State estimation for neural neutral-type networks with mixed time-varying delays and Markovian jumping parameters*

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This paper is concerned with a delay-dependent state estimator for neutral-type neural networks with mixed time-varying delays and Markovian jumping parameters. The addressed neural networks have a finite number of modes, and the modes may jump from one to another according to a Markov process. By construction of a suitable Lyapunov–Krasovskii functional, a delay-dependent condition is developed to estimate the neuron states through available output measurements such that the estimation error system is globally asymptotically stable in a mean square. The criterion is formulated in terms of a set of linear matrix inequalities (LMIs), which can be checked efficiently by use of some standard numerical packages.

Keywords: neural networks, state estimation, neutral delay, Markovian jumping parameters

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1. Introduction

Over the recent decades, various kinds of neural networks, such as Hopfield-type neural networks, cellular neural networks, Lotka–Volterra neural networks, Cohen–Grossberg neural networks, and bidirectional associative memory neural networks have received significant interest due to their extensive applications in signal processing, pattern recognition, static image processing, associative memory, and combinatorial optimization. For examples, refer to Refs. [1]–[3]. The problem of stability analysis of neural networks has been the central focus of numerous research activities. Numerous sufficient conditions for different types of stability, such as complete stability, asymptotic stability, and exponential stability, have been obtained for neural networks.[4−18]

Moreover, time delay is a natural phenomenon that is commonly encountered in various real systems.[19−21] Practically, such phenomena always appear in the study of existence of time-delays in neural network models. This indicates that time-delays are dependent on the past state. In fact, many practical delay systems can be modelled as differential systems of neutral-type, whose differential expression concludes not only the derivative term of the current state but also concludes the derivative of the past state, such as partial element equivalent circuits and transmission lines in electrical engineering, controlled constrained manipulators in mechanical engineering, population dynamics, see refer to Ref. [22] and the references therein. In many cases, it has been shown that the existing neural network models cannot characterize the properties of the neural reaction process precisely due to the complicated dynamic properties of the neural cells in the real world. Consequently it is natural and necessary that systems can contain some information about the derivative of the past state to further describe and model the dynamics for such complex neural reactions.[23]

Since the neuron states are not often fully available in the network outputs in many applications, the neuron state estimation problem becomes important for many applications to utilize the estimated neuron state.[24−28] Wang *et al.*[24] firstly investigated the state estimation problem for continuous-time neural

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networks with time-varying delay through available output measurements and derived some sufficient conditions for the existence of the desired estimators. Especially, further studies on the delay-dependent state estimation problem for neural networks with neutral delay or time-varying delays have been extensively investigated in recent years.[24−28]

On the other hand, systems with Markovian jump parameters, which are driven by a continuous-time Markov process, have been widely used to model many practical systems where they may experience abrupt changes in their structure and parameters.[29] It should be noted that only a few papers have dealt with the state estimation of neural networks with time-varying delays and Markovian jumping parameters.[30,31] To the best of the authors' knowledge, the state estimation of neural networks of neutral type with mixed time-varying delays and Markovian jumping parameters has not yet been investigated. This motivates the research work.

Based on the above discussions, an LMI approach for the state estimation problem for neural networks of neutral type with mixed time-varying delays and Markovian jumping parameters is considered. By constructing a new Lyapunov–Krasovskii functional and employing some analysis techniques, sufficient conditions for the network are derived in terms of LMIs, which can be easily calculated by MATLAB LMI control toolbox. Numerical examples are given to illustrate the effectiveness of the proposed method.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript T denotes the transposition and the notation $X\geq Y$ (respectively, $X>Y$), where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive-definite). I_n is the $n\times n$ identity matrix. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. Moreover, let $(\Omega,\mathscr{F},P)$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions. That is the filtration contains all P-null sets and is right continuous. Denote $L^P_{\mathscr{F}_0}([-\bar{h},0]$ the family of all $\mathscr{F}_0$-measurable $C([-\bar{h},0];\mathbb{R}^n)$-valued random variables $\xi=\{\xi(\theta):-\bar{h}\leq\theta\leq 0\}$ such that $\sup_{-\bar{h}\leq\theta\leq 0}\mathbb{E}|\xi(\theta)|^P<\infty$, where $\mathbb{E}$ stands for the mathematical expectation operator with respect to the given probability measure P. The notation $*$ always denotes the symmetric block in one symmetric matrix.

2. Problem description and preliminaries

As discussed in the previous section, neural networks of neutral type with Markovian jumping parameters are more appropriate to describe a class of neural networks with finite state representation, where the network dynamics can switch from one to another with the switch law being a Markov law. Based on the following models (1)–(6), we now introduce the neural networks of neutral type with time-varying delays and Markovian jumping parameters.

Let $\{r(t),t\geq 0\}$ be a right-continuous Markov process on the probability space which takes values in the finite space $S=\{1,2,\ldots,N\}$ with generator $\Gamma=(\gamma_{ij})$ $(i,j\in S)$ given by

$$\begin{aligned}P(r(t+\triangle)=j|r(t)=i)&\\ =\begin{cases}\gamma_{ij}\Delta+o(\Delta), & i\neq j,\\ 1+\gamma_{ii}\Delta+o(\Delta) & i=j,\end{cases}&\end{aligned}\tag{1}$$

where $\Delta>0$, $\lim_{\Delta t\to 0}(0(\Delta)/\Delta)=0$ and γ_{ij} is transition rate from mode i to mode j satisfying $\gamma_{ij}\geq 0$ for $i\neq j$ with $\gamma_{ii}=-\sum_{j=1,j\neq i}^{N}\gamma_{ij},i,j\in S$.

