A realizable multi-bit dense coding scheme with an Einstein–Podolsky–Rosen channel*

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We propose a multi-bit dense coding scheme by using only an Einstein–Podolsky–Rosen (EPR) channel and assistant qubits. It is shown that no matter how many classical bits there are, the quantum channel is always a Bell state. The present dense coding process can also prepare non-local multi-particle Greenberger–Horne–Zeilinger (GHZ) states at one of the participants. The quantum circuits for this dense coding process are constructed, the deterministic implementation method in an optical system based on the cross-Kerr nonlinearities is shown.

Keywords: dense coding, Greenberger–Horne–Zeilinger state, cross-Kerr nonlinearity

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1. Introduction

Quantum dense coding was first proposed by Bennett and Wiesner in 1992,[1] which shows that two bits of classical information can be encoded reliably in one spin-1/2 particle by performing local operations on one particle of the Einstein–Podolsky–Rosen (EPR) pair shared by participants Alice and Bob. The scheme was experimentally realized by Mattle *et al.* in 1996.[2] Since then, many theoretical schemes for dense coding have been studied extensively,[3−8] and dense coding has become an important branch of quantum information science. Many experimental schemes have been reported for different physical systems, such as nuclear magnetic resonance (NMR)[9,10] and cavity quantum electrodynamics (QED).[11−13] The basic idea of most existing schemes is that two bits are encoded in one qubit by using an EPR channel, that is, if one wants to send N bits of classical information, two participants have to share $N/2$ Bell states as the quantum channel. In recent years, some dense coding schemes using the multiparticle entanglement, including Greenberger–Horne–Zeilinger (GHZ) state,[14,15] W state,[16,17] and cluster state,[18] as the channel have also been studied. However, the main disadvantage of multiparticle entanglement-based schemes is that sending N bits of classical information needs two participants to share an N-qubit entangled state, which implies that many entanglement resources including different numbers of qubits are needed for the multi-bit dense coding schemes. Moreover, quantum noise is bound to have a greater effect on the quantum states in the process of distributing multiparticle entangled states.

Taking account of the above facts, in this paper, we propose a new scheme for the multi-bit dense coding by using only one Bell state as the quantum channel. We will introduce 3-bit dense coding in detail and discuss its implementation in an optical system via the cross-Kerr nonlinearity, which offers an ideal playground for deterministic optical quantum information processing and has been studied via electromagnetically induced transparency (EIT) in the past few years.[19−24]

This paper is organized as follows. In Section 2, we present the scheme for the multi-bit dense coding by using only one EPR channel and assistant particles. In Section 3, we first construct eight quantum circuits of the scheme for the 3-bit dense coding, then show how to realize the scheme in an optical system.

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Finally, the discussion and a summary are given.

2. Scheme for multi-bit dense coding using an EPR channel

For the sake of convenience, let us first consider a situation that sender Alice wishes to transmit a 3-bit message to receiver Bob. First, they need to share a Bell state

$$|\Phi\rangle_{12} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \tag{1}$$

where Alice possesses particle 1 and an auxiliary particle a in state $|0\rangle$, and Bob possesses particle 2. Therefore, the combined initial state is

$$|\varphi\rangle_{a12} = \frac{1}{\sqrt{2}}(|000\rangle + |011\rangle). \tag{2}$$

In order to achieve the dense coding of 3-bit messages $000, 001, \ldots, 111$, Alice needs to perform a set of two-qubit operations U_1, U_2, …, U_8 on her particles 1 and a to change the initial state into the eight GHZ states shown in Table 1.

Table 1. The final GHZ states corresponding to the local operations performed on particles a and 1 by Alice.

Classical message	Local operation	Final GHZ state
000	U_1	$\|\Phi^+\rangle = (1/\sqrt{2})(\|000\rangle + \|111\rangle)$
001	U_2	$\|\Phi^-\rangle = (1/\sqrt{2})(\|000\rangle - \|111\rangle)$
010	U_3	$\|\Psi_1^+\rangle = (1/\sqrt{2})(\|100\rangle + \|011\rangle)$
100	U_4	$\|\Psi_1^-\rangle = (1/\sqrt{2})(\|100\rangle - \|011\rangle)$
011	U_5	$\|\Psi_2^+\rangle = (1/\sqrt{2})(\|010\rangle + \|101\rangle)$
101	U_6	$\|\Psi_2^-\rangle = (1/\sqrt{2})(\|010\rangle - \|101\rangle)$
110	U_7	$\|\Psi_3^+\rangle = (1/\sqrt{2})(\|001\rangle + \|110\rangle)$
111	U_8	$\|\Psi_3^-\rangle = (1/\sqrt{2})(\|001\rangle - \|110\rangle)$

