Control of purity and entanglement for two spatially two-separated qubits via phase damping

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(Received 11 December 2011; revised manuscript received 18 May 2012)

Control of purity and entanglement of two two-qubits dispersively coupled to a field with a reservoir are investigated. Initially the qubits are entangled, while the field is either in a coherent state or a statistical mixture of two coherent states. For an alternative entanglement measure we calculate the negativity of the eigenvalues of the partially transposed density matrix. A measure related to the mutual entropy, namely the index of entropy, is employed to measure the entanglement. Its results agree well with the negativity. It is found that the entanglement and purity have strong sensitivity to phase damping. The asymptotic behaviour of the states of the field, the two two-qubits, and the total system fall into mixed states.

Keywords: entanglement, phase damping, purity, separated qubits

PACS: 03.67.Mn, 03.65.Vf, 03.65.Yz **DOI:** 10.1088/1674-1056/21/10/100310

1. Introduction

Entanglement is a remarkable feature of quantum mechanics and is the key resource of quantum computation and quantum information processing.[1] There are numerous applications of spatially[2,3] separated entangled pairs of particles, such as quantum cryptography teleportation,[4,5] superdense coding,[6,7] and so on. Several schemes have been proposed to generate the entanglement between two particles, such as entanglement swapping,[8,9] quantum repeating,[10] and entanglement transfer.[11,12] Various schemes have been proposed to generate the atomic singlet state,[13−15] and it has also been successfully prepared in some experiments.[16,17] One of the schemes is based on the exchange of a single photon between the atoms in a resonant cavity[13] and has been experimentally realized.[16] In a recent experiment, the prepared atomic singlet state has been successfully used to teleport the state of atomic qubits.[17] However, it is one of the most challenging tasks in quantum information processing to maintain the entanglement stored in two spatially separated atoms for a sufficiently long time in order to be used in the later steps of the protocols, because the coupling of the quantum system to the environment usually leads to decoherence. It is therefore of great importance to prevent the influence of environmental noise in quantum information processing. The problem of controlling the evolution of the entanglement and coherence between qubits, which interact with the same environment, has been widely studied in recent years.[18−21]

On the other hand, the connection of the entangle to the geometry of a fractal and random Hilbert space and El Naschie's E-infinity theory are discussed in Ref. [22]. Building upon the pioneering work of Bell[23] and Hardy's incredible result,[24] it was shown that the probability of quantum entanglement of two particles is a maximum of 9.0169945 percent.[24] The numerical result of Hardy's Gedanken experiment, namely 9.017 percent entanglement,[24] may also be viewed as relatively direct confirmation of the fractal nature of micro spacetime which was formally proposed in 1990 by El Naschie to be basically an infinite but hierarchal collection of elementary triadic random Cantor sets.[25,26] Although it has gone largely un-noticed for a long time, this result was essentially established independently in a much wider context by the present author almost two decades ago.[25−27] The author in Ref. [22] gives two fundamentally different derivations of Hardy's beautiful result leading to precisely the same general conclusion, namely that by virtue of

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the zero measure of the underlying Cantorian-fractal spacetime geometry, the notion of spatial separability in quantum physics is devoid of any meaning.[27] The first derivation is purely logical and uses a probability theory which combines the discrete with the continuum. The second derivation is purely geometrical and topological using the fundamental equations of a theory developed by the author and his collaborators frequently referred to as the E-infinity or Cantorian spacetime theory.[25−27]

So, in the present paper, we investigate the entanglement and coherence of a quantum system formed by two spatially two-separated qubits within two spatially separated and dissipative cavities in the dispersive limit by the negativity and entropies, respectively. The two spatially two-separated qubits are initially entangled and the cavities are initially prepared in two cases either by the statistical mixture of two coherent states or coherent states.

We show that the entanglement controlling phenomenon appears in the present system and the entanglement of two spatially two-separated qubits decreases with the time in a short range. We find that the control on the long-lived disentanglement and survival of the entanglement depends on the parameters of the phase damping. In other words, there is long-lived disentanglement in the presence of the dissipation of cavities, implying that the initial entanglement of two spatially two-separated qubits could be partially preserved even when they are put into dissipative cavities.

