Decoherence from a spin chain with Dzyaloshinskii–Moriya interaction*

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We study the dynamics of quantum discord and entanglement for two spin qubits coupled to a spin chain with Dzyaloshinsky–Moriya interaction. In the case of a two-qubit with an initial pure state, quantum correlations decay to zero at the critical point of the environment in a very short time. In the case of a two-qubit with initial mixed state, it is found that quantum discord may get maximized due to the quantum critical behavior of the environment, while entanglement vanishes under the same condition. Besides, we observed a sudden transition between classical and quantum decoherence when only a single qubit interacts with the environment. The effects of Dzyaloshinsky–Moriya interaction on quantum correlations are considered in the two cases. The decay of quantum correlations is always strengthened by Dzyaloshinsky–Moriya interaction.

Keywords: quantum discord, entanglement, decoherence

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1. Introduction

Entanglement is thought to be a fundamental resource for quantum computation and communication.[1] It is a kind of quantum correlation, which originates from the superposition principle of quantum mechanics. Recently, quantum discord has been realized as a different kind of quantum correlation other than entanglement. It arises from the difference between quantum mutual information and maximum of quantum conditional entropy.[2,3] This non-entanglement correlation can be applied to quantum computation, which is based on deterministic quantum computation with one quantum bit (DQC1) protocol.[4,5] Moreover, Brodutch *et al.* have shown that the changes in quantum discord indicates the failure of a local operations and classical communications (LOCC) implementation of the quantum gates.[6] Both entanglement and quantum discord characterize the similar quantumness features of quantum system with a pure state, namely, quantum discord reduces to entropy of entanglement. However, they become two different measures for a quantum system with a mixed state. For instance, quantum discord may be nonzero in disentangled states.[7,8]

The properties of quantum discord are intensively investigated for their potential application. It has been realized that quantum discord can be generated in the Heisenberg models at finite temperature.[9−15] Comparing with entanglement, though under the same condition, quantum discord behaves in a very different way as parameters vary. For instance, the situation where quantum discord increases with temperature while entanglement decreases was revealed.[13] In quantum critical phenomena, entanglement is used as an indicator of quantum phase transitions (QPTs).[16] As another kind of quantum correlation, it was examined that quantum discord plays the same role as entanglement. Sarandy has analyzed quantum discord in critical systems, such as XXZ and transverse field Ising models, and has shown that the quantum discord exhibits the signature of the QPT.[17] Furthermore, Werlang *et al.* have shown that quantum discord spotlights the critical points associated with QPT for the Heisenberg model even at finite temperature.[18] More recently, it has been found that the quantum discord detects the quantum critical points associated with first- and higher-order QPTs caused by field and three-spin interactions at finite temperature.[19]

Decoherence of a quantum system is the main obstacle to practical quantum computation on account of it interacting with the environment. Thus, it is necessary to reveal the dynamical properties of quantum discord so as to guard against decoherence. The

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dynamics of quantum discord both in the Markovian and nonMarkovian environment has been theoretically and experimentally investigated.[20−25] Interestingly, in the Morkovian environment, quantum discord disappears in the asymptotic time limit under the same condition where entanglement undergoes a sudden death.[20] In this sense, quantum discord is thought to be more robust than entanglement. Recently, there have been increasing investigations on decoherence from a spin environment,[26−32] such as the Ising-chain and the XY-chain. It is revealed that quantum coherence is dramatically destroyed at the critical point of the QPT of the external environment.[30]

In this paper, we shall investigate the dynamical behavior of quantum discord and entanglement for a two-qubit coupled to a spin chain with Dzyaloshinsky–Moriya (DM) interaction. The DM interaction arises from the spin–orbit coupling,[32] which often appears in models of low-dimensional magnetic materials. We start in Section 2 by introducing and diagonalizing the model, and we shall calculate the reduced density matrix of two-qubit. In Section 3, we shall evaluate quantum discord and entanglement for two-qubit, and present the main results and discussion. The last section is devoted to the conclusion.

2. Dynamics of a two-qubit coupled to an XY chain with DM interaction

We consider the environment to be an general XY chain with z-component DM interaction. The total Hamiltonian for two spin qubits transversely coupled to the chain is given by

$$H = H_{\rm E}^{(\lambda)} + H_{\rm I} \tag{1}$$

with

$$\begin{aligned} H_{\rm E}^{(\lambda)} = & -\sum_{l}^{N}\left(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z\right) \\ & -\sum_{l}^{N} D(\sigma_l^x\sigma_{l+1}^y - \sigma_l^y\sigma_{l+1}^x), \end{aligned} \tag{2}$$

$$H_{\rm I} = -g\left(\frac{1+\delta}{2}\sigma_A^z + \frac{1-\delta}{2}\sigma_B^z\right)\sum_{l}^{N}\sigma_l^z, \tag{3}$$

where $H_{\rm E}^{(\lambda)}$ is the self-Hamiltonian of the environment and $H_{\rm I}$ describes the interaction between a two-qubit and the environment. σ_l^α ($\alpha = x, y, z$) is the Pauli matrix on the l-th site of the chain. $\sigma_{A(B)}^z$ describes the two-qubit. γ measures anisotropy of exchange interaction in the XY plane, λ is the strength of the transverse magnetic field and D the strength of z-component DM interaction. $g(1\pm\delta)/2$ describes the coupling strength between qubits A, B and the surrounding chain. The parameter δ controls the anisotropy of coupling strength between each qubit and the environment. N is the total sites of the XY chain. The periodic boundary conditions $\sigma_{N+1}^\alpha = \sigma_1^\alpha$ are assumed.