In this paper, the following neural networks of neutral type with mixed time-varying delays and Markovian jumping parameter are considered:

$$\begin{aligned}\dot{x}(t)=&-A(r(t))x(t)+B_1(r(t))g(x(t))\\&+B_2(r(t))g(x(t-\tau(t)))\\&+B_3(r(t))\int_{t-r(t)}^{t}g(x(s))\mathrm{d}s\\&+E(r(t))\dot{x}(t-\sigma(t))+J(r(t)),\\ y(t)=&\,C(r(t))x(t)+D(r(t))f(t,x(t)),\end{aligned}\tag{2}$$

where $x(\cdot)=[x_1(\cdot),x_2(\cdot),\ldots,x_n(\cdot)]^{\mathrm{T}}\in\mathbb{R}^n$ is neuron state vector; $y(\cdot)$ is the output of the networks; $A(r(t))=\mathrm{diag}\{a_1(r(t)),\ldots,a_n(r(t))\}>0$ describes the rate with which each neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; the matrices $B_1(r(t))$, $B_2(r(t))$, $E(r(t))$, and $B_3(r(t))$ represent the connection weight matrices, the discretely and neutral delayed connection weight matrices, and distributively delayed connection weight matrix, respectively. The matrices $C(r(t))\in\mathbb{R}^{m\times n}$ and $D(r(t))\in\mathbb{R}^{m\times m}$ are the output weighting matrices. $g(x(\cdot))=[\,g_1(x_1(\cdot)),\ldots,g_n(x_n(\cdot))]^{\mathrm{T}}\in\mathbb{R}^n$ denotes the neuron activation function; $\tau(t)$, $\sigma(t)$, and $r(t)$ denote the discrete time-varying delays, neutral time-varying

delays and the distributed time-varying delays, respectively, and satisfy

$$0 \le h_1 \le \tau(t) \le h_2, \quad 0 < \sigma(t) \le \bar{\sigma},$$
$$0 < r(t) \le \bar{r}, \quad \dot{\tau}(t) \le \mu < \infty, \quad \dot{\sigma}(t) \le \eta < 1,$$

in which $h_1, h_2, \bar{\sigma}, \bar{r}, \mu$, and η are constants. $J(r(t)) = [J_1(r(t)), \ldots, J_n(r(t))]^{\mathrm{T}} \in \mathbb{R}^n$ is an external input vector and $f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^m$ is the neuron-dependent nonlinear disturbances on the network outputs which is assumed to satisfy the following underlying Assumption A.

Assumption A The neuron activation function $g(\cdot)$ and the neuron-dependent nonlinear disturbances $f(\cdot)$ in Eq. (2) are bounded and there exist four constant matrices $\phi^- = \mathrm{diag}\{\phi_1^-, \phi_2^-, \ldots, \phi_n^-\}$, $\phi^+ = \mathrm{diag}\{\phi_1^+, \phi_2^+, \ldots, \phi_n^+\}$, $\psi^- = \mathrm{diag}\{\psi_1^-, \psi_2^-, \ldots, \psi_m^-\}$, and $\psi^+ = \mathrm{diag}\{\psi_1^+, \psi_2^+, \ldots, \psi_m^+\}$, such that[31]

$$\phi_l^- \le \frac{g_l(a) - g_l(b)}{a - b} \le \phi_l^+, \tag{3}$$

$$\psi_r^- \le \frac{f_r(a, x(a)) - f_r(b, x(b))}{x(a) - x(b)} \le \psi_r^+ \tag{4}$$

for all $a, b \in \mathbb{R}$, $a \ne b$, $l = 1, 2, \ldots, n$ and $r = 1, 2, \ldots, m$.

For the delayed neural networks (2), we construct the full-order state estimation as follows:

$$\begin{aligned}
\dot{\hat{x}}(t) =\ & -A(r(t))\hat{x}(t) + B_1(r(t))g(\hat{x}(t)) \\
& + B_2(r(t))g(\hat{x}(t - \tau(t))) \\
& + B_3(r(t)) \int_{t-r(t)}^{t} g(\hat{x}(s))\mathrm{d}s \\
& + E(r(t))\dot{\hat{x}}(t - \sigma(t)) + J(r(t)) \\
& + K(r(t))(y - \hat{y}), \\
\hat{y}(t) =\ & C(r(t))\hat{x}(t) + D(r(t))f(t, \hat{x}(t)),
\end{aligned} \tag{5}$$

where $\hat{x}(t)$ is the estimation of the neuron state, $\hat{y}(t)$ is the output of the estimation of the neuron state and $K(r(t)) \in \mathbb{R}^{n \times m}$ is the estimator gain matrix to be designed later.

Notice that the Markov process $\{r(t), t \ge 0\}$ takes values in the finite space $S = \{1, 2, \ldots, N\}$. For convenience, each possible value of $r(t)$ is denoted by i, $i \in S$ in the sequel. Then we have

$$A(i) = A_i, \quad B_1(i) = B_{1i}, \quad B_2(i) = B_{2i},$$
$$B_3(i) = B_{3i}, \quad E(i) = E_i,$$
$$J(i) = J_i, \quad K(i) = K_i, \quad C(i) = C_i, \quad D(i) = D_i.$$

Define the error $e(t) = x(t) - \hat{x}(t)$, and $\phi(t) = g(x(t)) - g(\hat{x}(t))$, $\psi(t) = f(t, x(t)) - f(t, \hat{x}(t))$. Thus, the error-state system is given by

$$\begin{aligned}
\dot{e}(t) =\ & -(A_i + K_i C_i)e(t) + B_{1i}\phi(t) \\
& + B_{2i}\phi(t - \tau(t)) + B_{3i} \int_{t-r(t)}^{t} \phi(s)\mathrm{d}s \\
& + E_i \dot{e}(t - \sigma(t)) - K_i D_i \psi(t).
\end{aligned} \tag{6}$$

Obviously, $e(t)$ is bounded and continuously differential on $[-\bar{h}, 0]$, where $\bar{h} = \max\{h_2, \bar{\sigma}, \bar{r}\}$.