The present paper is organized as follows. In Section 2, we obtain an explicit analytical solution of the two spatially two-separated qubits interacting with a dissipative cavity in the dispersive limit. In Sections 3 and 4 the entanglement dynamics and coherence of the two spatially two-separated qubits are investigated by employing the negativity and entropy, respectively. Finally, we summarize our results in Section 5.

2. The model and source of decoherence in dispersive regime

We consider two spatially separated two-qubits and two dissipative cavities. We denote the two cavities, A and B, when the context requires us to differentiate between them, but otherwise they are supposed to be identical. Qubits 1 and 2 are trapped in cavity A, qubits 3 and 4 are trapped in cavity B. The ground and excited states for each qubit in cavity A or B are respectively denoted by $|0\rangle_i$ and $|1\rangle_i$ $(i = 1, 2)$. For the sake of simplicity, we suppose each of two qubits to be dispersively coupled to their cavity fields with the same coupling constant λ. Then, the Hamiltonian of the system in the rotating wave approximation is given by[28]

$$\hat{H}_{\rm eff} = \hat{H}_A + \hat{H}_B, \tag{1}$$

where

$$\begin{aligned}\hat{H}_A =\ & \lambda\Bigg[\sum_{i=1,2}\Big\{|1\rangle_{ii}\langle 1|\hat{a}\hat{a}^\dagger - |0\rangle_{ii}\langle 0|\hat{a}^\dagger\hat{a}\Big\} \\ & + g\Big(|1\rangle_{11}\langle 0|\otimes|0\rangle_{22}\langle 1| \\ & + |0\rangle_{11}\langle 1|\otimes|1\rangle_{22}\langle 0|\Big)\Bigg],\end{aligned} \tag{2}$$

$$\begin{aligned}\hat{H}_B =\ & \lambda\Bigg[\sum_{j=1,2}\Big\{|1\rangle_{jj}\langle 1|\hat{b}\hat{b}^\dagger - |0\rangle_{jj}\langle 0|\hat{b}^\dagger\hat{b}\Big\} \\ & + g\Big(|1\rangle_{11}\langle 0|\otimes|0\rangle_{22}\langle 1| \\ & + |0\rangle_{11}\langle 1|\otimes|1\rangle_{22}\langle 0|\Big)\Bigg],\end{aligned} \tag{3}$$

where $\hat{a}^\dagger$ (or $\hat{b}^\dagger$) and $\hat{a}$ (or $\hat{b}$) are respectively the creation and annihilation operators for the cavity field A (or B).

We suppose the damping constants in the two cavities to be denoted by γ_A and γ_B, respectively; then at zero temperature the density operator of the total system $\rho(t)$ satisfies the following master equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = (L_A + L_B)\rho(t), \tag{4}$$

where the two auxiliary superoperators L_A and L_B read

$$\begin{aligned}L_A^{(\cdot)} =\ & -\mathrm{i}[H_A, \cdot] \\ & + \gamma_A[2\hat{a}^\dagger\hat{a}\cdot\hat{a}^\dagger\hat{a} - (\hat{a}^\dagger\hat{a})^2\cdot - \cdot(\hat{a}^\dagger\hat{a})^2], \\ L_B^{(\cdot)} =\ & -\mathrm{i}[H_B, \cdot] \\ & + \gamma_B[2\hat{b}^\dagger\hat{b}\cdot\hat{b}^\dagger\hat{b} - (\hat{b}^\dagger\hat{b})^2\cdot - \cdot(\hat{b}^\dagger\hat{b})^2].\end{aligned} \tag{5}$$

Then, the density operator of the total system can be expressed as

$$\rho(t) = \exp(L_A + L_B)\rho(0). \tag{6}$$

To do this, we suppose the qubits and field are initially in the form

$$\rho(0) = \sum_{m,n=0}\chi_n\chi_m\sigma_{m,n}|m\rangle\langle n|\otimes|\nu\rangle\langle\nu|, \tag{7}$$

where

$$\chi_n = \exp\left(\frac{-|\alpha|^2}{2}\right)\frac{\alpha^n}{\sqrt{n!}},$$

and α is a complex number. And the field is initially in one of two cases: either a pure coherent state $|\alpha\rangle\langle\alpha|$, i.e., $\sigma_{m,n} = 1$ or a statistical mixture of coherent mixed state $(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)/2$, i.e., $\sigma_{m,n} = [1 + (-1)^{m+n}]/2$ with $g = 1$. Throughout this paper we suppose the two 2-qubits to be initially prepared in the Bell state

$$|\nu\rangle = \frac{1}{\sqrt{2}}(|1,1\rangle_A|1,1\rangle_B + |0,0\rangle_A|0,0\rangle_B). \quad (8)$$