The eigenstates of the operator

$$\left(\frac{1+\delta}{2}\sigma_A^z + \frac{1-\delta}{2}\sigma_B^z\right)$$

are simply given by

$$\begin{aligned} |\phi_1\rangle = |00\rangle, \quad & |\phi_2\rangle = |01\rangle, \\ |\phi_3\rangle = |10\rangle, \quad & |\phi_4\rangle = |11\rangle, \end{aligned} \tag{4}$$

where $|0\rangle$ and $|1\rangle$ denote spin up and down respectively. The total Hamiltonian (1) can be rewritten as

$$H = \sum_{\mu=1}^{4} |\phi_\mu\rangle\langle\phi_\mu| \otimes H_{\rm E}^{(\lambda_\mu)}, \tag{5}$$

where the parameters λ_μ are

$$\lambda_{1(4)} = \lambda \pm g, \quad \lambda_{2(3)} = \lambda \pm g\delta, \tag{6}$$

and $H_{\rm E}^{(\lambda_\mu)}$ is obtained from $H_{\rm E}^{(\lambda)}$ by replacing λ with λ_μ. Next we map the projected Hamiltonian $H_{\rm E}^{(\lambda_\mu)}$ into a one-dimensional spinless fermion system with creation and annihilation operators, $c_l^\dagger$ and c_l, via Jordan–Wigner transformation[33]

$$\begin{aligned} \sigma_l^x &= \prod_{s<l}(1-2c_s^\dagger c_s)(c_l + c_l^\dagger), \\ \sigma_l^y &= -{\rm i}\prod_{s<l}(1-2c_s^\dagger c_s)(c_l - c_l^\dagger), \\ \sigma_l^z &= 1 - 2c_l^\dagger c_l. \end{aligned} \tag{7}$$

Utilized Fourier transforms of the fermionic operator given by

$$d_k = \frac{1}{\sqrt{N}}\sum_{l}^{N} c_l\, {\rm e}^{-{\rm i}2\pi lk/N}$$

with $k = -M, \ldots, M$ and $M = (N-1)/2$, the projected Hamiltonian is transformed to the momentum space. Then performing Bogoliubov transformation

$$\eta_{k,\lambda_\mu} = \cos\frac{\theta_k^{(\lambda_\mu)}}{2} d_k - {\rm i}\sin\frac{\theta_k^{(\lambda_\mu)}}{2} d_{-k}^\dagger, \tag{8}$$

with

$$\theta_k^{(\lambda_u)} = \arctan\left(\frac{\gamma\sin(2\pi k/N)}{\lambda_\mu - \cos(2\pi k/N)}\right),$$

finally we gain the diagonalized Hamiltonian

$$H_{\rm E}^{(\lambda_\mu)} = \sum_k \Lambda_k^{(\lambda_\mu)} \left(\eta_{k,\lambda_\mu}^\dagger \eta_{k,\lambda_\mu} - \frac{1}{2} \right), \qquad (9)$$

where the spectrum $\Lambda_k^{(\lambda_\mu)}$ is

$$\Lambda_k^{(\lambda_\mu)} = 2 \left(\varepsilon_k^{(\lambda_\mu)} + 2D \sin \frac{2\pi k}{N} \right)$$

with

$$\varepsilon_k^{(\lambda_\mu)} = \sqrt{\left(\lambda_\mu - \cos \frac{2\pi k}{N} \right)^2 + \gamma^2 \sin^2 \frac{2\pi k}{N}}.$$

Suppose the total system is disentangled with $\rho_{\rm tot}(0) = \rho_{\rm AB}(0) \otimes \rho_{\rm E}(0)$, where $\rho_{\rm AB}(0)$ and $\rho_{\rm E}(0)$ are the initial state of two-qubit system and the environment, respectively. $\rho_{\rm E}(0) = |\psi_{\rm E}(0)\rangle\langle\psi_{\rm E}(0)|$ is assumed to be the groundstate of the environment. The evolution of the total system driven by H reads $\rho_{\rm tot}(t) = U(t)\rho_{\rm tot}(0)U^\dagger(t)$ with $U(t) = \exp(-{\rm i}Ht)$. The reduced density matrix of the two-qubit is obtained by tracing out the environment,

