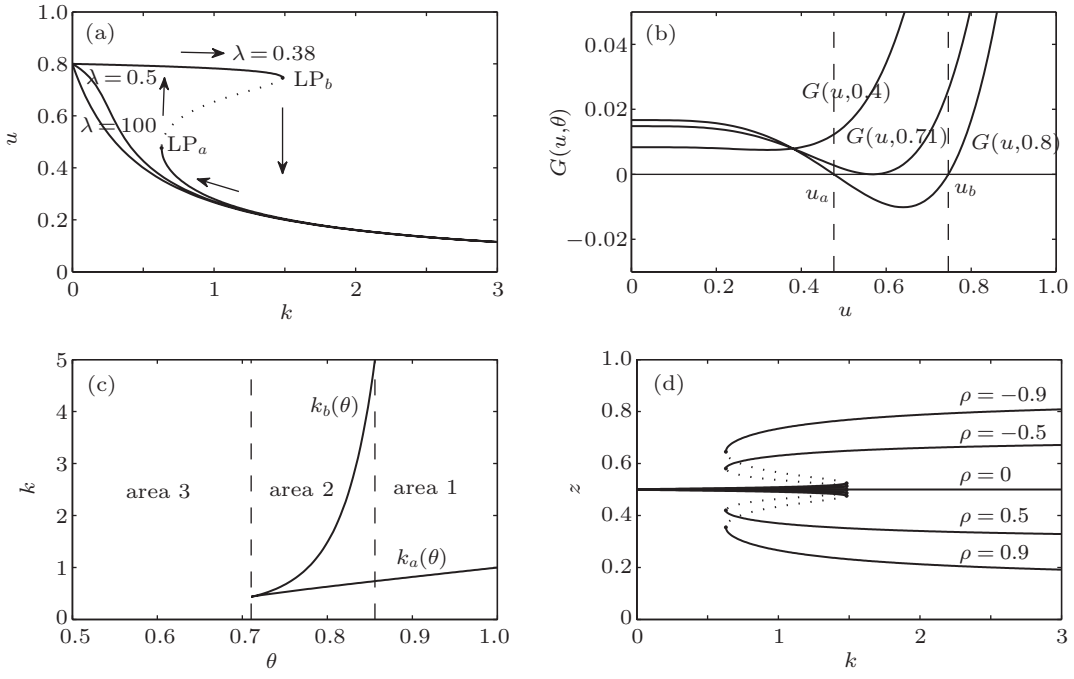


Fig. 1. Simulation results of opinion dynamics in the dyad case (solid/dotted line are for stable/unstable solution): panel (a) shows stable u versus k, panel (b) indicates $G(u,\theta)$ versus u, panel (c) displays k at bifurcation point versus θ, and panel (d) exhibits stable z versus k. Parameters are $\beta = 4$ and $\theta = 0.8$ in panels (a)–(d), and $\lambda = 0.38$ in panels (b)–(d).

Now we explore the dynamics of the GAO of PO. Substituting $B(u)$ in Eq. (10) into Eq. (8) and using $D_t z = 0$, we obtain

$$z - z_0 = -\rho(\theta - u)/2. \tag{12}$$

Equation (12) indicates that given that the group with more positive attitudes has more influence over the other group, the increase of z from the initial state to the steady state is positively related to the product of $-\rho$ and $(\theta - u)$. Since the steady discord is determined solely by k and θ, given that k remains unchanged, $z - z_0$ is proportional to $-\rho$ (Fig. 1(d)). Yet the abrupt change behaviours of z and u are both characterized by $k_a(\theta)$ and $k_b(\theta)$.

4. Opinion evolution among three groups

We consider triad interactions with two different network topologies: i) a broker network in which a broker group, group 2, is connected to the other two that do not interact directly; ii) a clique network in which all three groups interact with each other. In the context of a project, the two types of networks may describe the relationship between a client (group 1) with a contractor (group 2) and a subcontractor (group 3). In a broker network, only the contractor communicates with the client; the requirement of the client is conveyed to the subcontractor by the contractor, while in a clique network, the client interacts with both the contractor and his subcontractor. For both topologies, we assume that the natural bias is so distributed that group 2 is between groups 1 and 3: $\theta_1 - \theta_2 = \theta_2 - \theta_3 = \theta > 0$. We also investigate two different types of interactions in each pair: symmetric and asymmetric.