From Assumption A, we obtain the following inequalities

$$[\phi_l(t) - \phi_l^- e_l(t)]^{\mathrm{T}}[\phi_l(t) - \phi_l^+ e_l(t)] \le 0,$$
$$l = 1, 2, \ldots, n,$$
$$[\psi_r(t) - \psi_r^- e_r(t)]^{\mathrm{T}}[\psi_r(t) - \psi_r^+ e_r(t)] \le 0,$$
$$r = 1, 2, \ldots, m.$$

Let $e(t; \xi)$ be the trajectory of system (6) under the initial condition $e(\theta) = \xi(\theta)$ on $\{-\bar{h} \le \theta \le 0\}$ in $L^2_{\mathscr{F}_0}([-\bar{h}, 0], \mathbb{R}^n)$. It is obvious that the system (6) admits a trivial solution $e(t; 0) = 0$.

In order to derive our main results, the following definition and lemma are introduced.

Definition 1[31] For error-state system (6) and every $\xi \in L^2_{\mathscr{F}_0}([-\bar{h}, 0], \mathbb{R}^n)$, the trivial solution is asymptotically stable in the mean square if, for every network mode,

$$\lim_{t \longrightarrow \infty} \mathbb{E}|e(t; \xi)|^2 = 0.$$

Lemma 1[32] (Jensen's inequality) For any constant matrices $M \in \mathbb{R}^{m \times m}$, scalar $\gamma > 0$, vector function $\omega : [0, \gamma] \to \mathbb{R}^m$ such that the integrations concerned are well defined, then

$$\begin{aligned}
& \gamma \int_0^\gamma \omega^{\mathrm{T}}(s) M \omega(s)\mathrm{d}s \\
\ge\ & \Big(\int_0^\gamma \omega(s)\mathrm{d}s\Big)^{\mathrm{T}} M \Big(\int_0^\gamma \omega(s)\mathrm{d}s\Big).
\end{aligned}$$

3. Main results

In this section, we shall establish our main criterion based on linear matrix inequality (LMI) approach. For representation convenience, we introduce the following notations:

$$L_1 = \mathrm{diag}\{\phi_1^-\phi_1^+, \phi_2^-\phi_2^+, \ldots, \phi_n^-\phi_n^+, \},$$
$$L_2 = \mathrm{diag}\Big\{\frac{\phi_1^- + \phi_1^+}{2}, \frac{\phi_2^- + \phi_2^+}{2}, \ldots, \frac{\phi_n^- + \phi_n^+}{2}\Big\},$$
$$L_3 = \mathrm{diag}\{\psi_1^-\psi_1^+, \psi_2^-\psi_2^+, \ldots, \psi_n^-\psi_n^+, \},$$
$$L_4 = \mathrm{diag}\Big\{\frac{\psi_1^- + \psi_1^+}{2}, \frac{\psi_2^- + \psi_2^+}{2}, \ldots, \frac{\psi_n^- + \psi_n^+}{2}\Big\}.$$

Theorem 1 For given scalars $h_2 > h_1 \geq 0$, $\bar{\sigma}$, $\bar{r}$, $\eta < 1$, and $\mu < \infty$, the equilibrium point of error-state system (6) is globally asymptotically stable in the mean square, if there exist positive definite matrices $P_i = P_i^{\mathrm{T}} = \mathrm{diag}\{P_{1i}, P_{2i}, P_{3i}\} > 0$, $Q_l = Q_l^{\mathrm{T}} > 0$, $(l = 1, 2, \ldots, 5)$, $R_k = R_k^{\mathrm{T}} > 0$, $(k = 1, 2, 3, 4)$, $T_b = T_b^{\mathrm{T}} > 0$, $(b = 1, 2, 3)$, diagonal matrices $U > 0$, $V > 0$, $W > 0$, and any matrix X_i such that the following LMI holds