$$\begin{aligned} \rho_{\rm AB}(t) =& \ {\rm Tr}_{\rm E}[\,\rho_{\rm tot}(t)] \\ =& \sum_{\mu,\nu}^{4} F_{\mu\nu}(t) \langle\phi_\mu|\rho_{\rm AB}(0)|\phi_\nu\rangle |\phi_\mu\rangle\langle\phi_\nu| \end{aligned} \qquad (10)$$

with $F_{\mu\nu}(t) = \langle\psi_{\rm E}|U_{\rm E}^{\dagger(\lambda_\nu)}(t) U_{\rm E}^{(\lambda_\mu)}(t)|\psi_{\rm E}\rangle$, where $U_{\rm E}^{(\lambda_\mu)}(t) = \exp(-{\rm i}H_{\rm E}^{(\lambda_\mu)}t)$ is the projected time evolution operator.

Now we suppose that the two-qubit is initially prepared in an X-structure state,

$$\rho_{\rm AB}(0) = \frac{1}{4} \left(I_{\rm AB} + \sum_\alpha c_\alpha \sigma_A^\alpha \otimes \sigma_B^\alpha \right) \qquad (11)$$

with $I_{\rm AB}$ being the identity operator on the two-qubit, $\alpha = x, y, z$, and the parameters c_α are chosen to be real parameters that make the $\rho_{\rm AB}(0)$ a legal quantum state. This state is general to contain Bell-diagonal state and Werner state. According to Eqs. (10) and (11), in the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the reduced density matrix is given by

$$\rho_{\rm AB}(t) = \frac{1}{4} \begin{pmatrix} 1+c_z & 0 & 0 & \Gamma \\ 0 & 1-c_z & \Omega & 0 \\ 0 & \Omega^* & 1-c_z & 0 \\ \Gamma^* & 0 & 0 & 1+c_z \end{pmatrix}, \qquad (12)$$

with $\Gamma = (c_x - c_y)F_{14}(t)$ and $\Omega = (c_x + c_y)F_{23}(t)$. In order to evaluate quantum discord in this density matrix, one needs to compute the decoherence factor $F_{\mu\nu}(t)$. Let $|G\rangle_\lambda$ and $|G\rangle_{\lambda_\mu}$ denote the groundstates of the self-Hamiltonian $H_{\rm E}^{(\lambda)}$ and the projected-Hamiltonian, respectively. With the transformation:[28,29]

$$|G\rangle_\lambda = \prod_{k>0}^{M} (\cos\Theta_k^{(\lambda_\mu)} + {\rm i}\sin\Theta_k^{(\lambda_\mu)} \eta_{k,\lambda_\mu}^\dagger \eta_{-k,\lambda_\mu}^\dagger)|G\rangle_{\lambda_\mu}$$

with $\Theta^{(\lambda_\mu)} = (\theta_k^{(\lambda_\mu)} - \theta_k^{(\lambda)})/2$, following a tedious calculation, we obtain the decoherence factor as[30]

$$\begin{aligned} & |F_{\mu\nu}(t)| \\ =& \prod_{k>0}^{M} [1 - \sin^2(2\Theta_k^{(\lambda_\mu)}) \sin^2(\Lambda_k^{(\lambda_\mu)} t) - \sin^2(2\Theta_k^{(\lambda_\nu)}) \\ & \times \sin^2(\Lambda_k^{(\lambda_\nu)} t) + 2\sin(2\Theta_k^{(\lambda_\mu)}) \sin(2\Theta_k^{(\lambda_\nu)}) \\ & \times \sin(\Lambda_k^{(\lambda_\mu)} t) \sin(\Lambda_k^{(\lambda_\nu)} t) \cos(\Lambda_k^{(\lambda_\mu)} t - \Lambda_k^{(\lambda_\nu)} t) - 4 \\ & \times \sin(2\Theta_k^{(\lambda_\mu)}) \sin(2\Theta_k^{(\lambda_\nu)}) \sin^2(\Theta_k^{(\lambda_\mu)} - \Theta_k^{(\lambda_\nu)}) \\ & \times \sin^2(\Lambda_k^{(\lambda_\mu)} t) \sin^2(\Lambda_k^{(\lambda_\nu)} t)]^{1/2} \equiv \prod_{k>0}^{M} F_k(t). \end{aligned} \qquad (13)$$

When $|F_{\mu\nu}(t)| \to 1$, it indicates that the two-qubit is slightly disturbed by the environment. On the contrary, as $|F_{\mu\nu}(t)| \to 0$, the two-qubit undergoes strong decoherence. We present the detailed results and discussion in the next section.