The general dynamical system in the triad case is

$$\begin{aligned} D_t x_1 &= -(x_1 - \theta_1) + k_{12}B(x_2 - x_1) \\ &\quad + k_{13}B(x_3 - x_1), \\ D_t x_2 &= -(x_2 - \theta_2) + k_{21}B(x_1 - x_2) \\ &\quad + k_{23}B(x_3 - x_2), \\ D_t x_3 &= -(x_3 - \theta_3) + k_{31}B(x_1 - x_3) \\ &\quad + k_{32}B(x_2 - x_3). \end{aligned} \tag{13}$$

Performing the linear transformation as

$$\begin{bmatrix} z \\ u \\ v \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

and using the transformation method mentioned in the dyad case yield

$$\begin{aligned} D_t z &= -(z - z_0) + [(k_{21} - k_{12}B(u) + (k_{32} - k_{23})B(v) \\ &\quad + (k_{31} - k_{13})B(u + v)]/3, \\ D_t u &= -(u - \theta) - (k_{12} + k_{21})B(u) \\ &\quad + k_{23}B(v) - k_{13}B(u + v), \\ D_t v &= -(v - \theta) + k_{21}B(u) \\ &\quad - (k_{23} + k_{32})B(v) - k_{31}B(u + v), \end{aligned} \tag{14}$$

where $z_0 = \sum_{i=1}^{3} \theta_i/3$. From the first equation of expression (14), it can be concluded that if each pair has a symmetric coupling relationship ($k_{ij} = k_{ji}$), then the GAO of PO is an invariant of the opinion dynamics, but it is an unsatisfactory result in the context of a project because the performance of the PO will remain unimproved.

4.1. Opinion evolution in a symmetric network

Firstly, we explore the opinion dynamics in a symmetric broker network (k_{13}=k_{31}=0). Since in the symmetric systems $z(\infty) = z_0$, we focus only on the discord dynamics

$$\begin{cases} D_t u = -(u - \theta) - 2kB(u) + kB(v), \\ D_t v = -(v - \theta) + kB(u) - 2kB(v). \end{cases} \tag{15}$$

System (15) has a $\mathbb{Z}_2$-symmetry invariant under the involution $(u, v) \mapsto (v, u)$. In this system (see Fig. 2), there exists a symmetric branch of equilibrium curve in which each point (u, v, k) satisfies $u = v$ and

$$-(u - \theta) - kB(u) = 0. \tag{16}$$

Comparing Eq. (16) with Eq. (10), we find that the coefficient of $B(u)$ in Eq. (16) is only half that in Eq. (10), which suggests that more communication effort in a broker network is required to reach the same equilibrium state as in the dyad case. The Jacobian matrix of equilibrium points in the symmetric branch

$$DF|_{u=v} = \begin{bmatrix} -1 - 2kB'(u) & kB'(u) \\ kB'(u) & -1 - 2kB'(u) \end{bmatrix}$$

has two real eigenvalues λ_1 and λ_2:

$$\lambda_1 = -1 - kB'(u) = -1 - kB(u)(1 - (u/\lambda)^{\beta})/u,$$
$$\lambda_2 = -1 - 3kB'(u) = -1 - 3kB(u)(1 - (u/\lambda)^{\beta})/u.$$

Substituting $B(u)$ in Eq. (16) into the above two equations, we obtain

$$\lambda_1 = -\frac{1}{u\lambda^\beta}G_1(u,\theta) \quad \text{and} \quad \lambda_2 = -\frac{1}{u\lambda^\beta}G_2(u,\theta),$$

here, $G_2(u,\theta) = 3u^{\beta+1} - 3\theta u^\beta - 2\lambda^\beta u + 3\theta\lambda^\beta$.

According to the condition of stationary bifurcation point, the bifurcation point must have one zero eigenvalue. It can be proved that polynomials $G_1(u,\theta)$ and $G_2(u,\theta)$ both have no more than two real roots within range [0, 1]. Furthermore, we can distinguish between an LP and a branch point (BP) by studying whether $D_kF(u_0,k_0) \in \text{Range}(D_uF(u_0,k_0))$.[28] As for system (15), it means that if $\text{Rank}(D_uF(u_0,k_0)|D_kF(u_0,k_0)) = 1/2$, then it is a BP/LP, respectively. Since $D_kF(u_0,k_0) = -B(u)[1,1]^{\text{T}}$, $\text{Rank}(D_uF(u_0,k_0)|D_kF(u_0,k_0)) = 1$ suggests that

$$\text{Det}\left(\begin{bmatrix} -1-2kB'(u) & 1 \\ kB'(u) & 1 \end{bmatrix}\right) = 0.$$