$$\Xi_{14\times 14,i} < 0, \tag{7}$$

where

$$\begin{aligned}
&\Xi_{1,1,i} = Q_1 + Q_2 + Q_3 - P_{1i}A_i - A_i^{\mathrm{T}}P_{1i} - X_iC_i - C_i^{\mathrm{T}}X_i^{\mathrm{T}} - \frac{1}{\bar{\sigma}}R_2 - \frac{1}{h_2}R_3 - 4h_2^2T_1\\
&\qquad - 4(h_2 - h_1)^2T_2 - 4\bar{\sigma}^2T_3 - L_1U - L_3W + \sum_{j=1}^{N}\gamma_{ij}P_{1j},\\
&\Xi_{1,2,i} = \frac{1}{h_2}R_3, \quad \Xi_{1,5,i} = P_{1i}E_i, \quad \Xi_{1,6,i} = \frac{1}{\bar{\sigma}}R_2, \quad \Xi_{1,7,i} = -A_i^{\mathrm{T}}P_{1i} - C_i^{\mathrm{T}}X_i^{\mathrm{T}}, \quad \Xi_{1,8,i} = P_{1i}B_{1i} + L_2U,\\
&\Xi_{19,i} = P_{1i}B_{2i}, \quad \Xi_{1,10,i} = -X_iD_i + L_4W, \quad \Xi_{1,11,i} = P_{1i}B_{3i}, \quad \Xi_{1,12,i} = P_{2i}^{\mathrm{T}} + 4h_2T_1, \quad \Xi_{1,13,i} = 4(h_2 - h_1)T_2,\\
&\Xi_{1,14,i} = 4\bar{\sigma}T_3, \quad \Xi_{2,2,i} = -(1-\mu)Q_1 - \frac{1}{h_2}R_3 - \frac{1}{h_2 - h_1}(R_3 + R_4) - L_1V - \frac{1}{h_2 - h_1}R_4,\\
&\Xi_{2,3,i} = \frac{1}{h_2 - h_1}R_4, \quad \Xi_{2,4,i} = \frac{1}{h_2 - h_1}(R_3 + R_4), \quad \Xi_{2,9,i} = L_2V, \quad \Xi_{3,3,i} = -Q_2 - \frac{1}{h_2 - h_1}R_4, \quad \Xi_{3,13,i} = P_{3i}^{\mathrm{T}},\\
&\Xi_{4,4,i} = -Q_3 - \frac{1}{h_2 - h_1}(R_3 + R_4), \quad \Xi_{4,12,i} = -P_{2i}^{\mathrm{T}}, \quad \Xi_{4,13,i} = -P_{3i}^{\mathrm{T}}, \quad \Xi_{5,5,i} = -(1-\eta)Q_4, \quad \Xi_{5,7,i} = E_i^{\mathrm{T}}P_{1i}^{\mathrm{T}},\\
&\Xi_{6,6,i} = -\frac{1}{\bar{\sigma}}R_2, \quad \Xi_{7,7,i} = Q_4 + \bar{\sigma}R_2 + h_2R_3 + (h_2 - h_1)R_4 + h_2^4T_1 + (h_2^2 - h_1^2)^2T_2 + \bar{\sigma}^4T_3 - P_{1i} - P_{1i}^{\mathrm{T}},\\
&\Xi_{7,8,i} = P_{1i}B_{1i}, \quad \Xi_{7,9,i} = P_{1i}B_{2i}, \quad \Xi_{7,10,i} = -X_iD_i, \quad \Xi_{7,11,i} = P_{1i}B_{3i}, \quad \Xi_{8,8,i} = Q_5 + \bar{r}R_1 - U,\\
&\Xi_{9,9,i} = -(1-\mu)Q_5 - V, \quad \Xi_{10,10,i} = -W, \quad \Xi_{11,11,i} = -\frac{1}{\bar{r}}R_1,\\
&\Xi_{12,12,i} = -4T_1 + \sum_{j=1}^{N}\gamma_{ij}P_{2j}, \quad \Xi_{13,13,i} = -4T_2 + \sum_{j=1}^{N}\gamma_{ij}P_{3j}, \quad \Xi_{14,14,i} = -4T_3,
\end{aligned}$$

and other entries of Ξ are 0.

Moreover, the gain matrix of state estimator is given by

$$K_i = P_{1i}^{-1}X_i.$$

Proof Choose the Lyapunov–Krasovskii functional for system (6) as

$$\begin{aligned}
V(e(t), i) = {} & V_1(e(t), i) + V_2(e(t), i) + V_3(e(t), i)\\
& + V_4(e(t), i),
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
V_1(e(t), i) = {} & \zeta^{\mathrm{T}}(t)P_i\zeta(t),\\
V_2(e(t), i) = {} & \int_{t-\tau(t)}^{t} e^{\mathrm{T}}(s)Q_1e(s)\mathrm{d}s\\
& + \int_{t-h_1}^{t} e^{\mathrm{T}}(s)Q_2e(s)\mathrm{d}s\\
& + \int_{t-h_2}^{t} e^{\mathrm{T}}(s)Q_3e(s)\mathrm{d}s\\
& + \int_{t-\sigma(t)}^{t} \dot{e}^{\mathrm{T}}(s)Q_4\dot{e}(s)\mathrm{d}s\\
& + \int_{t-\tau(t)}^{t} \phi^{\mathrm{T}}(s)Q_5\phi(s)\mathrm{d}s,\\
V_3(e(t), i) = {} & \int_{-\bar{r}}^{0}\int_{t+\theta}^{t} \phi^{\mathrm{T}}(s)R_1\phi(s)\mathrm{d}s\mathrm{d}\theta\\
& + \int_{-\bar{\sigma}}^{0}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s)R_2\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\\
& + \int_{-h_2}^{0}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s)R_3\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\\
& + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s)R_4\dot{e}(s)\mathrm{d}s\mathrm{d}\theta,\\
V_4(e(t), i) = {} & 2h_2^2\int_{t-h_2}^{t}\int_{\theta}^{t}\int_{\lambda}^{t} \dot{e}^{\mathrm{T}}(s)T_1\dot{e}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta\\
& + 2[h_2^2 - h_1^2]\int_{t-h_2}^{t-h_1}\int_{\theta}^{t}\int_{\lambda}^{t} \dot{e}^{\mathrm{T}}(s)T_2\dot{e}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta\\
& + 2\bar{\sigma}^2\int_{t-\bar{\sigma}}^{t}\int_{\theta}^{t}\int_{\lambda}^{t} \dot{e}^{\mathrm{T}}(s)T_3\dot{e}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta,
\end{aligned}$$

where

$$\zeta^{\mathrm{T}}(t) = \Big[e^{\mathrm{T}}(t) \quad \int_{t-h_2}^{t} e^{\mathrm{T}}(s)\mathrm{d}s \quad \int_{t-h_2}^{t-h_1} e^{\mathrm{T}}(s)\mathrm{d}s\Big].$$

