A new bipartite entangled state describing the parametric down-conversion process and its applications in quantum optics*

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We construct a new bipartite entangled state (NBES), which describes both the squeezing and the entanglement involved in the parametric down-conversion process and can be produced using a symmetric beam splitter. Constructing asymmetric ket–bra integrations based on the NBES leads to some new squeezing operators, which clearly exhibit the relationships between squeezing and entangled state transformations. Moreover, an entangled Wigner operator with a definite physical meaning is also presented.

Keywords: new bipartite entangled state, quantum optics, beam splitter, squeezing operator, entangled Wigner operator

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1. Introduction

As is well known, parametric down-conversion (PDC)[1,2] is an important process since it exhibits squeezing and entanglement via a nonlinear interaction between light and certain types of nonlinear crystals. References [3] and [4] emphasized that signal and idler photons in the PDC process are entangled in the frequency domain, so the Hamiltonian describing this process can be expressed as

$$H = \omega_s a^\dagger a + \omega_i b^\dagger b + \epsilon(a^\dagger b^\dagger \, \mathrm{e}^{-\mathrm{i}2\omega_p t} + ab \, \mathrm{e}^{\mathrm{i}2\omega_p t}), \quad (1)$$

where $2\omega_p = \omega_s + \omega_i$, the indexes p, s, and i mean pump, signal, and idler respectively and ϵ is the coupling constant related to the amplitude of the pump and the second-order susceptibility of nonlinear crystals. Using the Heisenberg equation of motion in the interaction picture

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\mathrm{i}\epsilon b^\dagger, \qquad \frac{\mathrm{d}b^\dagger}{\mathrm{d}t} = -\mathrm{i}\epsilon a, \quad (2)$$

we obtain

$$a(t) = a(0)\cosh\epsilon t - \mathrm{i}b^\dagger(0)\sinh\epsilon t, \quad (3)$$

$$b(t) = b(0)\cosh\epsilon t - \mathrm{i}a^\dagger(0)\sinh\epsilon t, \quad (4)$$

which demonstrates that a squeezing transformation is involved in the PDC process. This naturally leads to an interesting question: is there a new entangled state representation which reveals both the squeezing mechanism and quantum entanglement between the signal mode and the idler mode in the PDC process? The answer is affirmative. In this work we propose a new bipartite entangled state $|\tau\rangle$ in two-mode Fock space, which describes both the squeezing and the entanglement involved in the PDC process, by noticing the commutative relation $[X_a - P_b, P_a - X_b] = 0$. It is well known that the beam splitter[5] is one of only a few experimentally-accessible devices which may act as an entangler, so another task of this paper is to discuss how to use a symmetric beam splitter to produce such an entangled state $|\tau\rangle$. Finally, we want to discuss some important applications in quantum optics.

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2. Explicit form of $|\tau\rangle$ and its properties

In order to provide a combinative view of both squeezing mechanism and quantum entanglement in the PDC process, we introduce the explicit form of $|\tau\rangle$ in two-mode Fock space

$$|\tau\rangle = \exp\left[-\frac{|\tau|^2}{2} + \tau a^\dagger - \mathrm{i}\tau^* b^\dagger + \mathrm{i}a^\dagger b^\dagger\right]|00\rangle\,, \quad (5)$$

where $\tau = \tau_1 + \mathrm{i}\tau_2$. Operating a and b on $|\tau\rangle$, respectively, we derive

$$(a - \mathrm{i}b^\dagger)\,|\tau\rangle = \tau\,|\tau\rangle\,, \qquad (a^\dagger + \mathrm{i}b)\,|\tau\rangle = \tau^*\,|\tau\rangle\,, \quad (6)$$

so $|\tau\rangle$ is the common eigenvector of $(a - \mathrm{i}b^\dagger)$ and $(a^\dagger + \mathrm{i}b)$. From Eq. (6) we find that real and imaginary part of $\sqrt{2}\tau$ is indeed the eigenvalue of $(X_a - P_b)$ and $(P_a - X_b)$, respectively, i.e.,

$$\begin{aligned}(X_a - P_b)\,|\tau\rangle &= \sqrt{2}\tau_1\,|\tau\rangle\,,\\ (P_a - X_b)\,|\tau\rangle &= \sqrt{2}\tau_2\,|\tau\rangle\,. \end{aligned} \quad (7)$$

