The excluded-volume effect in microscopic pedestrian simulations*

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We propose a pedestrian position update rule, which is added to a microscopic pedestrian model to avoid pedestrian overlap. In the rule, the step size of a pedestrian moving in a selected direction at each update is in inverse proportion to the repulsive actions imposed by other pedestrians moving in a direction with an exponential rate. The positions of the pedestrians are then updated in each small time interval. In this way, a barrier between the pedestrians can be generated, and after updating their positions the pedestrians do not overlap with each other. The modified model is compared to the original model through a simulation of the evacuation process of pedestrians in a closed area. The simulation results indicate that the modified model is superior to the original model in several aspects.

Keywords: pedestrian model, excluded-volume effect, discrete dynamic system

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1. Introduction

Understanding pedestrian behaviour in public facilities, such as stations, airports, squares and pavements, is very important when designing and improving such areas. Recently, the pedestrian flow problem has been investigated through mathematical models and computer simulations,[1,2] which play an important role in shedding light on and gaining an insight into the problem, and help make decisions in maintaining the level of service and assuring pedestrian safety in public facilities during accidents caused by crowds.

In general, pedestrian flow models are classified into two categories. One is continuum macroscopic models,[3−5] which are usually applied to a large crowd and treat the crowd as a whole or a fluid, formulating the flow conservation law in the form of partial differential equations. However, this class of models often oversimplifies and undermines inherent real-world complexity. The other category is the individual-based microscopic models, in which each pedestrian is considered as a discrete individual and the positional update of each pedestrian is formulated by a continuous or discrete dynamic system. This class of models mainly includes the social force model,[6−8] the cellular automata model,[9−17] the lattice gas model,[18−21] the optimal control model[22] and the discrete choice model.[23,24]

In reality, each pedestrian occupies a certain area of space. Thus, the excluded-volume effect[25,26] needs to be considered in microscopic pedestrian models. By the excluded-volume effect, we mean that each pedestrian is modeled as an entity with a certain size, and avoiding pedestrian overlap needs to be considered. Pedestrians are not allowed to either walk through each other or share the same space. Here, avoiding does not mean absolute avoidance. In fact, pedestrians do not have rigid bodies. In the process of movement, their figures and sizes are likely to be squeezed due to the actions of others, and the distance between them become less than the sum of their radii.

For some existing microscopic pedestrian models, e.g., the optimal control model[22] and the discrete choice model,[23,24] the excluded-volume effect is not taken into account and pedestrians are treated as particles, and hence they cannot guarantee that the positions of the pedestrians do not overlap with each other after the position update. When a number of pedestrians congregate near a bottleneck, a higher local density may occur. In this case, those models may

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become invalid. In the other models, the excluded-volume effect is implemented in the process of simulation through two classes of approaches.

In the first approach, it is determined whether the new position of each pedestrian after the position update is occupied by other pedestrians or by obstacles. In the microscopic pedestrian models with discrete space representations, such as the cellular automata model and the lattice gas model, the pedestrian space is discretized into square or hexagon lattices.[27] In each iterative time step, the positions of all the pedestrians are updated in a random sequence or synchronously. If the positions are updated in a random sequence, each pedestrian is only allowed to move into lattice sites that are not occupied by other pedestrians or obstacles.[14] If the pedestrian positions are updated synchronously, not only does each pedestrian's movement into an empty position need to be satisfied, but also the situation where two pedestrians target the same position, random one moves and the other one remains stationary should be avoided.[10] In the models with continuous space representations,[28,29] after updating the position of each pedestrian, a guarantee needs to made that the distances between the centre of every two pedestrians are more than the sum of their radii.

In the second approach, repulsive actions between pedestrians are defined and used to create an impenetrable potential barrier between any two pedestrians, thereby automatically guaranteeing the excluded-volume effect. In models using this approach,[6−8,30−32] the individual movement is generally formulated by the dynamic system, and the displacement or the velocity of the pedestrian is inversely related to the repulsive action between the pedestrians. It is assumed that the repulsive action between the pedestrians increases as the distance between them shortens, and when two pedestrians contact or are close to each other, the actions are very strong. In this way, a barrier between the pedestrians is generated so that after their movement at a very small time step, they do not overlap with each other. In fact, after their position update at each very small time step, if two pedestrians are very close to each other, their velocities become very small. Therefore, the repulsive actions between them take effect and prevent them from further approaching each other at the next time step.

In Ref. [29], a microscopic pedestrian model has been proposed. In the model, an update rule of the pedestrian position, which belongs to the first class of approaches, has been used to implement the excluded-volume effect and avoid pedestrian overlapping. The purpose of the present paper is to propose another update rule of pedestrian position, which belongs to the second class of approaches, and is added into the model to implement the excluded-volume effect. Comparing the two update rules is very important to understand the pedestrian microscopic model. Thus, both the original and the modified models are applied to simulate the evacuation of pedestrians in a specific scenario, and the results are compared. Using the two models, the effects of model parameters on the efficiency of evacuation and the distribution of pedestrians in a space during evacuation are investigated. We also investigate how the initial pedestrian distribution in a space affects the distribution of pedestrian step velocity using the modified model. In the next section, we will first introduce the model briefly, and then propose a position update rule belonging to class two.