Let $\mathscr{L}$ be the weak infinitesimal generator of the random process $\{e(t), t \geq 0\}$. Then, for each $r(t) = i, i \in S$, it can be shown that

$$\begin{aligned}\mathscr{L}V(e(t),i) =\ & \mathscr{L}V_1(e(t),i) + \mathscr{L}V_2(e(t),i) + \mathscr{L}V_3(e(t),i) + \mathscr{L}V_4(e(t),i)\\ & + 2\dot{e}^{\mathrm{T}}(t)P_{1i}\Big[-(A_i + K_iC_i)e(t) + B_{1i}\phi(t) + B_{2i}\phi(t-\tau(t))\\ & + B_{3i}\int_{t-r(t)}^{t}\phi(s)\mathrm{d}s + E_i\dot{e}(t-\sigma(t)) - K_iD_i\psi(t) - \dot{e}(t)\Big], \end{aligned} \tag{9}$$

where

$$\begin{aligned}\mathscr{L}V_1(e(t),i) =\ & 2\begin{bmatrix} e(t) \\ \int_{t-h_2}^{t} e(s)\mathrm{d}s \\ \int_{t-h_2}^{t-h_1} e(s)\mathrm{d}s \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_{1i} & 0 & 0 \\ 0 & P_{2i} & 0 \\ 0 & 0 & P_{3i} \end{bmatrix} \begin{bmatrix} \dot{e}(t) \\ e(t) - e(t-h_2) \\ e(t-h_1) - e(t-h_2) \end{bmatrix}\\ & + \sum_{j=1}^{N}\gamma_{ij}\zeta^{\mathrm{T}}(t)P_j\zeta(t),\\ \mathscr{L}V_2(e(t),i) \leq\ & e^{\mathrm{T}}(t)Q_1e(t) - (1-\mu)e^{\mathrm{T}}(t-\tau(t))Q_1e(t-\tau(t)) + e^{\mathrm{T}}(t)Q_2e(t)\\ & - e^{\mathrm{T}}(t-h_1)Q_2e(t-h_1) + e^{\mathrm{T}}(t)Q_3e(t) - e^{\mathrm{T}}(t-h_2)Q_3e(t-h_2)\\ & + \dot{e}^{\mathrm{T}}(t)Q_4\dot{e}(t) - (1-\eta)\dot{e}^{\mathrm{T}}(t-\sigma(t))Q_4\dot{e}(t-\sigma(t))\\ & + \phi^{\mathrm{T}}(t)Q_5\phi(t) - (1-\mu)\phi(t-\tau(t))Q_5\phi(t-\tau(t),\\ \mathscr{L}V_3(e(t),i) \leq\ & \bar{r}\phi^{\mathrm{T}}(t)R_1\phi(t) - \int_{t-r(t)}^{t}\phi^{\mathrm{T}}(s)R_1\phi(s)\mathrm{d}s\\ & + \dot{e}^{\mathrm{T}}(t)(\bar{\sigma}R_2 + h_2R_3 + (h_2-h_1)R_4)\dot{e}(t)\\ & - \int_{t-\sigma(t)}^{t}\dot{e}^{\mathrm{T}}(s)R_2\dot{e}(s)\mathrm{d}s - \int_{t-h_2}^{t}\dot{e}^{\mathrm{T}}(s)R_3\dot{e}(s)\mathrm{d}s - \int_{t-h_2}^{t-h_1}\dot{e}^{\mathrm{T}}(s)R_4\dot{e}(s)\mathrm{d}s,\\ \mathscr{L}V_4(e(t),i) =\ & \dot{e}^{\mathrm{T}}(t)[h_2^4T_1 + (h_2^2-h_1^2)^2T_2 + \bar{\sigma}^4T_3]\dot{e}(t) - 2h_2^2\int_{t-h_2}^{t}\int_{\theta}^{t}\dot{e}^{\mathrm{T}}(s)T_1\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\\ & - 2(h_2^2-h_1^2)\int_{t-h_2}^{t-h_1}\int_{\theta}^{t}\dot{e}^{\mathrm{T}}(s)T_2\dot{e}(s)\mathrm{d}s\mathrm{d}\theta - 2\bar{\sigma}^2\int_{t-\bar{\sigma}}^{t}\int_{\theta}^{t}\dot{e}^{\mathrm{T}}(s)T_3\dot{e}(s)\mathrm{d}s\mathrm{d}\theta. \end{aligned}$$

From Lemma 1, we have

$$-\int_{t-r(t)}^{t}\phi^{\mathrm{T}}(s)R_1\phi(s)\mathrm{d}s \leq -\frac{1}{\bar{r}}\Big[\int_{t-r(t)}^{t}\phi(s)\mathrm{d}s\Big]^{\mathrm{T}}R_1\Big[\int_{t-r(t)}^{t}\phi(s)\mathrm{d}s\Big], \tag{10}$$

$$-\int_{t-\sigma(t)}^{t}\dot{e}^{\mathrm{T}}(s)R_2\dot{e}(s)\mathrm{d}s \leq -\frac{1}{\bar{\sigma}}\Big[e(t) - e(t-\sigma(t))\Big]^{\mathrm{T}}R_2\Big[e(t) - e(t-\sigma(t))\Big], \tag{11}$$

$$-\int_{t-\tau(t)}^{t}\dot{e}^{\mathrm{T}}(s)R_3\dot{e}(s)\mathrm{d}s \leq -\frac{1}{h_2}\Big[e(t) - e(t-\tau(t))\Big]^{\mathrm{T}}R_3\Big[e(t) - e(t-\tau(t))\Big], \tag{12}$$

$$\begin{aligned}-\int_{t-h_2}^{t-\tau(t)}\dot{e}^{\mathrm{T}}(s)(R_3+R_4)\dot{e}(s)\mathrm{d}s \leq\ & -\frac{1}{h_2-h_1}\Big[e(t-\tau(t)) - e(t-h_2)\Big]^{\mathrm{T}}(R_3+R_4)\\ & \times\Big[e(t-\tau(t)) - e(t-h_2)\Big], \end{aligned} \tag{13}$$

$$\begin{aligned}-\int_{t-\tau(t)}^{t-h_1}\dot{e}^{\mathrm{T}}(s)R_4\dot{e}(s)\mathrm{d}s \leq\ & -\frac{1}{h_2-h_1}\Big[e(t-h_1) - e(t-\tau(t))\Big]^{\mathrm{T}}R_4\\ & \times\Big[e(t-h_1) - e(t-\tau(t))\Big], \end{aligned} \tag{14}$$