Through a simple calculation, we can obtain the eight translation matrices, which take the following forms

$$U_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$U_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

$$U_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_6 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_7 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

$$U_8 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \tag{3}$$

After performing one of the above operations on her two particles, Alice sends the two particles to Bob. In order to obtain the three-bit information, Bob has to find out which operation Alice has performed by

performing a GHZ state analysis on the three particles. The GHZ-state analysis schemes have been presented for different systems.[25,26] Using these analyzers, Bob can completely distinguish the eight GHZ states and obtain the coded message. For example, if the measurement result of Bob is that the three particles is in the state $|\Phi^-\rangle$, he knows that the operation performed by Alice is U_2, that is, the densely coded classical information is 001. This is true for the other cases, i.e., Bob's measurement results correspond to different GHZ states and different messages sent by Alice, the corresponding relationship is also shown in Table 1. A concrete implementation method of the above eight operations will be presented in the next section. In the scheme, Alice transmits 3 bits of classical information by sending 2 qubits, so the capacity of the quantum channel is 3/2.

Straightforwardly, the above scheme can be generalized to the multi-bit case. If Alice wants to transmit N bits of classical information to Bob, they just need to share the same entangled state as that in the 3-bit case, but Alice needs to possess $N-2$ assistant particles in state $|0\rangle$. That is, the combined initial state is

$$|\varphi\rangle_{a_1a_2\cdots a_{N-2}12} = \frac{1}{\sqrt{2}}|0\rangle^{\otimes N-2}(|00\rangle + |11\rangle)_{12}. \quad (4)$$

Similar to the 3-bit case, Alice can encode the N-bit information by performing local operations on her $N-1$ particles. After the local operations, the 2^N N-bit message and the 2^N N-particle GHZ states form a one-to-one relationship. Therefore, Alice sends her $N-1$ particles to Bob, and the latter can obtain N bits of classical information by performing an N-qubit GHZ-state analysis on the N particles. In this case, the capacity of the quantum channel becomes $N/(N-1)$, which is the same as that in the scheme based on the GHZ state channel in Refs. [12] and [14].

3. Physical implementation of the dense coding scheme in an optical system

In order to realize the above dense coding scheme in a physical system, we should first construct the quantum circuits of the eight matrices in Eq. (3). We know that the two-level unitary gates are universal for quantum computation,[27] while U_5, U_6, U_7, and U_8 are not two-level unitary matrices. Therefore, we have to decompose these matrices into products of two-level unitary matrices in the first step. Through calculation and analysis, we obtain

$$U_5 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_6 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_7 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$U_8 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}. \quad (5)$$

It is well known that single qubit operations and controlled-NOT (CNOT) gates together can be used to implement an arbitrary two-level unitary operation.[27] In Fig. 1, we give the circuits of operations U_1, U_2, ..., U_8.

As particles must be sent in the scheme and photons are most suitable for remote transmission owing to their high speed and robustness against decoherence, the optical system is the optimal candidate for implementing the dense coding. On the other hand, the interaction between the photons can be achieved by using the cross-Kerr nonlinear medium. Here, we discuss the implementation of the scheme with giant cross-Kerr nonlinearity, which involving the vertically polarization states has a Hamiltonian of the form

$$\hat{H}_K = \hbar\chi\hat{n}_{1V}\hat{n}_{2V}, \quad (6)$$

where $\hat{n}_{1V}$ and $\hat{n}_{2V}$ are the vertically polarized photon number operators of spatial modes 1 and 2, respectively, and χ is the nonlinearity coupling strength, which is decided by the property of the nonlinear material. Suppose that two photons in an arbitrary polarization state enter into the cross-Kerr medium from