3. Correlations for two-qubit

In a bipartite quantum system with subsystems A and B, quantum discord for its state $\rho_{\rm AB}$ is defined as[2]

$$Q(\,\rho_{\rm AB}) = I(\,\rho_{\rm AB}) - C(\,\rho_{\rm AB}), \qquad (14)$$

where $I(\,\rho_{\rm AB}) = S(\,\rho_A) + S(\,\rho_B) - S(\,\rho_{\rm AB})$ with $S(\,\rho) = -\,{\rm Tr}(\,\rho \log_2 \rho)$ is quantum mutual information, which measures total correlation in the state $\rho_{\rm AB}$. $\rho_{A(B)}$ is the reduced density matrix of $\rho_{\rm AB}$. $C(\,\rho_{\rm AB})$, measure of classical correlation, is the maximum of quantum conditional entropy obtained by performing one-side measurement,[3]

$$C(\,\rho_{\rm AB}) = \max_{\{\Pi_B^{(j)}\}} \left\{ S(\,\rho_A) - \sum_j p_j S(\,\rho_A^{(j)}) \right\}, \qquad (15)$$

where $\{\Pi_B^{(j)}\}$ denotes a set of von Neumann measurements of B, and $\rho_A^{(j)} = {\rm Tr}_B(\Pi_B^{(j)} \rho_{\rm AB} \Pi_B^{(j)})/p_j$ is the state of A after obtaining outcome j on B, with $p_j = {\rm Tr}_{\rm AB}(\Pi_B^{(j)} \rho_{\rm AB} \Pi_B^{(j)})$.

To obtain quantum discord of the density matrix given by Eq. (12), we need to calculate quantum mutual information and classical correlation. The former

quantity is directly calculated,

$$I[\rho_{\rm AB}(t)] = 2 + \sum_{n=1}^{4} \omega_n \log_2 \omega_n, \tag{16}$$

with $\omega_1 = \frac{1}{4}(1 - c_z + |\Omega|)$, $\omega_2 = \frac{1}{4}(1 - c_z - |\Omega|$, $\omega_3 = \frac{1}{4}(1 + c_z + |\Gamma|)$, and $\omega_4 = \frac{1}{4}(1 + c_z - |\Gamma|)$. Noticing that the condition $S[\rho_A(t)] = S[\rho_B(t)]$ is satisfied, we have the unique value of classical correlation irrespective of whether performing measurement on subsystem A or B. Therefore, we propose a complete set of von Neumann measurements of subsystem B, $\{\Pi_B^{(j)}|j = 1, 2\} = \{|\phi^{(1)}\rangle\langle\phi^{(1)}|, |\phi^{(2)}\rangle\langle\phi^{(2)}|\}$, with $|\phi^{(1)}\rangle = \cos\theta|0\rangle + {\rm e}^{{\rm i}\varphi}\sin\theta|1\rangle$ and $|\phi^{(2)}\rangle = {\rm e}^{-{\rm i}\varphi}\sin\theta|0\rangle - \cos\theta|1\rangle$, the parameters $\theta, \varphi \in [0, 2\pi]$. Then we have the remaining state of subsystem A with an outcome j,

$$\rho_A^{(j)} = \begin{pmatrix} \frac{1}{2}[1 - (-1)^j c_z \cos(2\theta)] & \frac{(-1)^{j+1}}{4}({\rm e}^{-{\rm i}\varphi}\Omega + {\rm e}^{{\rm i}\varphi}\Gamma)\sin(2\theta) \\ \frac{(-1)^{j+1}}{4}({\rm e}^{{\rm i}\varphi}\Omega^* + {\rm e}^{-{\rm i}\varphi}\Gamma^*)\sin(2\theta) & \frac{1}{2}[1 + (-1)^j c_z \cos(2\theta)] \end{pmatrix} \tag{17}$$

and the respective probability $p_j = 1/2$. Consequently, we obtain the classical correlation of Eq. (12) as

$$C[\rho_{\rm AB}(t)] = \frac{1+\vartheta}{2}\log_2(1+\vartheta) + \frac{1-\vartheta}{2}\log_2(1-\vartheta), \tag{18}$$

where

$$\vartheta = \max\left\{|c_z|, \frac{|\Omega| + |\Gamma|}{2}\right\}.$$

Finally, quantum discord is obtained as

$$\begin{aligned} Q[\rho_{\rm AB}(t)] = \frac{1}{4}[&(1 - c_z + |\Omega|)\log_2(1 - c_z + |\Omega|) \\ &+ (1 - c_z - |\Omega|)\log_2(1 - c_z - |\Omega|) \\ &+ (1 + c_z - |\Gamma|)\log_2(1 + c_z - |\Gamma|) \\ &+ (1 + c_z + |\Gamma|)\log_2(1 + c_z + |\Gamma|)] \\ &- C[\rho_{\rm AB}(t)]. \end{aligned} \tag{19}$$