Using the normal ordered product of the two-mode vacuum projector[6,7] $|00\rangle\langle 00| = :\exp\left[-a^\dagger a - b^\dagger b\right]:$, we find that $|\tau\rangle$ possesses the completeness relation

$$\begin{aligned}\int \frac{\mathrm{d}^2\tau}{\pi}\,|\tau\rangle\langle\tau| &= \int \frac{\mathrm{d}^2\tau}{\pi}:\,\mathrm{e}^{-(\tau^* - a^\dagger - \mathrm{i}b)(\tau - a + \mathrm{i}b^\dagger)}:\\ &= \int \frac{\mathrm{d}^2\tau}{\pi}:\,\exp\left[-\left(\tau_1 - \frac{X_a - P_b}{\sqrt{2}}\right)^2 - \left(\tau_2 - \frac{P_a - X_b}{\sqrt{2}}\right)^2\right]:\,= 1, \end{aligned} \quad (8)$$

and the orthogonal property

$$\langle\tau'\,|\tau\rangle = \pi\delta(\tau - \tau')\delta(\tau^* - \tau'^*). \quad (9)$$

Therefore, the set of $|\tau\rangle$ is qualified to make up a new complete and orthogonal representation. Making Fourier transformation for $|\tau = \tau_1 + \mathrm{i}\tau_2\rangle$ with respect to τ_2 we have

$$\begin{aligned}&\int_{-\infty}^{\infty} \frac{\mathrm{d}\tau_2}{\sqrt{2}\pi}\,|\tau = \tau_1 + \mathrm{i}\tau_2\rangle\,\mathrm{e}^{\mathrm{i}\sqrt{2}\tau_2 x}\\ &= \left|y = \frac{\tau_1}{\sqrt{2}} - x\right\rangle_a \left|p = -\frac{\tau_1}{\sqrt{2}} - x\right\rangle_b, \end{aligned} \quad (10)$$

where the single-mode state $|y\rangle_a$ is the set of coordinate eigenvectors of X_a

$$|y\rangle_a = \pi^{-1/4}\exp\left[-\frac{1}{2}y^2 + \sqrt{2}ya^\dagger - \frac{1}{2}a^{\dagger 2}\right]|0\rangle_a \quad (11)$$

and $|p\rangle_b$ is the set of momentum eigenvectors of P_b

$$|p\rangle_b = \pi^{-1/4}\exp\left(-\frac{1}{2}p^2 + \mathrm{i}\sqrt{2}pb^\dagger + \frac{1}{2}b^{\dagger 2}\right)|0\rangle_b\,. \quad (12)$$

Further, performing the inverse Fourier transformation we derive

$$\begin{aligned}|\tau\rangle \;=\; &\mathrm{e}^{\mathrm{i}\tau_1\tau_2}\int_{-\infty}^{\infty}\mathrm{d}x\left|y = \sqrt{2}\tau_1 - x\right\rangle_a\\ &\otimes\,|p = -x\rangle_b\,\mathrm{e}^{-\mathrm{i}\sqrt{2}\tau_2 x}, \end{aligned} \quad (13)$$

which is actually the Schmidt decomposition of $|\tau = \tau_1 + \mathrm{i}\tau_2\rangle$, so the state $|\tau = \tau_1 + \mathrm{i}\tau_2\rangle$ is an entangled state.[8]

3. Generation scheme of $|\tau\rangle$

As we know, beam splitters can play an important role in producing quantum entanglement. So we now discuss how to produce the state $|\tau\rangle$ by using a symmetric beam splitter as an experiment setup. The role of a symmetric beam splitter operator with free-phase can be expressed by[8,9]

$$B(\theta) = \exp\left[\mathrm{i}\frac{\theta}{2}(a^\dagger b + ab^\dagger)\right]. \quad (14)$$

Letting the two zero-momentum eigenstates $|p = 0\rangle_a = \exp[(1/2)a^{\dagger 2}]\,|0\rangle_a$ in mode a and $|p = 0\rangle_b = \exp[(1/2)b^{\dagger 2}]\,|0\rangle_b$ in mode b respectively enter the two input ports of a symmetric beam splitter and get overlapped, when $\theta = \pi/2$, then the output state is

$$B(\pi/2)\,|p = 0\rangle_1 \otimes |p = 0\rangle_2 = \exp(\mathrm{i}a^\dagger b^\dagger)\,|00\rangle\,, \quad (15)$$