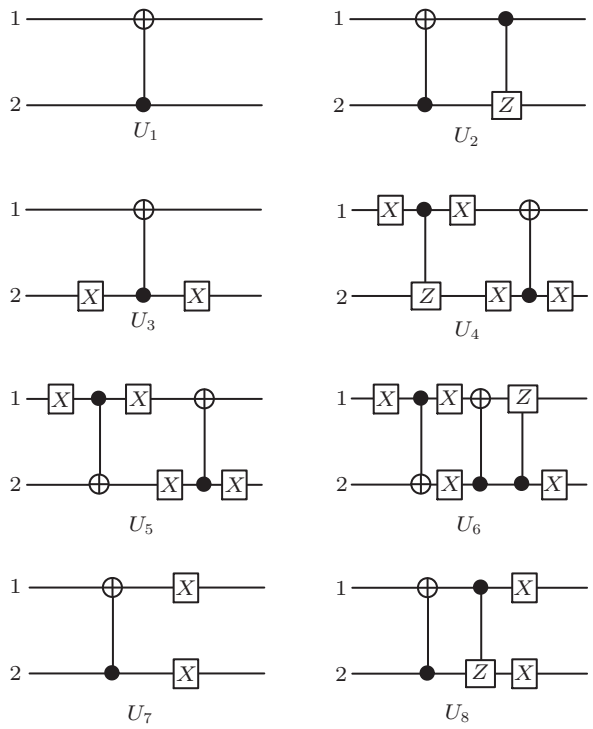


Fig. 1. Quantum circuits of the eight operations corresponding to the eight 3-bit classical messages. The quantum circuits can be regarded as entanglers from the separable state $|\varphi\rangle_{a12} = (1/\sqrt{2})(|000\rangle + |011\rangle)$ to the eight GHZ states, respectively.

the two different spatial modes, respectively, the combined system will evolve as

$$\begin{aligned}&\exp(-\mathrm{i}\chi t\hat{n}_{1V}\hat{n}_{2V})(\alpha|HH\rangle+\beta|HV\rangle\\&+\gamma|VH\rangle+\delta|VV\rangle)\\=&\,\alpha|HH\rangle+\beta|HV\rangle+\gamma|VH\rangle\\&+\gamma\exp(-\mathrm{i}n_{1V}n_{2V}\theta)|VV\rangle,\end{aligned}\tag{7}$$

where $|H\rangle$ and $|V\rangle$ denote the horizontally and the vertically polarized one-photon states, respectively, $\theta = \chi t$ denotes the phase shift induced by the cross-Kerr nonlinearity, and t is the interaction time for the two optical fields in the cross-Kerr medium. As described in Ref. [28], when θ equals π, the cross-Kerr nonlinearity will directly implement a controlled-Z gate between the two photons. A CNOT gate can be implemented by using a controlled-Z gate and two Hadamard gates, the latter can be realized by a half-wave plate (HWP) oriented at 22.5°. The other single-qubit gates in Fig. 1 can also be achieved by optical plates and phase shifters.

Furthermore, after receiving the photons sent by Alice, Bob has to perform a joint GHZ measurement on all the photons to distinguish the eight GHZ states and extract the three bits of classical information. So another important device needed in the scheme is the GHZ-state analyzer, which has been implanted via weak cross-Kerr nonlinearities in Ref. [25]. In their scheme, the authors designed a GHZ-state analyzer with two parity meters, six photon detectors, several HWPs, and a polarization beam splitter. The analyzer can completely distinguish the eight GHZ states. So Bob's measuring device is shown in Fig. 2 of Ref. [25].

Now we take an example to explain the measurement strategy. Assume the quantum channel is

$$|\Phi\rangle_{23} = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle). \tag{8}$$

The auxiliary photon is $|H\rangle$. Alice densely codes the three-bit messages on photons 1 and 2 via the above eight local operations, and sends the two photons to Bob. Then Bob performs the GHZ-state analysis on the three photons using the device shown in Fig. 2 of Ref. [25]. If the parity measurement results are $P_{12} = P_{23} = 0$, and the detectors clicked are D_{H1}, D_{H2}, D_{H3} (or $H_1V_2V_3$, $V_1H_2V_3$, $V_1V_2H_3$), i.e., the state after coding is $|\Phi^+\rangle$, Bob can obtain the classical information 000 coded by Alice. The other seven cases are similar. The corresponding relationship between Alice's densely coded messages and Bob's measurement results is shown in Table 2. So, the present scheme can be implemented in an optical system by representing the state $|0\rangle(|1\rangle)$ in the previous section as the horizontal polarization state $|H\rangle$ (vertical polarization $|V\rangle$).