$$\begin{aligned}-\int_{t-h_2}^{t}\int_{\theta}^{t}\dot{e}^{\mathrm{T}}(s)T_1\dot{e}\mathrm{d}s\mathrm{d}\theta \leq\ & -\frac{2}{h_2^2}\Big(\int_{t-h_2}^{t}\int_{\theta}^{t}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\Big)^{\mathrm{T}}T_1\Big(\int_{t-h_2}^{t}\int_{\theta}^{t}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\Big)\\ \leq\ & -\frac{2}{h_2^2}\Big(h_2e(t) - \int_{t-h_2}^{t}e(s)\mathrm{d}s\Big)^{\mathrm{T}}T_1\Big(h_2e(t) - \int_{t-h_2}^{t}e(s)\mathrm{d}s\Big), \end{aligned} \tag{15}$$

$$-\int_{t-h_2}^{t-h_1}\int_{\theta}^{t}\dot{e}^{\mathrm{T}}(s)T_2\dot{e}\mathrm{d}s\mathrm{d}\theta \le -\frac{2}{h_2^2-h_1^2}\Big(\int_{t-h_2}^{t-h_1}\int_{\theta}^{t}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\Big)^{\mathrm{T}}T_2\Big(\int_{t-h_2}^{t-h_1}\int_{\theta}^{t}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\Big)$$
$$\le -\frac{2}{h_2^2-h_1^2}\Big((h_2-h_1)e(t)-\int_{t-h_2}^{t-h_1}e(s)\mathrm{d}s\Big)^{\mathrm{T}}T_2$$
$$\times\Big((h_2-h_1)e(t)-\int_{t-h_2}^{t-h_1}e(s)\mathrm{d}s\Big), \quad (16)$$

$$-\int_{t-\bar{\sigma}}^{t}\int_{\theta}^{t}\dot{e}^{\mathrm{T}}(s)T_3\dot{e}\mathrm{d}s\mathrm{d}\theta \le -\frac{2}{\bar{\sigma}^2}\Big(\int_{t-\bar{\sigma}}^{t}\int_{\theta}^{t}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\Big)^{\mathrm{T}}T_3\Big(\int_{t-\bar{\sigma}}^{t}\int_{\theta}^{t}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\Big)$$
$$\le -\frac{2}{\bar{\sigma}^2}\Big(\bar{\sigma}e(t)-\int_{t-\sigma(t)}^{t}e(s)\mathrm{d}s\Big)^{\mathrm{T}}T_3\Big(\bar{\sigma}e(t)-\int_{t-\sigma(t)}^{t}e(s)\mathrm{d}s\Big). \quad (17)$$

For positive diagonal matrices $U>0$, $V>0$, and $W>0$, we can obtain from Assumption A that

$$\Upsilon_1 = \begin{bmatrix} e(t) \\ \phi(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} L_1U & -L_2U \\ -L_2U & U \end{bmatrix} \begin{bmatrix} e(t) \\ \phi(t) \end{bmatrix} \le 0, \quad (18)$$

$$\Upsilon_2 = \begin{bmatrix} e(t-\tau(t)) \\ \phi(t-\tau(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} L_1V & -L_2V \\ -L_2V & V \end{bmatrix} \times \begin{bmatrix} e(t-\tau(t)) \\ \phi(t-\tau(t)) \end{bmatrix} \le 0, \quad (19)$$

$$\Upsilon_3 = \begin{bmatrix} e(t) \\ \psi(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} L_3W & -L_4W \\ -L_4W & W \end{bmatrix} \begin{bmatrix} e(t) \\ \psi(t) \end{bmatrix} \le 0. \quad (20)$$

Using expressions (10)–(17) in Eq. (9), subtracting Eqs. (18)–(20) from Eq. (9) and using the relationship $P_{1i}K_i = X_i$, we have

$$\mathscr{L}V(e(t),i) \le \xi^{\mathrm{T}}(t)\Xi_i\xi(t), \quad (21)$$

where

$$\xi^{\mathrm{T}}(t) = \Big[e^{\mathrm{T}}(t)\ e^{\mathrm{T}}(t-\tau(t))\ e^{\mathrm{T}}(t-h_1)\ e^{\mathrm{T}}(t-h_2) \times \dot{e}^{\mathrm{T}}(t-\sigma(t))\ e^{\mathrm{T}}(t-\sigma(t))\ \dot{e}^{\mathrm{T}}(t)\ \phi^{\mathrm{T}}(t) \cdot \phi^{\mathrm{T}}(t-\tau(t))\ \psi^{\mathrm{T}}(t) \times \int_{t-r(t)}^{t}\phi^{\mathrm{T}}(s)\mathrm{d}s\ \int_{t-h_2}^{t}e^{\mathrm{T}}(s)\mathrm{d}s \times \int_{t-h_2}^{t-h_1}e^{\mathrm{T}}(s)\mathrm{d}s\ \int_{t-\sigma(t)}^{t}e^{\mathrm{T}}(s)\mathrm{d}s\Big].$$

Taking the mathematical expectation of both sides of expression (21), we obtain

$$\mathbb{E}\Big[\mathscr{L}V(e(t),i)\Big] \le \mathbb{E}(\xi^{\mathrm{T}}(t)\Xi_i\xi(t)) \le -\lambda_{\min}(-\Xi_i)\mathbb{E}|e(t,\xi)|^2. \quad (22)$$

Therefore, from Definition 1 and using the Lyapunov stability theory, it is concluded that the error-state system (6) is globally asymptotically stable in the mean square. This completes the proof of Theorem 1. □

Remark 1 Park *et al.*[25–27] discussed state estimation for neural networks of neutral-type with time-varying delays for the first time. Different from those papers, the state estimation problem for neural networks of neutral type with mixed time-varying delays and Markovian jumping parameters via LMI approach has been extended in this paper.