Then we can write $\rho(0)$ in the form,

$$\begin{aligned}\rho(0) = \frac{1}{2}(&\rho_{11}^A(0)\otimes\rho_{11}^B(0) + \rho_{10}^A(0)\otimes\rho_{10}^B(0)\\ &+\rho_{01}^A(0)\rho_{01}^B(0) + \rho_{00}^A(0)\otimes\rho_{00}^B(0)),\end{aligned} \quad (9)$$

where

$$\begin{aligned}\rho_{ij}^r(0) = &\sum_{m,n=0}\chi_n\chi_m\sigma_{m,n}|m\rangle\langle n|\otimes|i,i\rangle_r\langle j,j|_r,\\ &(i,j=0,1;r=A,B).\end{aligned}$$

From Eqs. (6) and (7), we obtain the density operator of the total system at time t

$$\begin{aligned}\rho(t) = \frac{1}{2}(&\rho_{11}^A(t)\otimes\rho_{11}^B(t) + \rho_{10}^A(t)\otimes\rho_{10}^B(t)\\ &+\rho_{01}^A(t)\rho_{01}^B(t) + \rho_{00}^A(t)\otimes\rho_{00}^B(t)),\end{aligned} \quad (10)$$

where

$$\rho_{ij}^r(t) = \exp(L_r t)\rho_{ij}^r(0), \quad (i,j=0,1;r=A,B). \quad (11)$$

First we consider the interaction between qubits 1, 2 and cavity A, from Eq. (11) after some calculations we can obtain

$$\begin{aligned}\rho_{11}^A(t) &= \frac{1}{\sqrt{2}}\sum_{m,n=0}^{\infty}\sigma_{m,n}\chi_m\chi_n^*\beta_{m,n}\\ &\quad\times \mathrm{e}^{-2\mathrm{i}\lambda t(m-n)}|1,1,m\rangle\langle 1,1,n|,\\ \rho_{10}^A(t) &= (\rho_{01}^A(t))^\dagger = \frac{1}{\sqrt{2}}\sum_{m,n=0}^{\infty}\sigma_{m,n}\chi_m\chi_n^*\beta_{m,n}\\ &\quad\times \mathrm{e}^{-2\mathrm{i}\lambda t(1+m+n)}|1,1,m\rangle\langle 0,0,n|,\\ \rho_{00}^A(t) &= \frac{1}{\sqrt{2}}\sum_{m,n=0}^{\infty}\sigma_{m,n}\chi_m\chi_n^*\beta_{m,n}\\ &\quad\times \mathrm{e}^{-2\mathrm{i}\lambda t(n-m)}|0,0,m\rangle\langle 0,0,n|,\end{aligned} \quad (12)$$

where the term $\beta_{m,n} = \exp(-\gamma_A t(m-n)^2)$ depending on the phase damping parameter controls the decay of elements of the matrix. Similar results can be obtained for qubits 3, 4 which are trapped in cavity B.

Then, from Eq. (10), by tracing over the field variables we obtain the reduced density operator of the qubits as follows:

$$\begin{aligned}\rho_q(t) = \frac{1}{2}\sum_{m,n=0}^{\infty}&[\langle m|\rho_{11}^A(t)|n\rangle\otimes\langle m|\rho_{11}^B(t)|n\rangle\\ &+\langle m|\rho_{10}^A(t)|n\rangle\otimes\langle m|\rho_{10}^B(t)|n\rangle\\ &+\langle m|\rho_{01}^A(t)|n\rangle\otimes\langle m|\rho_{01}^B(t)|n\rangle\\ &+\langle m|\rho_{00}^A(t)|n\rangle\otimes\langle m|\rho_{00}^B(t)|n\rangle].\end{aligned} \quad (13)$$