In order to investigate the relation between quantum discord and entanglement, we choose the entanglement of formation (E) for quantifying the amount of entanglement of the two-qubit. The E is a monotonically increasing function of concurrence (C'),[35]

$$E = -f(C')\log_2 f(C') - [1 - f(C')]\log_2[1 - f(C')] \tag{20}$$

with $f(C') = (1+\sqrt{1 - C'^2})/2$. The concurrence C' is defined as $C' = \max\{0, 2\xi_{\max} - {\rm Tr}[\sqrt{\rho_{\rm AB}\tilde{\rho}_{\rm AB}}]\}$, where $\tilde{\rho}_{\rm AB} = \sigma_A^y \otimes \sigma_B^y \rho_{\rm AB}^* \sigma_A^y \otimes \sigma_B^y$, $\xi_{\max}$ is the maximum eigenvalue of $\sqrt{\rho_{\rm AB}\tilde{\rho}_{\rm AB}}$. And one can directly calculate the concurrence of Eq. (12),

$$C' = \max\left\{0, \frac{|\Gamma| + c_z - 1}{2}, \frac{|\Omega| - c_z - 1}{2}\right\}.$$

3.1. The case with an initial pure state

Let us consider the case that the two-qubit is initially prepared in pure state. We set the parameters $c_x = -c_y = c_z = 1$, then the initial state becomes a Bell state $(1/\sqrt{2})(|00\rangle + |11\rangle)$. Substituting these values into Eq. (19), and noticing that $\vartheta = 1$, i.e., classical correlation always equals 1, we obtain the quantum discord for this case,

$$\begin{aligned} Q = &\frac{1 - |F_{14}(t)|}{2}\log_2(1 - |F_{14}(t)|) \\ &+ \frac{1 + |F_{14}(t)|}{2}\log_2(1 + |F_{14}(t)|). \end{aligned} \tag{21}$$

Obviously, Q is a monotonically increasing function of $|F_{14}(t)|$ ($0 \le |F_{14}(t)| \le 1$), and its maximum is $Q = 1$ with $|F_{14}(t)| = 1$ and minimum $Q = 0$ with $|F_{14}(t)| = 0$. Yuan *et al.* revealed that $|F(t)|$

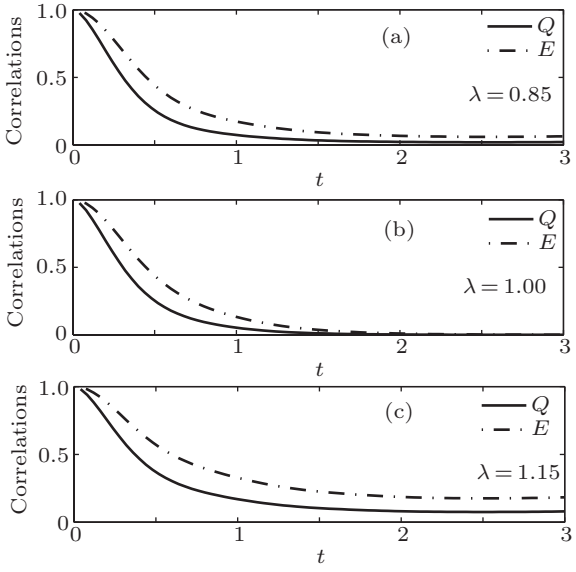


Fig. 1. Quantum discord (Q) and entanglement (E) as a function of time t, (a) $\lambda = 0.85$, (b) $\lambda = 1.00$, (c) $\lambda = 1.15$. Other parameters are set as $\gamma = 1$, $g = 0.05$, $\delta = 0$, $D = 0$, and $N = 600$.

decays to zero in a short time evolution when λ approaches the critical point $\lambda_c = 1$ in the weak coupling regime ($g \ll 1$).[29] Thus one can expect that Q just vanishes at the critical point. To evaluate the entanglement, we need to calculate concurrence. And it is $C' = |F_{14}(t)|$ for this case. In Fig. 1, we plot the time evolution of quantum discord and entanglement for Ising chain ($\gamma = 1$), with other parameters $g = 0.05$, $\delta = 0$, $D = 0$, $N = 600$, and different values of $\lambda, 0.85, 1.00, 1.15$. Entanglement behaves as similarly as the quantum discord. The quantum discord is always less than entanglement in the process of evolution. At point $\lambda = 1$, the quantum discord and entanglement decay to zero simultaneously.

Now, we examine the effects of DM interaction on quantum correlations when the environment approaches the critical point. Following the similar procedure given in Ref. [31], one can introduce a cutoff number K_c and by a tedious calculation, one has $|F_{14}(t)| \approx e^{-(\tau_1+\tau_2)t^2}$ when $\lambda \to 1$, with $\tau_1 = 32\pi^2\gamma^2 g^2/(\lambda-1)^2 \sum_k^{K_c} k^2/N^2$ and $\tau_2 = 256\pi^3\gamma^2 gD/(\lambda-1)^2 \sum_k^{K_c} k^3/N^3$. Consequently, the decay of the quantum discord and entanglement is enhanced owing to the DM interaction. Figure 2 shows the effects of the DM interaction, which is consistent with the theoretical computing. It is found that the smaller γ, the stronger the effect of the DM interaction on the decay of the decoherence factor.[31,37] It implies that the effects of the DM interaction on quantum correlations can be controlled by the parameter γ.