Suppose if we left Markovian jumping parameters in error-state system (6), we obtain the following system

$$\dot{e}(t) = -(A+KC)e(t)+B_1\phi(t) + B_2\phi(t-\tau(t)) + B_3\int_{t-r(t)}^{t}\phi(s)\mathrm{d}s + E\dot{e}(t-\sigma(t)) - KD\psi(t). \quad (23)$$

For system (23), we have the following result.

Corollary 1 For given scalars $h_2>h_1\ge 0$, $\bar{\sigma}$, $\bar{r}$, $\eta<1$, and $\mu<\infty$, the equilibrium point of error-state system (23) is globally asymptotically stable, if there exist positive-definite matrices $P=P^{\mathrm{T}}=\mathrm{diag}\{P_1,P_2,P_3\}>0$, $Q_l=Q_l^{\mathrm{T}}>0$ $(l=1,2,3,4,5)$, $R_k=R_k^{\mathrm{T}}>0$ $(k=1,2,3,4)$, $T_b=T_b^{\mathrm{T}}>0$ $(b=1,2,3)$, diagonal matrices $U>0$, $V>0$, $W>0$, and any matrix X such that the following LMI holds

$$\Pi_{14\times 14}<0, \quad (24)$$

where

$$\Pi_{1,1} = Q_1+Q_2+Q_3-P_1A-A^{\mathrm{T}}P_1-XC-C^{\mathrm{T}}X^{\mathrm{T}}-\frac{1}{\bar{\sigma}}R_2-\frac{1}{h_2}R_3$$

$$
\begin{aligned}
&-4h_2^2T_1-4(h_2-h_1)^2T_2-4\bar{\sigma}^2T_3-L_1U-L_3W,\\
&\Pi_{1,2}=\frac{1}{h_2}R_3,\quad \Pi_{1,5}=P_1E,\quad \Pi_{1,6}=\frac{1}{\bar{\sigma}}R_2,\quad \Pi_{1,7}=-A^{\mathrm{T}}P_1-C^{\mathrm{T}}X^{\mathrm{T}},\\
&\Pi_{1,8}=P_1B_1+L_2U,\quad \Pi_{1,9}=P_1B_2,\quad \Pi_{1,10}=-XD+L_4W,\quad \Pi_{1,11}=P_1B_3,\\
&\Pi_{1,12}=P_2^{\mathrm{T}}+4h_2T_1,\quad \Pi_{1,13}=4(h_2-h_1)T_2,\quad \Pi_{1,14}=4\bar{\sigma}T_3,\\
&\Pi_{2,2}=-(1-\mu)Q_1-\frac{1}{h_2}R_3-\frac{1}{h_2-h_1}(R_3+R_4)-L_1V-\frac{1}{h_2-h_1}R_4,\\
&\Pi_{2,3}=\frac{1}{h_2-h_1}R_4,\quad \Pi_{2,4}=\frac{1}{h_2-h_1}(R_3+R_4),\quad \Pi_{2,9}=L_2V,\quad \Pi_{3,3}=-Q_2-\frac{1}{h_2-h_1}R_4,\\
&\Pi_{3,13}=P_3^{\mathrm{T}},\quad \Pi_{4,4}=-Q_3-\frac{1}{h_2-h_1}(R_3+R_4),\quad \Pi_{4,12}=-P_2^{\mathrm{T}},\quad \Pi_{4,13}=-P_3^{\mathrm{T}},\\
&\Pi_{5,5}=-(1-\eta)Q_4,\quad \Pi_{5,7}=E^{\mathrm{T}}P_1^{\mathrm{T}},\quad \Pi_{6,6}=-\frac{1}{\bar{\sigma}}R_2,\\
&\Pi_{7,7}=Q_4+\bar{\sigma}R_2+h_2R_3+(h_2-h_1)R_4+h_2^4T_1+(h_2^2-h_1^2)^2T_2+\bar{\sigma}^4T_3-P_1-P_1^{\mathrm{T}},\\
&\Pi_{7,8}=P_1B_1,\quad \Pi_{7,9}=P_1B_2,\quad \Pi_{7,10}=-XD,\quad \Pi_{7,11}=P_1B_3,\quad \Pi_{8,8}=Q_5+\bar{r}R_1-U,\\
&\Pi_{9,9}=-(1-\mu)Q_5-V,\quad \Pi_{10,10}=-W,\quad \Pi_{11,11}=-\frac{1}{\bar{r}}R_1,\quad \Pi_{12,12}=-4T_1,\\
&\Pi_{13,13}=-4T_2,\quad \Pi_{14,14}=-4T_3,
\end{aligned}
$$

and other entries of Π are zeros.

Moreover, the state estimator gain matrix is given by $K=P_1^{-1}X$.

Proof The remaining parts of the proof immediately follows from Theorem 1. □

In order to show the effectiveness of our method, we have the following two examples.

Example 1 Consider system (6) with the following parameters

$$
A_1=\begin{bmatrix}8.2 & 0\\ 0 & 7.2\end{bmatrix},\quad A_2=\begin{bmatrix}6.7 & 0\\ 0 & 7\end{bmatrix},
$$

$$
B_{11}=\begin{bmatrix}-1.2 & 0.9\\ 0.5 & -1\end{bmatrix},\quad B_{12}=\begin{bmatrix}0.7 & 0.5\\ 0.8 & -1\end{bmatrix},
$$

$$
B_{21}=\begin{bmatrix}-0.5 & 0.4\\ -1 & -0.5\end{bmatrix},\quad B_{22}=\begin{bmatrix}-0.5 & 0.6\\ -0.9 & -0.5\end{bmatrix},
$$

$$
B_{31}=\begin{bmatrix}0.9 & 1.1\\ 0.3 & 0.4\end{bmatrix},\quad B_{32}=\begin{bmatrix}0.6 & 1\\ 0.5 & 0.9\end{bmatrix},
$$

$$
C_1=\begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix},\quad C_2=\begin{bmatrix}0.35 & 0\\ 0 & 0.35\end{bmatrix},
$$

$$
D_1=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\quad D_2=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},
$$

$$
E_1=E_2=\begin{bmatrix}-0.1 & 0\\ 0 & 0.1\end{bmatrix},
$$

$$
\Gamma=\begin{bmatrix}-0.1 & 0.1\\ 0.2 & -0.2\end{bmatrix},\quad J_1=J_2=\begin{bmatrix}0.5\\ 0.5\end{bmatrix}.
$$