We have

$$-1 - 3kB'(u) = \lambda_2 = -\frac{1}{u\lambda^\beta}G_2(u,\theta) = 0,$$

which means that roots of $G_2(u,\theta)$ within range [0,1] can only be BPs. Similarly, we can also derive that the roots of $G_1(u,\theta)$ within range [0,1] are LPs. From these, we can infer that when the bifurcation conditions are met, bifurcation points will appear on the symmetric equilibrium branch, which makes part of symmetric equilibrium curve unstable and causes so-called "symmetry breaking" phenomenon: one obtains asymmetric equilibrium solutions in a symmetric system. This conclusion is supported by our numerical simulations of system (15) computed using the software package Matcont, a MATLAB numerical continuation package for the bifurcation analysis of dynamical systems.[29,30] Figure 2 presents equilibrium curves of system different values of θ. In both of the cases ($\theta = 0.3$ and 0.4), the equilibrium curve is composed of two types of branches: the symmetric equilibrium branch along which $u = v$ and the asymmetric equilibrium branch along which $u \neq v$. In Fig. 2(a), the symmetric branch starts from (u,v,k)=(0.3,0.3,0), and on which there exist two BPs. Checking the eigenvalues of the equilibrium points on the symmetric branch, we find that the segment between the two BPs is unstable while the other parts of the branch are stable. The BPs correspond to pitchfork bifurcation points. Two asymmetric equilibrium branches bifurcate vertically from the two BPs. However, one branch is mapped onto another by the involution (Fig. 2(c)). From Fig. 2(a), we see that the two BPs correspond to two distinct pitchfork bifurcations. The BP with a smaller value of k ($k^{(b)}_{\text{BP1}}$), BP_1, corresponds to a supercritical pitchfork bifurcation point, while the other BP_2, to a subcritical one. The stable asymmetric branch of the supercritical pitchfork collides with the unstable asymmetric branch of the subcritical pitchfork at two LPs. We denote the LP with a larger/smaller value of u by $\text{LP}_{A1}/\text{LP}_{A2}$, and they correspond to the same value of k ($k^{(b)}_{\text{LP}}$). If k is greater than $k^{(b)}_{\text{LP}}$, consensus among the three groups may occur. Assuming that system (15) is integrated for increasing k, the final opinions for a given k are used as the initial conditions for the next simulation run with the subsequent increment of k. For $k < k^{(b)}_{\text{BP1}}$, discords u and v decrease with the increase of k. When k increases to above $k^{(b)}_{\text{BP1}}$, u and v change in opposite directions, meaning that the whole PO tends to be split into two clusters. If groups 1 and 2 turn closer gradually to form a cluster, then group 3 will diverge from it. However, when initial discord θ is not too large (Fig. 2(a)), the divergence between two clusters can be eliminated by increasing the value of k. When k grows above $k^{(b)}_{\text{LP}A}$, the stable solution jumps abruptly back onto the symmetric branch again, suggesting that with strong enough communication the three groups will converge together eventually. Figure 2(b) shows that if θ is becoming even larger, this kind of jumping back to consensus will never take place due to unacceptable cost of communication even though the segment of the symmetric branch with a greater value of k is composed of stable solutions. Besides, Fig. 2(b) also reveals that when k grows above $k^{(b)}_{\text{BP1}}$, the conversion from converging together to diverging apart into two clusters will take place abruptly at $k = k^{(b)}_{\text{BP1}}$. Figures 2(c) and 2(d) show that the maximum discord δ remains at a high level even though it decreases with k increasing if $k \in (k^{(b)}_{\text{BP1}},\ k^{(b)}_{\text{LP}A})$. This is not surprising because under this condition only two of the groups converge sharply together while the discord between these two groups and the third diverge apart dramatically.