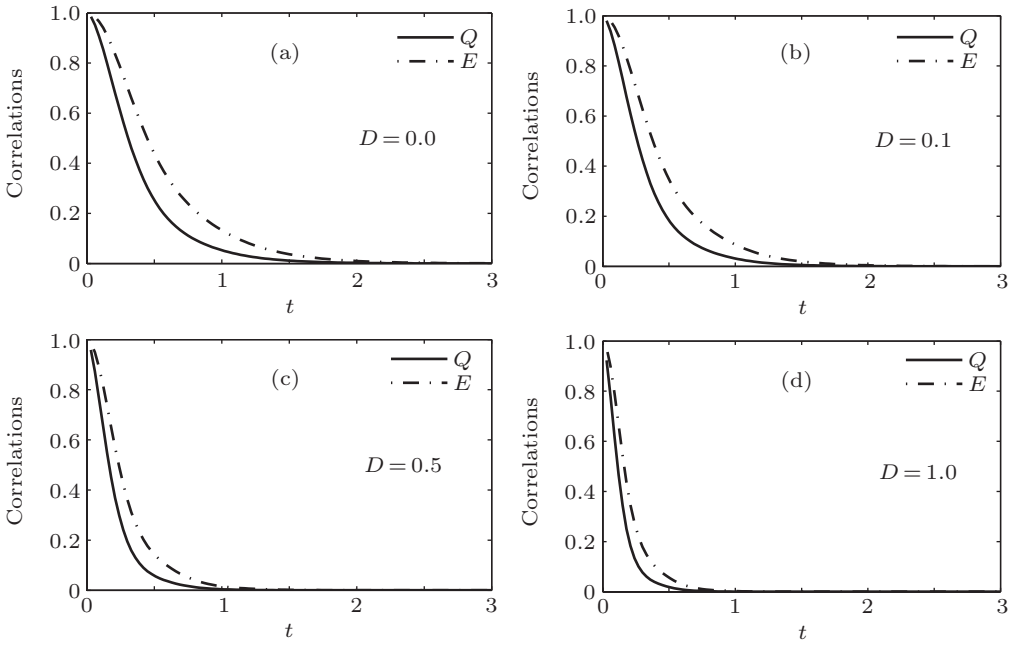


Fig. 2. Quantum discord (Q) and entanglement (E) as a function of time t, (a) $D = 0.0$, (b) $D = 0.1$, (c) $D = 0.5$, (d) $D = 1.0$. Other parameters are set as $\gamma = 1$, $\lambda = 1$, $g = 0.05$, $\delta = 0$, and $N = 600$.

3.2. The case with initial mixed state

In this subsection, we study the time evolution of quantum correlations when the two-qubit is initially prepared in a mixed state. We choose the parameters $c_x = 1$, $-c_y = c_z = 0.2$, thus the initial state becomes a mixed state

$$\rho_{AB}(0) = 0.6|\Phi^+\rangle\langle\Phi^+| + 0.4|\Psi^+\rangle\langle\Psi^+|, \qquad (22)$$

where $|\Phi^+\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$ and $|\Psi^+\rangle = (1/\sqrt{2})(|01\rangle + |10\rangle)$. It is easy to check that equation (11) is a family of the Bell-diagonal state for $c_x = 1$, $c_y = -c_z$, $c_z \in [0, 1]$. To illustrate the dynamical properties of quantum discord and entanglement, we perform the numerical calculation for exact expressions (19) and (20). In Fig. 3, the quantum discord and entanglement are plotted as a function of time t and λ with the parameters $g = 0.05$, $\delta = 0$, $\lambda = 1$, $D = 0$, $N = 600$. It is observed that when $\lambda \to 1$, the entanglement vanishes quickly while the quantum discord rapidly increases and gets maximized at $\lambda = 1$. This phenomenon is different from the case with a pure initial state. It indicates that the environment

enhances the quantum discord of the two-qubit.

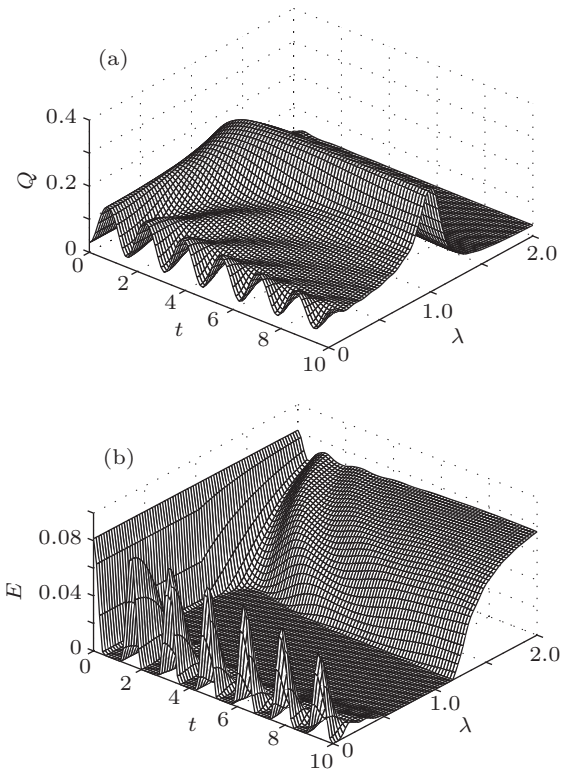


Fig. 3. (a) The quantum discord and (b) the entanglement as a function of time t and λ. Other parameters are set as $\gamma = 1$, $g = 0.05$, $\delta = 0$, $D = 0$, and $N = 600$.