By tracing over the qubit variables, the reduced density operator of the field is given by

$$\begin{aligned}&\rho_{\mathrm{f}}(t)\\ &= \frac{1}{2}\sum_{m,n=0}^{\infty}[{}_A\langle 1,1|\rho_{11}^A(t)|1,1\rangle_A\otimes_B\langle 1,1|\rho_{11}^B(t)|1,1\rangle_B\\ &\quad+{}_A\langle 0,0|\rho_{00}^A(t)|0,0\rangle_A\otimes_B\langle 0,0|\rho_{00}^B(t)|0,0\rangle_B].\end{aligned} \quad (14)$$

It is noted that the two cavity fields are initially in a product state and not correlated. As time evolves the state of the fields becomes a statistical mixture of two coherent states, which is separable, and the two modes are classically correlated. As expected, due to the dissipation the intensity of the fields will decay exponentially.

3. The control on entanglement

Entanglement is at the heart of quantum information theory. Various attempts have been made to characterize the entanglement properties of quantum systems qualitatively and quantitatively. This has been motivated by the progress in the experimental techniques aiming to create entangled states, which provide phenomena that are very different from classical physics. Here we quantify the control on entanglement of the final state (Eq. (10)). So, for our case in which $\rho(t)$ describes mixed states, we use the negativity to measure the amount of entanglement which is based on the Peres–Horodecki[29,30] criterion for entanglement and is defined by the following formula

$$E = \max\left(0, -2\sum_i \mu_i\right). \quad (15)$$

The value $E = 1$ corresponds to maximum entanglement between the qubits while $E = 0$ describes completely separated qubits. To get the negativity for qubits N_q, the sum is taken over the negative eigenvalues μ_i of the partial transposition of the reduced density matrix $\rho_q(t)$ of the qubits, and to get the total negativity N_{t} the sum is taken over the negative eigenvalues μ_i of the partial transposition of the density matrix $\rho_(\mathrm{t})$ of the total system.

Firstly, we study the effect of the phase damping on the entanglement, where the field is initially in coherent state. In Figs. 1(a) and 1(b), we plot the negativity for qubits N_q and the total negativity $N_{\rm t}$ as a function of the scaled time λt with different values for the phase damping parameters γ_A and γ_B. It is observed that the phase damping has practically no influence on the time evolution of the negativity of the qubits. This accords to Eq. (12), only diagonal matrix elements (i.e., $m = n$) in the photon number representation contribute to the qubit reduced density operator $\rho_q(t)$ hence phase damping has no effect on these diagonal matrix elements. This explains why the qubit is entangled with and disentangled from the field as if phase dissipation of the cavity did not exist. If we set $\gamma_A = \gamma_B = 0.1$ and $\gamma_A = \gamma_B = 0.5$ we find that $N_{\rm t}$ decreases with the increase of damping parameter. This means the total negativity quite vanishes and the states finally go in to a pure state and its coherence is lost completely, i.e., the phase damping destroys the entanglement of the states in this case. So, one can determine a particular region in which there is no entanglement between the qubit and the fields due to the complete destruction of coherence in the presence of the phase damping. If we put $\gamma_A = 0.1$, $\gamma_B = 0.0$, the phase damping has no influence on the total negativity (see Fig. 2(a)) but if γ_B increasing, i.e., ($\gamma_B = 0.5$ see Fig. 2(b)) the entanglement returns to decreasing. From the above results we can control entanglement by using different input values for the phase damping parameters γ_A, γ_B.

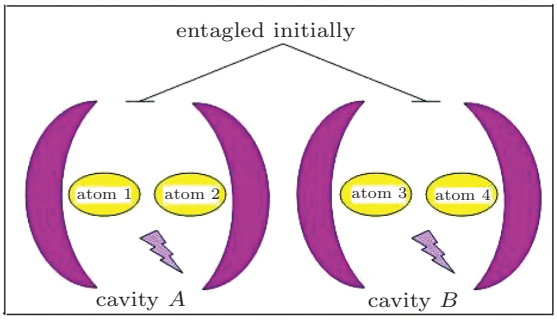


Fig. 1. (colour online) The schematic picture of two spatially two-separated qubits within two spatially separated and dissipative cavities. Each two qubits are put into a single-mode cavity and interacts with its own cavity field locally. Note that there is no direct interaction between two spatially two-separated qubits once they have been put into cavities.