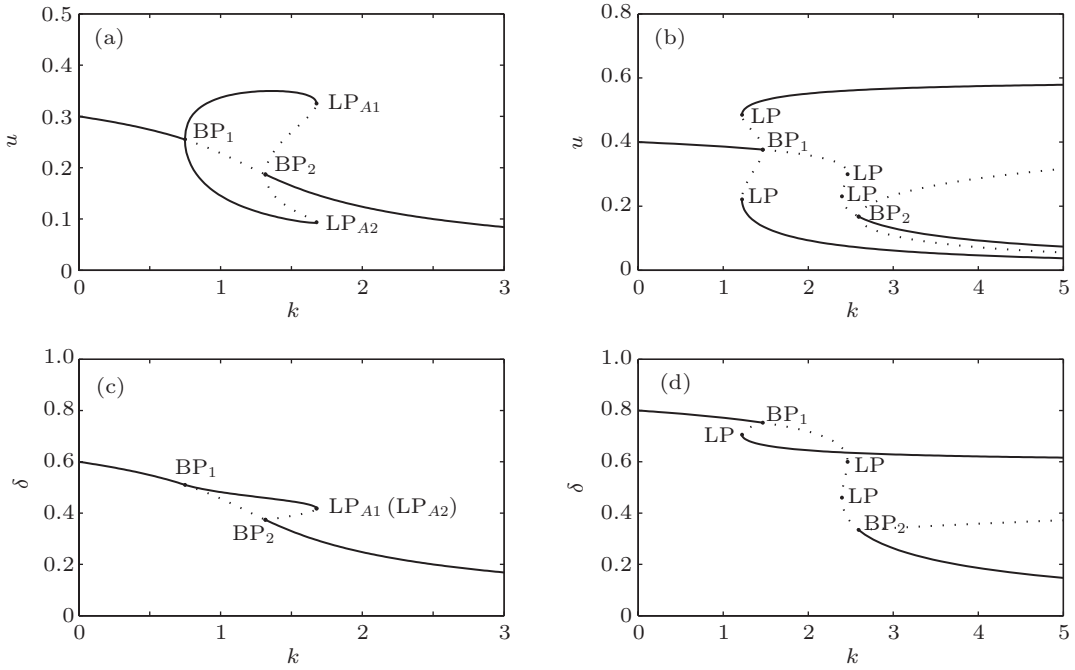


Fig. 2. (Simulation results of opinion dynamics in a broker network (solid/dotted line are for stable/unstable solution). Panels (a) and (b) each show discord u as a function of k; panels (c) and (d) each indicate maximum discord δ as a function of k. Parameters are $\lambda = 0.15$, $\beta = 2$ in panels (a)–(d); $\theta = 0.3$ in panels (a) and (c), and $\theta = 0.4$ in panels (b) and (d).

Figure 3 shows the simulation results of opinion dynamics in a symmetric broker and clique network. To compare their efficacies in reducing discord, we should place the two network topologies on an equal footing according to the equal total communication costs (TCCs). The TCC is $4k^{(b)}$ in the broker network, yet $6k^{(c)}$ in the clique one. We let $k^{(b)} = k$ while $k^{(c)} = 2k/3$ to guarantee the equal TCCs. It can be seen that the opinion dynamics in the symmetric clique network has similar behaviours to in the symmetric broker network. Figure 3(a) shows that each bifurcation curve in the (θ, u) space has a critical value of θ (i.e., $\theta_{c1} = 0.3897, \theta_{c2} = 0.2829/\theta_{c3} = 0.3119, \theta_{c4} = 0.2888$ for the LP and BP curves in the symmetric broker/clique network, respectively) and there exist four different bifurcation portfolios: for $\theta \in (\theta_{c2}, \theta_{c4})$, only in a broker network can BP bifurcation take place; for $\theta \in (\theta_{c4}, \theta_{c3})$, in both networks only BP bifurcation can come into being; for $\theta \in (\theta_{c3}, \theta_{c1})$, BP bifurcation can occur in both networks, while LP bifurcation can appear only in the clique network; and for $\theta \in (\theta_{c1}, 0.5)$, in both networks two types of bifurcation will happen. Figure 3(b) shows an example for $\theta \in (\theta_{c3}, \theta_{c1})$, which corresponds to the simulation condition of Gabby model (*Physica* A **378** (2007) p. 125 Fig. 5). His simulation results only included the symmetric branch of the equilibrium curve, and suggested that as if the consensus among three groups can be achieved via increasing k above $k^{(c)}_{\mathrm{LP1}}$, the behaviour of the system is just like that in dyad case. However, our results in Fig. 3(b) argue that the behaviours of system S2 in triads interaction are fundamentally different from those in dyad case due to symmetry breaking. It is true that if one chooses a relatively short duration of interaction (t_{f}), for instance, $t_{\mathrm{f}} = 50$ and $k \in (k^{(b)}_{\mathrm{BP1}},\ k^{(b)}_{\mathrm{LP}A})$, the simulation results will be on the symmetric branch of the equilibrium curve just like the results of Gabbay's. However, when checking the eigenvalues of these points, one will find that they are always unstable for some of them are saddle points, and the others are unstable nodes. In fact, the two BPs on the symmetric branch make the curve segment between the two points unstable, thereby suggesting that under this condition the convergence of all three groups seldom takes place.