Let us turn to study the effects of δ on quantum correlations. For $\delta = 0$, the decoherence factor $|F_{23}(t)| = 1$, otherwise $|F_{23}(t)|$ depends on t. Especially when $\delta = 1$, it is worth while noting that $|F_{23}(t)| = |F_{14}(t)|$. Thus, by tuning δ, the evolution of the anti-diagonal elements of Eq. (12) would become different. In Fig. 4, we plot the quantum discord and the entanglement against time t with δ equals to $0.0, 0.1, 0.5, 1.0$, respectively. It is observed that the decay of quantum discord can be enhanced by increasing δ, however, the entanglement is not sensitive to δ. When $\delta = 1$, only one qubit interacts with the surrounding chain and the other is free from the environment. $|F_{14}(t)|$ decreases to zero in a very short time under some certain conditions,[30] and $|F_{23}(t)|$ decreases as well. Thus, the anti-diagonal elements in expression (12) vanish simultaneously in a very short time, in other words, the coherence of the two-qubit quickly disappears. As plotted in Fig. 4(d), the quantum discord remains constant for an interval of time, and then decays to zero. This situation indicates that there is a sudden transition between classical and quantum decoherences, which has been discussed in detail in Ref. [36]. In Fig. 6, we plot the total and classical correlation and quantum discord as a function of time t according to the condition presented in Fig. 4(d). On the left-hand side of the dash–dotted line, one can observe that the classical correlation decreases and the quantum discord remains constant. Whereas, in the other side of the line, the classical correlation does not change with time and the quantum discord decreases. Figure 5 shows that the situation is indeed the transition.

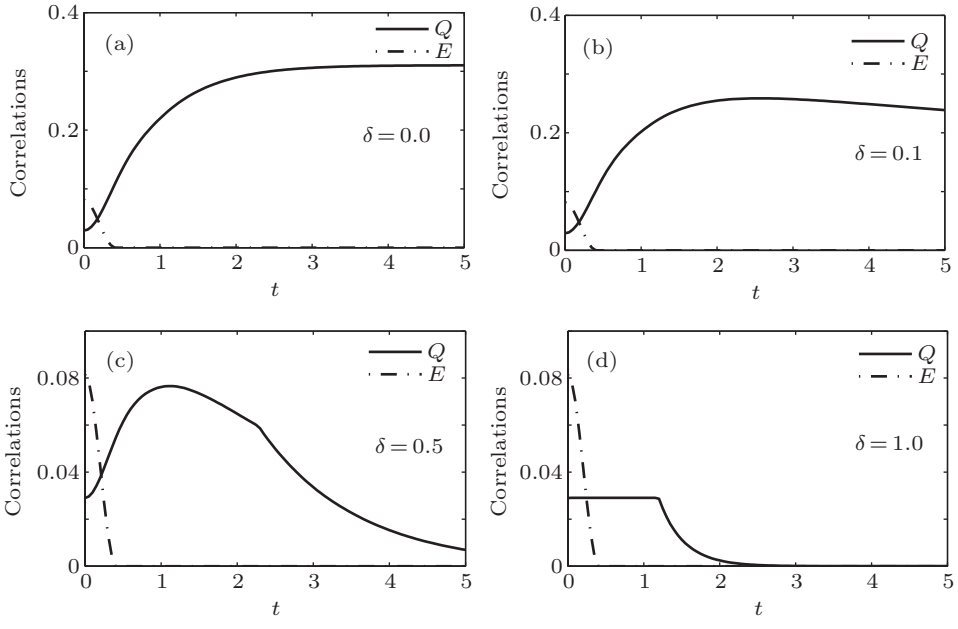


Fig. 4. The quantum discord (Q) and entanglement (E) as a function of time t, (a) $\delta = 0.0$, (b) $\delta = 0.1$, (c) $\delta = 0.5$, (d) $\delta = 1.0$. Other parameters are set as $\gamma = 1$, $\lambda = 1$, $g = 0.05$, $D = 0$, and $N = 600$.