which is the entangled state $|\tau = 0\rangle$. Then we operate the local oscillator laser beam, denoted by the displacement operator $D(\tau) = \exp(\tau a^\dagger - \tau^* a)$, on the output state $|\tau = 0\rangle$ of a symmetric beam splitter, and the result is

$$D(\tau)\exp(\mathrm{i}a^\dagger b^\dagger)\,|00\rangle = D(\tau)\,|\tau = 0\rangle = |\tau\rangle\,, \quad (16)$$

where the relation $D(\tau)\,a^\dagger D^\dagger(\tau) = a^\dagger - \tau^*$ is used. Thus, the ideal state $|\tau\rangle$ can be produced because the displacement $D(\tau)$ can be realized experimentally by reflecting the light field $|\tau = 0\rangle$ from an almost perfect reflecting mirror and adding through the mirror a field with phase and amplitude modulated according to the value $\tau \equiv |\tau|\,\mathrm{e}^{\mathrm{i}\phi}$.

4. Squeezing operators gained via the $|\tau\rangle$ representation

With the help of the newly introduced entangled state $|\tau\rangle$ representation, in this section we derive some new squeezing operators. Through making the classical dilation transformation $\tau \to \tau/k$ in $|\tau\rangle$, we can construct the following asymmetric ket–bra integration which leads to

$$\int \frac{\mathrm{d}^2\tau}{\pi k}|\tau/k\rangle\langle\tau| = \int \frac{\mathrm{d}^2\tau}{\pi k}{:}\ \exp\left[-\left(1+\frac{1}{k^2}\right)\frac{|\tau|^2}{2}+\tau\left(\frac{a^\dagger}{k}+\mathrm{i}b\right)+\tau^*\left(a-\mathrm{i}\frac{b^\dagger}{k}\right)+\mathrm{i}a^\dagger b^\dagger-\mathrm{i}ab-a^\dagger a-b^\dagger b\right]{:}$$
$$= \operatorname{sech}\gamma{:}\ \exp[\mathrm{i}a^\dagger b^\dagger\tanh\gamma+(\operatorname{sech}\gamma-1)(a^\dagger a+b^\dagger b)+\mathrm{i}ab\tanh\gamma]{:}\,, \quad (17)$$

where $k=\mathrm{e}^\gamma$ is a squeezing parameter and $\operatorname{sech}\gamma = 2k/(k^2+1)$, $\tanh\gamma=(k^2-1)/(k^2+1)$. Using the formula[10] $:\exp\left[(\mathrm{e}^f-1)a^\dagger a\right]:\ =\mathrm{e}^{fa^\dagger a}$, we can further rewrite Eq. (17) as

$$\int\frac{\mathrm{d}^2\tau}{\pi k}\,|\tau/k\rangle\,\langle\tau| = \exp(\mathrm{i}a^\dagger b^\dagger\tanh\gamma)$$
$$\times\ \exp[(a^\dagger a+b^\dagger b+1)\ln\operatorname{sech}\gamma]$$
$$\times\ \exp(\mathrm{i}ab\tanh\gamma)$$
$$= \exp\left[\mathrm{i}\gamma(a^\dagger b^\dagger+ab)\right]\equiv S(k), \quad (18)$$

we see that $S(k)$ is a new two-mode squeezing operator by comparing $S(k)\,|00\rangle$ (see Eq. (24)) with the usual two-mode squeezed state $\exp\left[\gamma(a^\dagger b^\dagger+ab)\right]|00\rangle$.[11] From Eqs. (8) and (9) we have

$$S(k)S^\dagger(k) = \int\frac{\mathrm{d}^2\tau}{\pi k^2}\,|\tau/k\rangle\,\langle\tau|\int\frac{\mathrm{d}^2\tau'}{\pi}\,|\tau'\rangle\,\langle\tau'/k|$$
$$= 1 = S^\dagger(k)S(k), \quad (19)$$

which means that $S(k)$ defined by Eq. (18) is unitary. On the other hand, the squeezing operator $S(k)$ causes the following squeezing transformation

$$S(k)aS^{-1}(k) = a\cosh\gamma-\mathrm{i}b^\dagger\sinh\gamma, \quad (20)$$
$$S(k)bS^{-1}(k) = b\cosh\gamma-\mathrm{i}a^\dagger\sinh\gamma, \quad (21)$$