In an actual project context, there always exist some kinds of disturbances. Figure 4 shows the simulation results of the stochastic system constructed by adding independent white noise terms into different equations corresponding to the broker network.

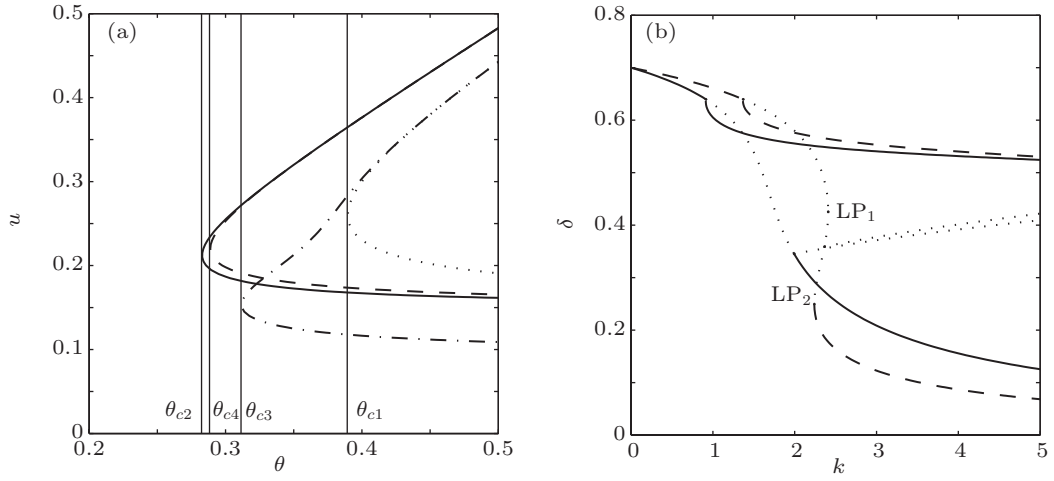


Fig. 3. (a) Curves of simulated BP (solid/dashed) and LP (dotted/dash–dot) versus θ in a broker/clique network, and (b) curves of simulated δ (solid/dashed) versus k in a broker/clique network with $\theta = 0.35$. Dotted line in panel (b) is for unstable solution. Parameters are $\lambda = 0.15$ and $\beta = 2$.