To see the effect of the initial state of the field, we plot in Figs. 3–5 the negativity when the field is an initially statistical mixture of coherent state $\rho(t)$. These figures clearly show that the death of entanglement depends on a proper manipulation of the damping parameter. A remarkable property of such an initial state setting is that entanglement can fall abruptly to zero for a very long time and the entanglement will not be recovered, i.e., the state will stay separable. From the above result we can conclude that when the field is in a statistical mixture of coherent states the disentanglement occurs faster than the field in coherent states. So we can control entanglement by the initial state of the field.

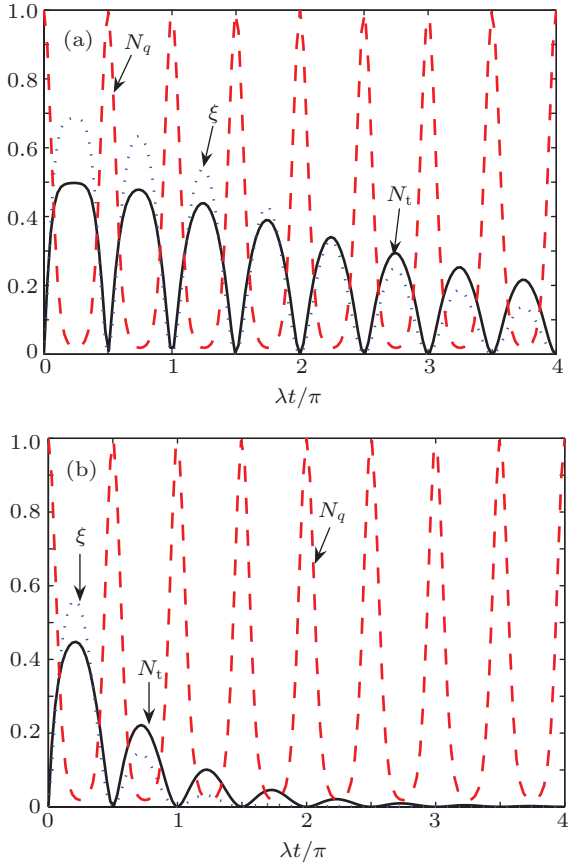


Fig. 2. (colour online) The time evolution of the entanglement when the field in a coherent state with $\bar{n} = 1$ for various values of the damping parameter: (a) $\gamma_A = \gamma_B = 0.1$ and (b) $\gamma_A = \gamma_B = 0.5$.

In most of the previous studies on the entanglement, quantum mutual entropy is adapted to measure the degree of entanglement of the qubit and the field. The quantum mutual entropy (also called the index of correlation[31,32] for a given density matrix is defined by

$$I(\rho) = {\rm Tr}[\rho(\ln\rho - \ln(\rho_q \otimes \rho_f))]. \qquad (16)$$

It follows that $I(\rho)$ represents a distance between ρ and the product of marginals $\rho_q \otimes \rho_f$, i.e., it measures all correlations including classical as well as quantum

correlation and does not discriminate the purely quantum entanglement from the classical correlation.[32,33] It is evident that the mutual entropy is bounded $0 \leq I(\rho) \leq 2\min[S_q, S_f]$, and a bipartite density matrix with excessive mutual entropy, i.e., $\min[S_q, S_f] \leq I(\rho) \leq 2\min[S_q, S_f]$, is entangled. This occurs precisely for quantum entangled systems and is forbidden for classical systems, therefore, a necessary condition for separability is $0 \leq I(\rho) \leq \min[S_q, S_f]$. This means that $I(\rho)$ does not fulfill the first property that we need from an entanglement measure; that is, for any separable state the measure of entanglement should be zero.