which is just the squeezing transformations involved in the PDC process in Eqs. (3) and (4). So for the PDC process the squeezing operator is just $S(k)=\exp\left[\mathrm{i}\gamma(a^\dagger b^\dagger+ab)\right]$, whose neat entangled state $|\tau\rangle$ representation is $\displaystyle\int\frac{\mathrm{d}^2\tau}{\pi k}\,|\tau/k\rangle\,\langle\tau|$. In this way, we also see that the PDC process not only involves squeezing but also is an entanglement process. From Eqs. (20) and (21) we have

$$S(k)(X_a-P_b)S^{-1}(k) = \mathrm{e}^\gamma(X_a-P_b), \quad (22)$$
$$S(k)(P_a-X_b)S^{-1}(k) = \mathrm{e}^\gamma(P_a-X_b). \quad (23)$$

From Eq. (18) we know that $S(k)$ squeezes $|00\rangle$ in a natural way

$$S(k)\,|00\rangle = \operatorname{sech}\gamma\exp(\mathrm{i}a^\dagger b^\dagger\tanh\gamma)\,|00\rangle$$
$$= \operatorname{sech}\gamma\sum_{n=0}^{\infty}(-1)^{n/2}\tanh^n\gamma\,|n,n\rangle\,, \quad (24)$$

which is both a two-mode squeezed state and an entangled state as the outcome of the PDC process. Comparing Eq. (16) with Eq. (24) we see that the state $S(k)\,|00\rangle$ will be useful in constructing some new bipartite entangled states. Moreover, we also find that $S(k)\,|00\rangle$ can be produced at the output state of a symmetric beam splitter when the input states are two one-mode squeezed states. On the other hand, by virtue of the $|\tau\rangle$ representation we search for another new squeezing operator which squeezes $|\tau_1,\tau_2\rangle$ to $1/\sqrt{k}\,|\tau_1,\tau_2/k\rangle$. Using Eq. (10) we see

$$(X_a+P_b)\,|\tau\rangle = \int_{-\infty}^{\infty}\mathrm{d}x(\sqrt{2}\tau_1-2x)|\sqrt{2}\tau_1-x\rangle_a$$
$$\otimes\ |p=-x\rangle_b\ \mathrm{e}^{-\mathrm{i}\tau_2(\sqrt{2}x-\tau_1)}$$
$$= -\mathrm{i}\sqrt{2}\frac{\partial}{\partial\tau_2}\,|\tau\rangle\,. \quad (25)$$

Combining Eq. (7) with Eq. (29) and letting $\tau_2=\mathrm{e}^\gamma$, we have

$$\langle\tau|\,\frac{1}{2}(X_a+P_b)(P_a-X_b) = \mathrm{i}\tau_2\frac{\partial}{\partial\tau_2}\,\langle\tau|$$
$$= \mathrm{i}\frac{\partial}{\partial\gamma}\,\langle\tau_1,\tau_2=\mathrm{e}^\gamma|\,. \quad (26)$$

Owing to

$$\exp\left(-\gamma\frac{\partial}{\partial\lambda}\right)f(\lambda) = f(\lambda-\gamma),$$

then we have

$$\langle\tau|\ \mathrm{e}^{\mathrm{i}\gamma(X_a+P_b)(P_a-X_b)/2}$$
$$= \exp\left(-\gamma\frac{\partial}{\partial\lambda}\right)\langle\tau_1,\tau_2=\mathrm{e}^\lambda| = \langle\tau_1,\tau_2=\mathrm{e}^{\lambda-\gamma}|$$
$$= \left\langle\tau_1,\mathrm{e}^{-\gamma}\tau_2\right|, \quad (27)$$

which shows that the unitary operator $S'(k) = \mathrm{e}^{[\mathrm{i}\gamma(X_a+P_b)(P_a-X_b)-\gamma]/2}$ squeezes $\langle\tau|$ as

$$\langle\tau|\,S'(k) = \mathrm{e}^{-\gamma/2}\left\langle\tau_1,\mathrm{e}^{-\gamma}\tau_2\right|, \quad (28)$$

so $S'(k)$ is called as the one-side squeezing operator due to making the classical dilation transformation $\tau_2\longrightarrow\mathrm{e}^{-\gamma}\tau_2$ in the $|\tau_1,\tau_2\rangle$ representation. Using the completeness relations (8) and (28) we obtain