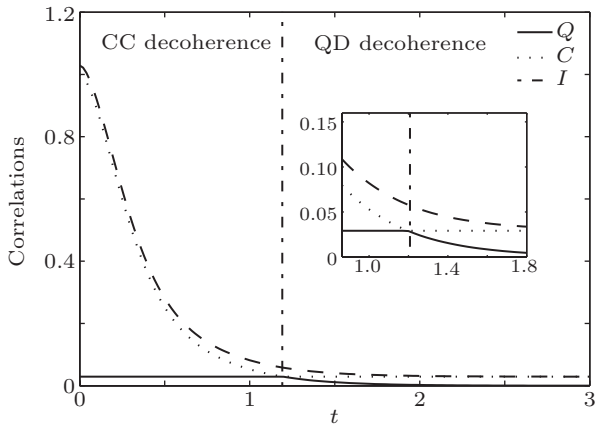


Fig. 5. The quantum discord (Q), the classical correlation (C), and the total correlation (I) as a function of time t. Other parameters are set as $\gamma = 1$, $\lambda = 1$, $g = 0.05$, $\delta = 1.0$, $D = 0$, and $N = 600$.

To end this subsection, let us focus on how the DM interaction affects quantum correlations. In Fig. 6, we plot the time evolution of quantum discord and entanglement when the two-qubit coupled to a general XY chain under different D. For $D = 0$, by decreasing values of γ, the decay of quantum discord can be slightly enhanced, however, the decay of the entanglement can be slightly weakened. Nevertheless, the entanglement vanishes in a much shorter time than the quantum discord. For $D \neq 0$, the decay of the quantum discord is remarkably strengthened. Figure 6 also shows that the effects of the DM interaction on the quantum discord is more remarkable than those on entanglement.

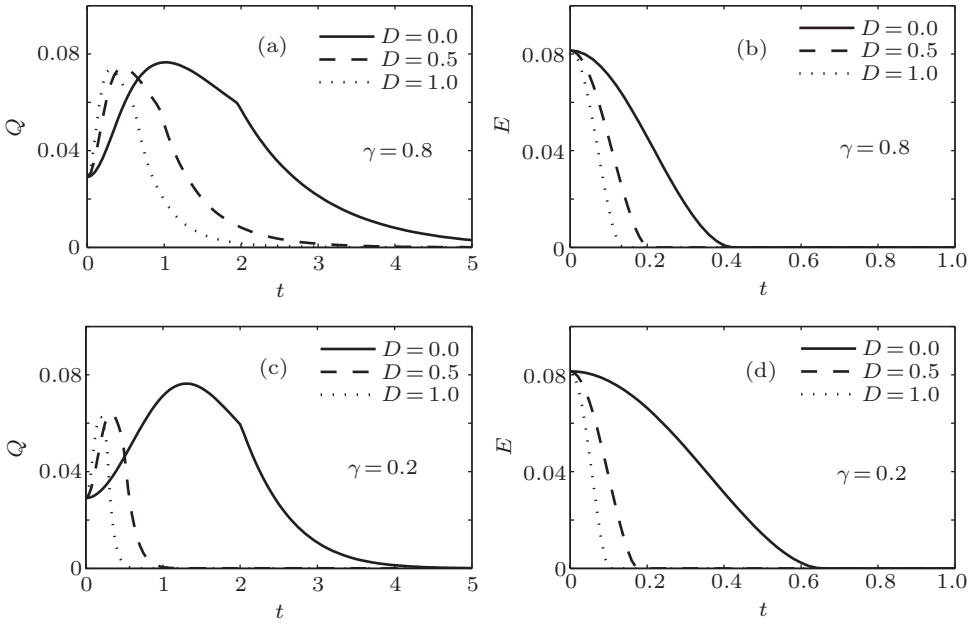


Fig. 6. Panels (a) and (c) are plots of the quantum discord (Q) as a function of time t, and (b) and (d) plots of the entanglement (E) as a function of time t, (a) and (b) $\gamma = 0.8$, (c) and (d) $\gamma = 0.2$. Other parameters are set as $g = 0.05$, $\delta = 0.5$, and $N = 600$.

4. Conclusion

In summary, we have studied the time evolution of the quantum discord and the entanglement for a two-qubit coupled to a spin chain with the DM interaction. We evaluated the quantum discord and the entanglement for a two-qubit prepared in an X-structure state. The difference of the quantum discord and the entanglement becomes drastic in the case of two-qubit with initial mixed state. When two-qubit is initially in a pure state, it is found that quantum correlations rapidly decay to zero as the environment approaches the critical point of its QPT. Interestingly, the quantum discord may get maximized for a two-qubit is initially in the mixed state when the environment at the critical point. In contrast to quantum discord, the entanglement vanishes in a short time when the environment approaches the critical point. We examined the effects of anisotropy of the coupling strength where each qubit is coupled to the environment. It has been shown that the coherence of a two-qubit rapidly vanishes when only a single qubit interacts with the environment. Besides, we have observed a sudden transition between classical and quantum decoherences as only one qubit interacts with the surrounding chain. The DM interaction promotes the process of decay of

quantum correlations irrespective of whether the two-qubit is initially in pure or mixed state. Furthermore, the effect of the DM interaction on the quantum discord can be strengthened by the anisotropy parameter. Besides, the DM interaction has slight effects on the entanglement.

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