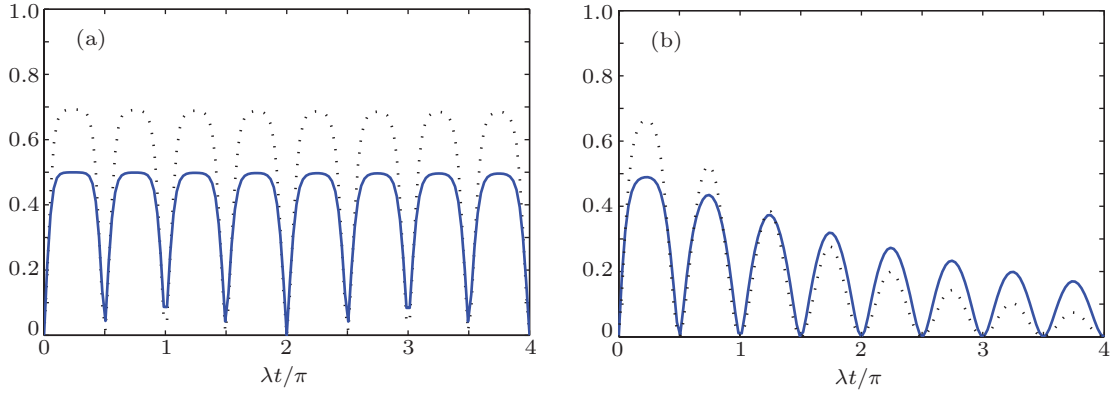


Fig. 3. (colour online) The time evolution of the entanglement $\xi(\rho)$ (dotted curve), $N_{\rm t}$ (solid curve) when the field is in a coherent state with $\bar{n} = 1$ for various values of the damping parameter: (a) $\gamma_A = 0.1$, $\gamma_B = 0.0$, and (b) $\gamma_A = 0.1$, $\gamma_B = 0.5$.

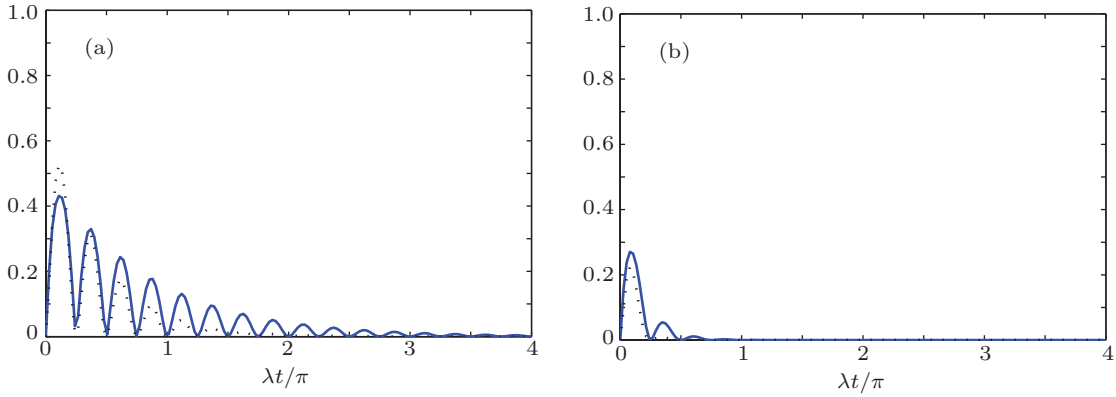


Fig. 4. (colour online) The time evolution of the entanglement $\xi(\rho)$ (dotted curve), $N_{\rm t}$ (solid curve) when the field in statistical mixture of coherent state with $\bar{n} = 1$ for various values of the damping parameter: (a) $\gamma_A = \gamma_B = 0.1$ and (b) $\gamma_A = \gamma_B = 0.5$.