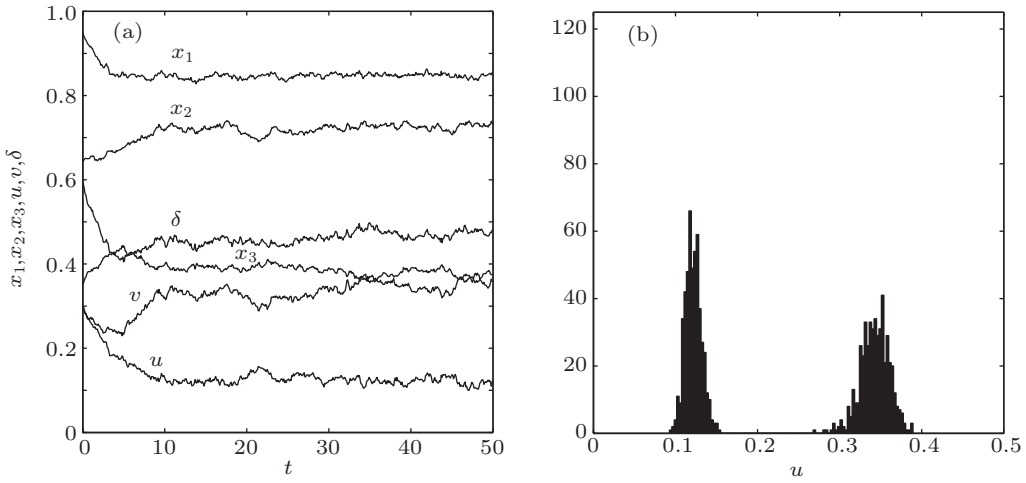


Fig. 4. (a) Temporal evolutions of the opinion value (x_1,x_2,x_3) of each group, discords between groups 1, 2(u) and 2, 3(v), and maximum discord (δ) in the disturbed simulation of dyad interactions in a broker network, and (b) histogram of the trajectories of discord u at the end-time ($t_{\rm f} = 50$) in 1000 simulation runs. Parameter are $[\theta_1, \theta_2, \theta_3] = [0.95, 0.65, 0.35]$, $\lambda = 0.15$ and $k = 1.2$.

Figure 4(a) shows one of sample paths of the stochastic system. Clearly at the beginning of the coupling evolution, all the discrepancies of the three groups decline, however, the opposite trends of opinion evolution appear after a short period of time, during which u gradually becomes smaller while v becomes larger due to the noises. After a not too long period of time, u approaches to 0.1, a discrepancy that is much smaller than the initial one, while v diverges to about 0.35. Figure 4(b) indicates that groups 1, 2 and groups 2, 3 have equal chances to form a cluster. So it can be concluded that in an actual project context, if the initial discords are high, even in a short period of time organization polarization may happen rather than convergence of all three groups as Gabbay expected.

4.2. Leader group-induced opinion evolution

Although triad interaction in a symmetric network can shed light on how the discords among three groups evolve with time, these symmetric relationships can be considered only as the case of self-organized cooperation among three groups with an equal coupling strength. As described above, the GAO of a PO, z, keeps unchanged, which does not help improve the performance of the PO. In most cases, a

leader group is needed to improve the situation. A leader group is a group that can impose more influence on other groups than *vice versa*. In the context of a PO, the client or a major contractor who takes charge of the whole PO may act as a leader group. We assume that group 1 acts as the leader group, the other two groups as the follower groups. One example is that the client (group 1) who demands high quality of project deliverables gives impetus to continual improvement of quality in the product and service of the contractor (group 2) and the subcontractor (group 3). We explore two scenarios: 1) in a broker network, group 1 imposes strong influence only on group 2 and the relationship between groups 2 and 3 keeps two sides equal; 2) in a clique network, group 1 imposes strong influence on both group 2 and group 3 simultaneously, while the relationship between groups 2 and 3 also keeps two sides equal. Although there are many other scenarios in triad interactions with a leader group, we discuss and compare only the two scenarios. In both scenarios, ρ and ρ_λ represent the asymmetric level of coupling strength and the latitude of acceptance between the leader group and the follower groups, respectively.

4.2.1. Asymmetric level of coupling strength ρ

To reveal the influence of ρ on opinion evolution, two simulations for scenario 1, ($k_{12} = (1+\rho)k$, $k_{21} = (1-\rho)k$, $k_{13} = k_{31} = 0$, $k_{23} = k_{32} = k$), and scenario 2 ($k_{12} = k_{13} = 2(1+\rho)k/3$, $k_{21} = k_{31} = 2(1-\rho)k/3$, $k_{23} = k_{32} = 2k/3$) are performed. Note that all the coupling strengths in scenario 2 have been processed to ensure the equal TCCs in scenarios 1 and 2. Figures 5(a) and 5(b) show the numerical results of scenarios 1 and 2, respectively. It can be observed from Fig. 5 that in both scenarios, the z and δ of three-group PO with large initial discord increase along with $-\rho$, i.e., the more the influence of leader group (group 1) on the follower group/groups (group 2/groups 2 and 3), the higher the GAO of PO and the larger the maximum discord δ is. This can be interpreted as the fact that since group 1 draws the opinion of group 2 closer towards his while his opinion changes little, which in turn weakens the influences of groups 1 and 2 on group 3, so δ (i.e., the discord between groups 1 and 3) becomes larger. In scenario 2, although group 1 interacts with group 3 directly, the influence of group 1 on group 3 is still limited by the deep gap between them. Therefore, in both scenarios, if the initial discord is high, the increase of $-\rho$ may cause the PO to be split into two distinct camps, which is not a good news for the leader group because each participant group is responsible for the quality of project deliverables.