Table 2. The corresponding relationship between Alice's densely coded messages and Bob's measurement results.

Alice's message	Bob's measurement result	Bob's photon-detection result
000	$P_{12}=0, P_{23}=0$	D_{H1}, D_{H2}, D_{H3} (or $H_1V_2V_3, V_1H_2V_3, V_1V_2H_3$)
001	$P_{12}=0, P_{23}=0$	D_{H1}, D_{H2}, D_{V3} (or $H_1V_2H_3, V_1H_2H_3, V_1V_2V_3$)
010	$P_{12}=1, P_{23}=0$	D_{H1}, D_{H2}, D_{H3} (or $H_1V_2V_3, V_1H_2V_3, V_1V_2H_3$)
100	$P_{12}=1, P_{23}=0$	D_{H1}, D_{H2}, D_{V3} (or $H_1V_2H_3, V_1H_2H_3, V_1V_2V_3$)
011	$P_{12}=1, P_{23}=1$	D_{H1}, D_{H2}, D_{H3} (or $H_1V_2V_3, V_1H_2V_3, V_1V_2H_3$)
101	$P_{12}=1, P_{23}=1$	D_{H1}, D_{H2}, D_{V3} (or $H_1V_2H_3, V_1H_2H_3, V_1V_2V_3$)
110	$P_{12}=0, P_{23}=1$	D_{H1}, D_{H2}, D_{H3} (or $H_1V_2V_3, V_1H_2V_3, V_1V_2H_3$)
111	$P_{12}=0, P_{23}=1$	D_{H1}, D_{H2}, D_{V3} (or $H_1V_2H_3, V_1H_2H_3, V_1V_2V_3$)

It is worthwhile to note that the process of dense coding in the present scheme is actually the process of entanglement preparation, i.e., preparing non-local GHZ states by using the initial state $(1/\sqrt{2})|H\rangle^{\otimes N-2}(|HH\rangle + |VV\rangle)$. In this sense, the quantum circuits in Fig. 1 can be treated as entanglers, which can be used for preparing specified non-local GHZ states with the separable state $(1/\sqrt{2})(|HHH\rangle + |HVV\rangle)$.

4. Discussion and summary

Now we give a brief discussion on the main advantage and the experimental feasibility of the present scheme. It is clear that the capacity of the quantum channel is $N/(N-1)$ in our dense coding scheme, which is the same as that in the scheme based on the GHZ channel shown in Refs. [14] and [15]. Hence the capacity will decrease with the increase of the bit number, which means that the scheme is more efficient in the case of small N. However, compared with Refs. [14] and [15], the significant advantage of our scheme is that participants Alice and Bob can encode different numbers of bits by sharing the same entanglement resource, which will simplify the quantum channel (economize the entanglement resource) for the multi-bit dense coding and more efficiently depress the impact of the noise on the quantum channel owing to fewer particles being shared.

For the experiments, the significant element in our scheme is the giant cross-Kerr nonlinearity that generates the π phase modulation between two optical modes. The giant cross-Kerr nonlinearity is considered to be difficult to achieve, but some feasible methods to obtain the π phase shift have been reported in recent years.[19−22] Especially, in Ref. [22], Sun *et al.* pointed out that the π cross phase can be achieved between two ultraweak optical fields in a semiconducting carbon nanotube quantum dot structure with strong spin–orbit coupling based on the EIT when the coupling Rabi frequency is much smaller than the exciton recombination rate. Nevertheless, the experimental performance is affected by many factors,[29] such as dispersion and self-phase modulation of the optical fields in the cross-Kerr medium. The present scheme is probabilistic even with these factors. In fact, figure 1 gives the general quantum circuits for encoding 3-bit messages in the eight GHZ states, so the dense coding can also be implemented in any other physical systems, as long as the single-qubit operations and the two-qubit CNOT gate can be realized in those physical systems.

In summary, we have proposed a feasible scheme for multi-bit dense coding and presented the 3-bit case in detail. In the scheme, two participants only need a Bell state as the quantum channel no matter how many classical bits will be encoded. We have also constructed the quantum circuits of the eight operations corresponding to the eight 3-bit classical messages and discussed the experimental implementation in an optical system. As a by-product, the eight circuits in Fig. 1 can be used as eight entanglers for generating non-local GHZ states with a specific initial state. Moreover, the scheme can be achieved deterministically in the ideal case.

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