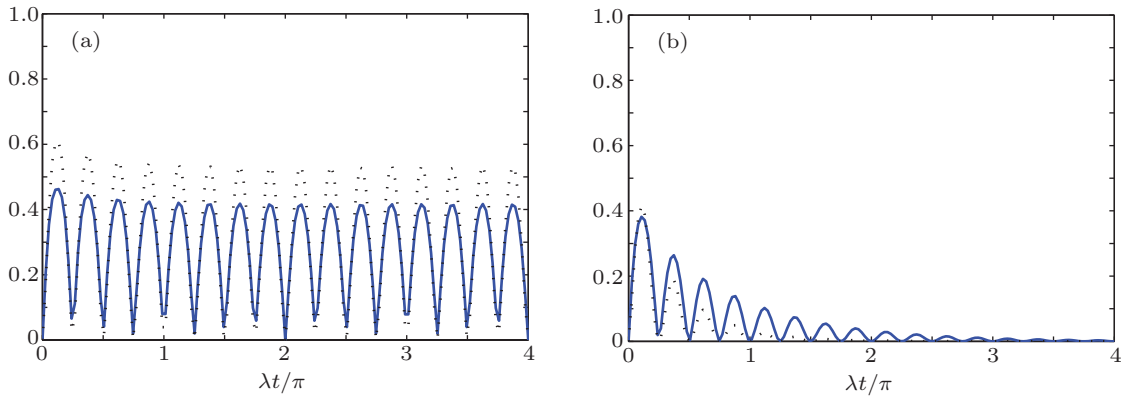


Fig. 5. (colour online) The time evolution of the entanglement $\xi(\rho)$ (dotted curve), $N_{\rm t}$ (solid curve) when the field in statistical mixture of coherent state with $\bar{n} = 1$ for various values of the damping parameter: (a) $\gamma_A = 0.1$, $\gamma_B = 0.0$, and (b) $\gamma_A = 0.1$, $\gamma_B = 0.5$.

When two subsystems q and f are classically maximally correlated, the classical upper bound $\tilde{I}(\rho) = \min[S_q, S_f]$ is saturated, although the system is quantum mechanically disentangled. This implies that the negativity gives a more correct estimation of the quantum entanglement under the effect of the phase damping. Now, we suggest a new measure from the mutual entropy, which we call index of entropy, to measure the entanglement. It is defined by

$$\xi(\rho) = I(\rho) - \tilde{I}(\rho). \tag{17}$$

This measurement, plotted in Figs. 1–5, shows almost complete agreement between E and this measurement $\xi(\rho)$ for this case. So, the index of entropy may be taken as indicator to measure the entanglement and its results for the present case agrees well with the negativity.

4. The coherence properties

Here we use the von Neumann entropy to investigate the purity of the state of the total system caused by the phase damping. It is given by

$$S = -\sum_{i=1}^{\infty} \pi_i \ln(\pi_i),$$

and may be used as a measure of the entanglement of the states of the whole system, where π_i are the eigenvalues of the density matrix of the system $\rho(t)$.

Also we use the entropies of the sub-systems to investigate their purity, which can be defined through their respective reduced density matrix as[35]

$$S_i(t) = -\operatorname{Tr}_i(\hat{\rho}_i(t)\ln\hat{\rho}_i(t)), \quad (i = q \ \text{ or } \ f),$$

where $\hat{\rho}_{q(f)}(t) = \operatorname{Tr}_{f(q)}[\hat{\rho}(t)]$. The entropies of a general two-component quantum system are linked by a remarkable theorem presented by Araki and Lieb,[36] which states $|S_q - S_f| \leq S \leq S_q + S_f$.

It is worth while noting that if the phase-damping parameter $\gamma_A = \gamma_B = 0$, then $\rho(t)$ given by Eq. (6) is governed by a unitary time evolution and consequently the total entropy S is time-independent. Since we have assumed that the qubits and the field are initially in a disentangled pure state, the total entropy S of the qubit-field system is zero. We can see that the reduced entropies of the two subsystems are identical, namely $S_q(t) = S_f(t)$.

But with the phase damping parameter γ_A, $\gamma_B \neq 0$, the total entropy S is time-dependent and the field entropy splits up from the qubits entropy and hence cannot be used as a measure for entanglement. In this case we use $S_i(t)$ to quantify the purity loss of its states. So, the temporal evolution of the purity loss of the marginal density matrix of the field is investigated by $S_f(t)$. But $S_q(t)$ reflects the purity loss of the state of the four qubits. On the other hand, the interaction of the field subsystem with the environment also induces purity loss, and this process is usually called decoherence. In what follows, we shall reserve the word decoherence to mean coherence loss induced by the environment.