The activation function satisfies Assumption A with $\phi^-=-0.5I$, $\phi^+=0.4I$, and the nonlinear disturbance also satisfies Assumption A with $\psi^-=0.2I$, $\psi^+=0.4I$. Thus, we can obtain the following parameters

$$
L_1=\begin{bmatrix}-0.2 & 0\\ 0 & -0.2\end{bmatrix},\quad L_2=\begin{bmatrix}-0.05 & 0\\ 0 & -0.05\end{bmatrix},
$$

$$
L_3=\begin{bmatrix}0.08 & 0\\ 0 & 0.08\end{bmatrix},\quad L_4=\begin{bmatrix}0.3 & 0\\ 0 & 0.3\end{bmatrix}.
$$

Let $h_1=1$, $h_2=1.3$, $\bar{r}=0.2$, $\bar{\sigma}=0.5$, $\mu=1.2$, $\eta=0.5$. By making use of MATLAB LMI toolbox to solve the LMIs in Theorem 1, we obtain the corresponding gain matrices as follows:

$$
K_1=P_{11}^{-1}X_1=\begin{bmatrix}11.1513 & -0.8626\\ -0.8626 & 13.4529\end{bmatrix},
$$

$$
K_2=P_{12}^{-1}X_2=\begin{bmatrix}11.5485 & 0.0358\\ 0.0358 & 15.2435\end{bmatrix}.
$$

Therefore, it follows from Theorem 1, that the error-state system (6) is globally asymptotically stable in the mean square.

Example 2 Consider the error-state system (23)

with the following parameters

$$A = \begin{bmatrix} 5.2 & 0 \\ 0 & 4.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 & -1 \\ 0.7 & -1.3 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 1.1 & 0.8 \\ -1 & -0.5 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.9 & 1.1 \\ 0.3 & 0.45 \end{bmatrix},$$
$$E = \begin{bmatrix} -0.15 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad J = \begin{bmatrix} 2\sin t + 0.03t^2 \\ 2\cos t - 0.03t^2 \end{bmatrix}.$$

Let the activation function be $g(x(t)) = \tanh x(t)$ and then, Assumption A yields $\phi^- = 0$, $\phi^+ = I$. Thus, we can obtain the following parameters

$$L_1 = \mathrm{diag}\{0, 0\}, \quad L_2 = \mathrm{diag}\{0.5, 0.5\}.$$

The parameters for output signals of the networks are given as

$$C = \begin{bmatrix} 0.25 & 0 \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$L_3 = \begin{bmatrix} 0.08 & 0 \\ 0 & 0.08 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

In this example, the nonlinear disturbance is taken in the form $f(t, x(t)) = 0.1\cos x(t) + 0.3$. Assumption A gives $\psi^- = 0.2I$, $\psi^+ = 0.4I$. Let $h_1 = 0.5$, $h_2 = 1.5$, $\bar{r} = 0.5$, $\bar{\sigma} = 0.3$, $\mu = 0.5$, $\eta = 0.2$. By solving the LMIs in Corollary 1, we obtain the corresponding gain matrix as follows:

$$K = P_1^{-1}X = \begin{bmatrix} 51.1804 & -42.0186 \\ -42.0186 & 53.4178 \end{bmatrix}.$$

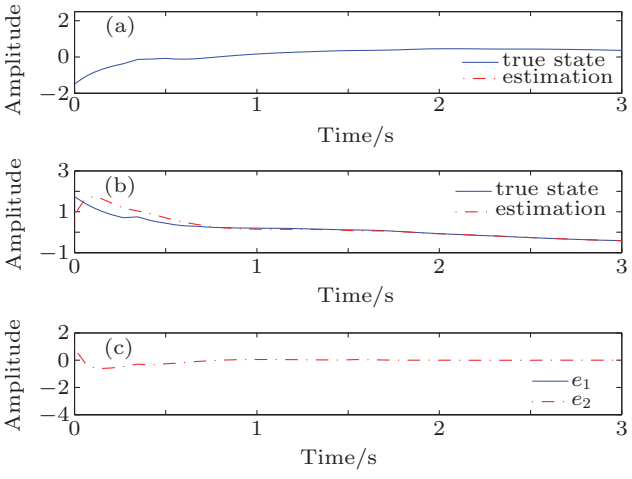


Fig. 1. (colour online) The error trajectories are converging to zero in Example 1: (a) state x_1 and its estimation, (b) state x_2 and its estimation, (c) error states.

The responses of the state dynamics for the error-state system (23) which converges to zero with the above estimator gain matrix K is shown in Fig. 1. Therefore, it follows from Corollary 1 and Fig. 1, that the error-state system (23) is globally asymptotically stable.

4. Conclusions

In this paper, we have dealt with the problem of state estimation for neutral-type neural networks with mixed time-varying delays and Markovian jumping parameters. By defining a Lyapunov–Krasovskii functional, LMI-based stability conditions are derived and the desired state estimators are designed. The effectiveness of the proposed criteria is demonstrated through numerical examples with simulation results. On the other hand, the passivity theory is an another effective tool to analyze the stability of a nonlinear system. Passivity framework is a promising approach to the stability analysis of neural networks. By taking this into account, state estimation of neural networks based on the passivity theory may be considered in the near future.

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