$$
\begin{aligned}
S'(k) &= \exp\left(-\frac{\gamma}{2}\right)\int\frac{\mathrm{d}^2\tau}{\pi}\,|\tau\rangle\,\langle\tau_1,\,\mathrm{e}^{-\gamma}\tau_2| \\
&= \mathrm{sech}^{1/2}\gamma\colon\,\exp\left\{-\frac{\tanh\gamma}{4}[(a^\dagger+\mathrm{i}b^\dagger)^2-(a-\mathrm{i}b)^2]+\frac{1}{2}(a^\dagger+\mathrm{i}b^\dagger)(a-\mathrm{i}b)(\mathrm{sech}\,\gamma-1)\right\}\colon \\
&= \mathrm{sech}^{1/2}\gamma\exp\left[-\frac{\tanh\gamma}{4}(a^\dagger+\mathrm{i}b^\dagger)^2\right]\exp\left[\frac{1}{2}(a^\dagger+\mathrm{i}b^\dagger)(a-\mathrm{i}b)\ln\mathrm{sech}\,\gamma\right]\exp\left[\frac{\tanh\gamma}{4}(a-\mathrm{i}b)^2\right]. \quad (29)
\end{aligned}
$$

It then follows the two-mode one-side squeezed state

$$
S'(k)\,|00\rangle = \mathrm{sech}^{1/2}\gamma\exp\left[-\frac{\tanh\gamma}{4}(a^\dagger+\mathrm{i}b^\dagger)^2\right]|00\rangle\,, \quad (30)
$$

which can also be produced when a pair of incoming modes (one is a single-mode squeezed state and the other is a vacuum state) impinge on a beam splitter. Similarly, by making the transformation $\tau_1 \longrightarrow \mathrm{e}^{-\gamma}\tau_1$ in the $\langle\tau_1,\tau_2|$ representation we can obtain thus a one-side squeezing operator which squeezes $|\tau_1,\tau_2\rangle$ to $1/\sqrt{k}\,|\tau_1/k,\tau_2\rangle$, i.e.,

$$
\begin{aligned}
S''(k) &= \exp[-\mathrm{i}\gamma(P_a+X_b)(X_a-P_a)-\gamma]/2 \\
&= \exp\left(-\frac{\gamma}{2}\right)\int\frac{\mathrm{d}^2\tau}{\pi}\,|\tau\rangle\,\langle\,\mathrm{e}^{-\gamma}\tau_1,\tau_2| \\
&= \mathrm{sech}^{1/2}\gamma\exp\left[-\frac{\tanh\gamma}{4}(a^\dagger-\mathrm{i}b^\dagger)^2\right] \\
&\quad\times\exp\left[\frac{1}{2}(a^\dagger-\mathrm{i}b^\dagger)(a+\mathrm{i}b)\ln\mathrm{sech}\,\gamma\right] \\
&\quad\times\exp\left[\frac{\tanh\gamma}{4}(a+\mathrm{i}b)^2\right]. \quad (31)
\end{aligned}
$$

5. Entangled Wigner operator in $|\tau\rangle$ representation and its definite physical meaning

In terms of the EPR argument in 1935,[12] investigating the Wigner operator from the point of view of entanglement is of necessity for an associated two-particle system. Now, in terms of the $|\tau\rangle$ state we introduce the following operator integration

$$
\Delta(\rho,\sigma) = \int\frac{\mathrm{d}^2\tau}{\pi^3}\,|\rho-\tau\rangle\,\langle\rho+\tau|\exp(\tau\sigma^*-\tau^*\sigma), \quad (32)
$$

where the parameters ρ and σ are complex, $\rho = \rho_1+\mathrm{i}\rho_2$, $\sigma = \sigma_1+\mathrm{i}\sigma_2$. Performing the integration in Eq. (32) gives its explicit normally ordered form

$$
\Delta(\rho,\sigma) = \pi^{-2}\colon\,\exp[-(\rho^*-a^\dagger-\mathrm{i}b)(\rho-a+\mathrm{i}b^\dagger)-(\sigma^*-a^\dagger+\mathrm{i}b)(\sigma-a-\mathrm{i}b^\dagger)]\colon\,, \quad (33)
$$

which is indeed two-mode Wigner operator in $|\tau\rangle$ representation. Letting $\rho=\alpha-\mathrm{i}\beta^*$ and $\sigma=\alpha+\mathrm{i}\beta^*$, where $\alpha=(q_a+\mathrm{i}p_a)/\sqrt{2}$ and $\beta=(q_b+\mathrm{i}p_b)/\sqrt{2}$, we can rewrite Eq. (33) as