We plotted the previous functionals versus the scaled time λt with $\bar{n} = 1$, and the field in coherent states. To see what happens to the temporal evolution of the measures S_q, S_f, S due to of phase damping against the scaled time λt we display in Fig. 6(a). Since the entropies depend on the off-diagonal terms of the densities matrices, we expect that these quantities will be affected by the phase damping, the states are changed from a pure state to a statistical mixture as time develops. We note that the total entropy increases monotonically and no longer equals zero, we observe that the field entropy splits from the atomic entropy, and the amplitudes of the fast oscillations diminish. It is remarked that the field attains higher entropy than the atom. But with increasing the damping parameters (see Figs. 6(a) and 6(b), it is to be noted that the previous measures of degree of entanglement depend on the atomic damping and are dephased. Also in these curves, the total entropy S increases as time develops when the parameters are increased, and hence decoherence is more effective. With the increasing of the parameters, we observe rapid deterioration of the revivals of the field and atom sub-entropies. Also in these curves, the total entropy S increases as time develops when the parameters of the phase damping increase, and hence decoherence is more effective.

Then it is observed that the phase damping has practically no influence on the time evolution of the entropy of the qubits. Consequently, the qubit's coherence is lost and regained regularly and periodically even as $t \to \infty$. According to Eq. (6), only diagonal matrix elements (i.e., $m = n$) in the photon number representation contribute to the reduced qubit density operator ρ_q hence phase damping has no effect on these diagonal matrix elements. This explains why the atom is entangled with and disentangled from the field as if phase dissipation of the cavity did not exist.

We note that the factors of the damping γ_A, γ_B, reflect the presence of the reservoir and show that they affect the coherence properties, so the total entropy

loses coherence monotonically due to phase damping, and finally coherence is lost and fall completely into a statistically mixed state. From the above result we can control purity by the phase damping parameters.

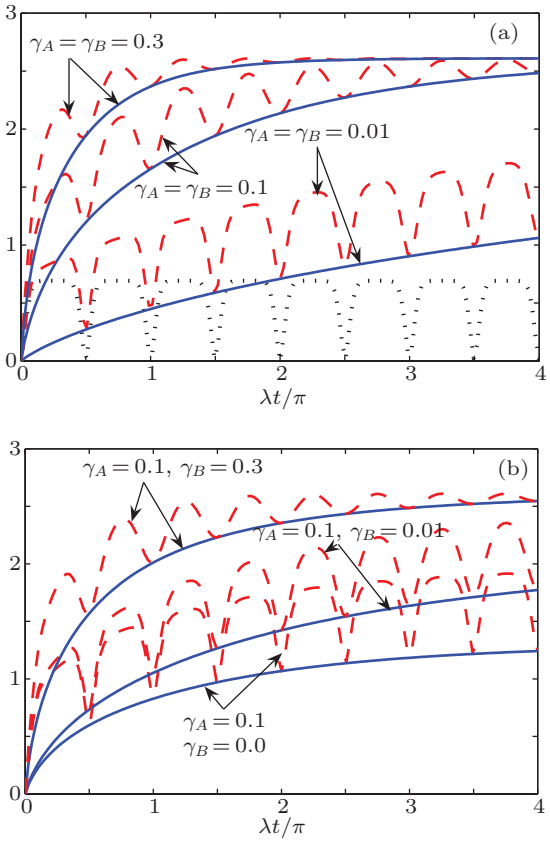


Fig. 6. (colour online) The time evolution of the S_q (dotted curve), S (solid curve), and S_f (dashed curve) with $\bar{n}=1$.

5. Conclusion

In the present paper, we have investigated the entanglement and coherence of a quantum system formed by two two-qubits interacting with two spatially separated and dissipative cavities in the dispersive limit. We obtained an explicit expression of the reduced density matrix of the two two-qubits and calculated the entanglement and coherence of them by employing the negativity and entropies, respectively. In a short-term regime, the disentanglement phenomenon could be obtained, and the entanglement of the two two-qubits decreases with the time development. However, in the long term, the entanglement of the two two-qubits tends to a fixed value. This value depends only on the damping parameter. Particularly, we found that there is long-lived disentanglement in the presence of the dissipation of the fields. And we can control the purity and entanglement of the system by the phase damping parameters.

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