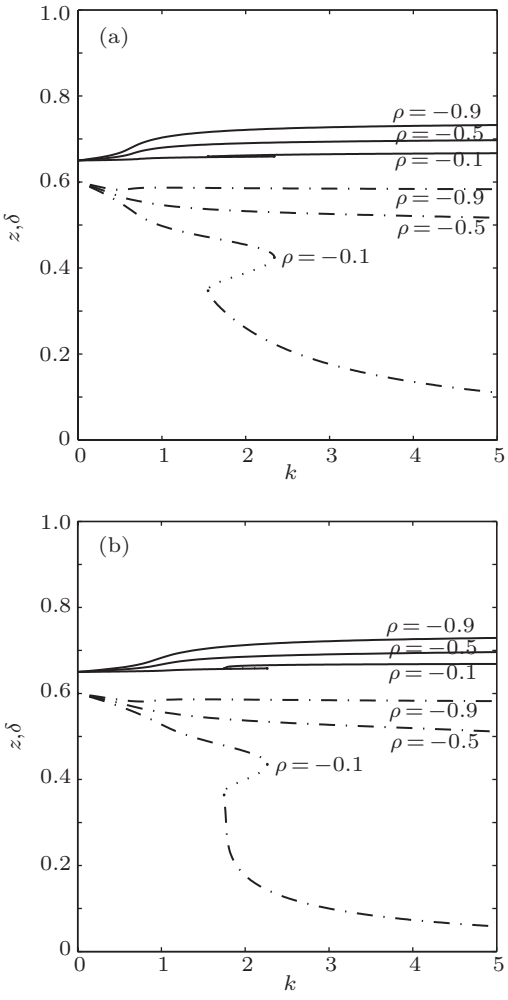


Fig. 5. Simulation results of opinion dynamics in (a) a broker and (b) clique network (solid line is for z; dash–dot line is for δ; dotted line is for unstable solution). Parameters are $\lambda = 0.15$, $\beta = 2$, $\theta = 0.3$, $z_0 = 0.65$, and $\delta(0) = 0.6$.

4.2.2. Asymmetric level of latitude of acceptance ρ_λ

In the above discussion, we assumed that all the groups have equal latitudes of acceptance, i.e., $\lambda_j = \lambda$. Under this assumption, increasing $-\rho$ between the leader group and the follower groups may cause organization polarization. Now we consider the case that the leader group and the follower groups have different latitudes of acceptance. It is reasonable that the leader group more often will not change his opinion while the follower groups tend to adjust their opinions to some extent in order to meet the needs of the client and the leader group. Figure 6 displays the simulation result of opinion dynamics among the

heterogeneous groups. It is assumed that the follower groups have the equal latitude of acceptance, λ_b, while the leader group has a latitude of acceptance, λ_a. So $\rho_\lambda = (\lambda_a - \lambda_b)/(\lambda_a + \lambda_b)$. From Fig. 6, we can see that in both networks with other conditions remaining unchanged, the change of $-\rho_\lambda$ will have a significant influence on the opinion dynamics. The increase of $-\rho_\lambda$ will improve the steady GAO, $z(\infty)$, of the PO. Besides, unlike the case of equal latitude of acceptance, $z(\infty)$ does not always monotonically increase with k increasing. For example, if $\rho_\lambda = -1/3$ in a clique network (Fig. 6(b)), there is a maximum GAO when k approaches to 2. This may be due to the greater compromise the leader group makes if he is subjected to strong enough coupling force from the follower groups. On the other hand, the effect of ρ_λ on steady maximum discord $\delta(\infty)$ is subtle. When $-\rho_\lambda$ increases from 0 to 1/3, $\delta(\infty)$ reduces dramatically, indicating that with the increase of k, consensus can be achieved. When $-\rho_\lambda$ grows even larger, if k is below a certain threshold, $\delta(\infty)$ can be reduced further. Meanwhile, if k is above the threshold, $\delta(\infty)$ may become larger.