$$
\Delta(\rho,\sigma) = \pi^{-2}\colon\,\exp[-2(\alpha^*-a^\dagger)(\alpha-a)-2(\beta^*-b^\dagger)(\beta-b)]\colon\,=\Delta_a(\alpha,\alpha^*)\Delta_b(\beta,\beta^*), \quad (34)
$$

where $\Delta_a(\alpha,\alpha^*)$ and $\Delta_b(\beta,\beta^*)$ are just the normal ordering forms of the single-mode Wigner operator.[13] Performing the one-side integration of $\Delta(\rho,\sigma)$ leads to the two marginal distributions,

$$
\int\mathrm{d}^2\sigma\Delta(\rho,\sigma) = \frac{1}{\pi}\colon\,\exp[-(\rho^*-a^\dagger-\mathrm{i}b)(\rho-a+\mathrm{i}b^\dagger)]\colon\,=\frac{1}{\pi}\,|\tau\rangle\,\langle\tau|_{\tau=\rho}\,, \quad (35)
$$

$$
\int\mathrm{d}^2\rho\Delta(\rho,\sigma) = \frac{1}{\pi}\colon\,\exp[-(\sigma^*-a^\dagger+\mathrm{i}b)(\sigma-a-\mathrm{i}b^\dagger)]\colon\,=\frac{1}{\pi}\,|\chi\rangle\,\langle\chi|_{\chi=\sigma}\,, \quad (36)
$$

where $|\chi\rangle$ conjugated to $|\tau\rangle$ is the common eigenvector of X_a+P_b and P_a+X_b, i.e., $|\chi\rangle=\exp[-|\chi|^2/2+\chi a^\dagger+\mathrm{i}\chi^*b^\dagger-\mathrm{i}a^\dagger b^\dagger]\,|00\rangle$, $\chi=\chi_1+\mathrm{i}\chi_2$, so the corresponding marginal distributions of Wigner function of a bipartite state $|\phi\rangle$ are

$$
\int\mathrm{d}^2\rho\,\langle\phi|\,\Delta(\rho,\sigma)\,|\phi\rangle = \frac{1}{\pi}\,|\langle\chi\,|\phi\rangle|^2_{\chi=\sigma}\,, \quad (37)
$$

$$
\int\mathrm{d}^2\sigma\,\langle\phi|\,\Delta(\rho,\sigma)\,|\phi\rangle = \frac{1}{\pi}\,|\langle\tau\,|\phi\rangle|^2_{\tau=\rho}\,, \quad (38)
$$

where $|\langle\chi\,|\phi\rangle|^2_{\chi=\sigma}$ and $|\langle\tau\,|\phi\rangle|^2_{\tau=\rho}$ are proportional to

the measurement probability for the combined observables of two particles. Specifically, it means that the outcomes measuring two particles compatible observables $X_a + P_b$ and $P_a + X_b$ are $\sqrt{2}\sigma_1$ and $\sqrt{2}\sigma_2$, while $|\langle\tau\,|\phi\rangle|^2_{\tau=\rho}$ means the outcomes measuring $X_a - P_b$ and $P_a - X_b$ of two particles are respectively $\sqrt{2}\rho_1$ and $\sqrt{2}\rho_2$. In other words, from the Wigner operator $\Delta(\rho,\sigma)$ in $|\tau\rangle$ representation to derive two marginal distributions of Wigner function can give the probability of finding the particles in an entangled way. Therefore, from this point of view we can name $\Delta(\rho,\sigma)$ the entangled Wigner operator.

In summary, in this work we have introduced a new bipartite entangled state $|\tau\rangle$ describing the PDC process, whose set makes up a new complete and orthogonal representation. In experiment the EPR light field described by the state $|\tau\rangle$ can be built from the two single-mode zero-momentum eigenstates combined at a symmetric beam splitter. The asymmetric ket–bra integrations based on the state $|\tau\rangle$ lead to some new squeezing operators, which provide a natural representation of the squeezing operator and reveal the inherent relation between entanglement and squeezing. In addition, from the state $|\tau\rangle$ we introduce the entangled Wigner operator $\Delta(\rho,\sigma)$, which is endowed with a definite physical meaning, i.e., whose marginal distributions are proportional to the measurement probability for the combined observables of two particles in the entangled state representation. According to the above results, we expect that the state $|\tau\rangle$ will have possible practical physical applications in developing quantum entanglement and quantum information theory in the near future.

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