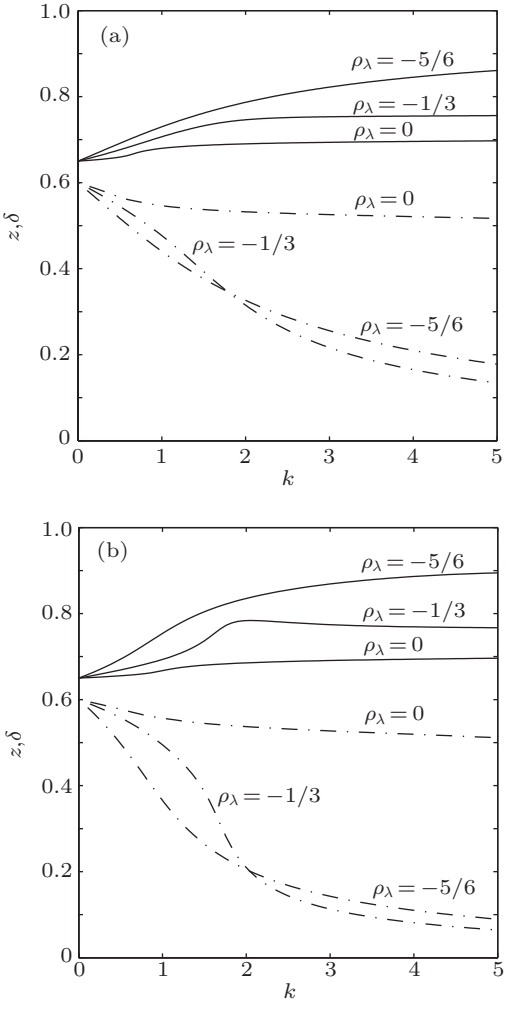


Fig. 6. Simulation results of opinion dynamics among three groups with different latitudes of acceptance (solid line is for z, and dash–dot line is for δ) in (a) a broker and (b) clique network. Parameters are $\rho = -0.5$; $\rho_\lambda = 0, -1/3, -5/6$; $\lambda = 0.15$; $\theta = 0.3$; $z_0 = 0.65$; $\delta(0) = 0.6$.

5. Conclusions

In a cooperative project with the involvement of multiple companies, each of the participant groups is responsible for the quality of the project deliverables. So both consensus and a high adoption rate of positive work attitude (measured as the GAO of the PO) are essential for the improvement of project performance. Since the classic BC models based on a pure compromise strategy neglect the influence of participant's company culture on opinion evolution, they fail to explain the frequent occurrence of conflicts among different groups in a PO. A generalized BC with self-bias model is proposed to explore the opinion dynamics in a multi-group project in which each group's opinion is affected by both a cohesive force tending to restore the opinion back towards its initial status because of its company culture and coupling forces with other groups attempting to bring opinions closer due to collaboration willingness. In the case of a two-group project, the model behaves like a cusp catastrophe model, as the initial discord θ is the key variable that specifies the conditions under which coupling scale k will affect the opinion change in a continuous fashion, or in an abrupt way. The critical value of θ and the bifurcation condition are derived, which identify three distinctive evolutionary regimes, i.e., a convergence regime when θ is below a critical value, a bifurcation regime when the initial opinion difference is above this critical value and the cost of communication is acceptable, and a deadlock regime when the cost of communication to reduce the large discord becomes unacceptable. In the dyad case, the GAO of PO can be raised simply by enhancing the leader group's influence on the follower and its resistance to the influence of the followers. In the case of three-group project, the GAO of a PO remains unchanged as long as each pair has a symmetry coupling relationship. As for the discord dynamics, there still exist three distinctive regimes as in the case of dyad interaction. However, there are richer and more diverse forms of bifurcation in this case than

in the case of dyad interaction. If the system has a symmetric coupling relationship among three groups and the initial discord among them is high, two BPs may appear on the symmetric branch of equilibrium curve which may cause symmetry to break and split the whole PO into two clusters. This "organization polarization" phenomenon ignored by Gabbay has a significant influence on the characteristic of opinion dynamics. It brings about more difficulty in reaching consensus among three groups than between two groups. We can conclude that apart from the inherent divergence in the interests, the serious initial discords and nonlinear interaction among multi-groups can also make conflicts and disputes so common and even inevitable in cooperation projects. The GAO is also hard to improve if there are two distinctive clusters in the PO. In this particular circumstance, strong influence of the leader group on the follower groups can only improve the GAO at the expense of enlarging the maximum discord. One way to raise the GAO consensually is to choose the right candidate groups having large latitude of acceptance as the follower groups and the group having strong faith in advanced management philosophy as the leader group.

Although our study goes a step further in the analysis of dynamic behaviours of a multiple-group PO, our model has its limitations. In the model, the influence of external environment is not taken into account. Further study should explore the opinion dynamics with the external disturbance using the stochastic differential equation. Besides, although only nonlinear interactions on a small social network are considered here, our model can be extended to the investigation of the opinion dynamics on a large complex network.

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