2. The microscopic pedestrian model

In the microscopic pedestrian model, each pedestrian first determines his or her direction of movement, and then moves a certain step size in that direction at each time step, Δt. The model considers the size of the pedestrians, and is thus suited for simulating the movement of dense crowds. First, the formula for selecting the movement direction is introduced.

2.1. Formula for selecting movement direction

The current position and possible direction of movement of a pedestrian are shown in Fig. 1. In the figure, pedestrian n, denoted by the grey circle, is moving in the desired direction e_n. The radius of pedestrian n is r_n. It is assumed that the movement of this pedestrian is affected by the movements of others and by the obstacles in a surrounding circular area, the radius of which is $R_n = 10r_n$. The pedestrian can move along direction d_i^n $(i = 1, \ldots, 12)$ or remain stationary. The angles β_i^n $(i = 1, \ldots, 12)$ between the 12 directions and the desired direction e_n are π, $3\pi/4$, $\pi/2$, $3\pi/8$, $\pi/4$, $\pi/8$, 0, $-\pi/8$,$-\pi/4$, $-3\pi/8$, $-\pi/2$ and

$-3\pi/4$, respectively.

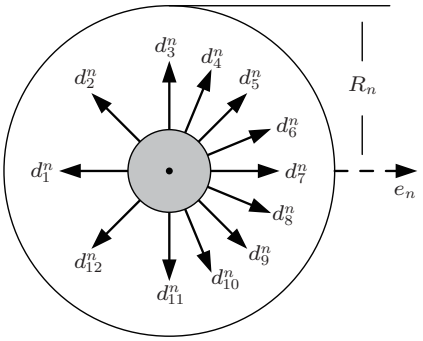


Fig. 1. The possible movements of a pedestrian.

The probability of moving along each of the 12 directions is denoted by P_i^n $(i = 1, \ldots, 12)$, and the probability of remaining stationary is denoted by P_0^n. The effect that pedestrian m has on the movement of pedestrian n in the directions toward pedestrian m is illustrated in Fig. 2. If direction d_i^n of pedestrian n is in the radial cone between radials l_1 and l_2, with parallel segments l_3 and l_4, respectively, then his or her movement in that direction will be affected. The effect of a wall on pedestrian n can be defined in a similar fashion. If the distance between a wall and pedestrian n is less than $R_n - r_n$, then the wall will affect the movement of pedestrian n in the directions towards the wall. As shown in Fig. 3, the movement of pedestrian n in the directions in the radial cone between radials l_1 and l_2 will be affected by the wall. If the movement of pedestrian n in a direction is affected by another pedestrian or a wall, then the latter imposes a repulsive action on the former in that direction.

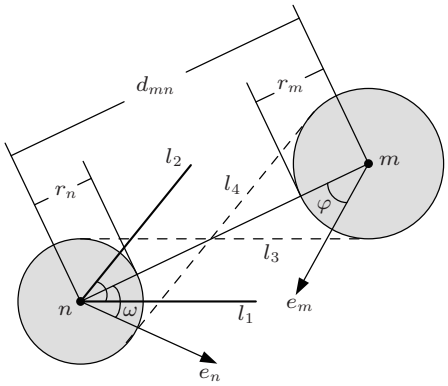


Fig. 2. The actions of two pedestrians.

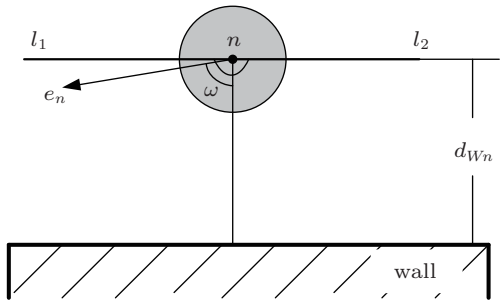


Fig. 3. The actions of a pedestrian encountering a wall.

The following logit-based formulae are used to compute movement probability P_i^n:

$$P_i^n = \frac{\exp\left[\lambda\left(\max\{0, \cos\beta_i^n\} - \sum\limits_m f_i^{mn} - \sum\limits_W f_i^{Wn}\right)\right]}{1 + \sum\limits_{k=1}^{12} \exp\left[\lambda\left(\max\{0, \cos\beta_k^n\} - \sum\limits_m f_k^{mn} - \sum\limits_W f_k^{Wn}\right)\right]}, \quad i = 1, \cdots, 12, \tag{1}$$

$$P_0^n = \frac{1}{1 + \sum\limits_{k=1}^{12} \exp\left[\lambda\left(\max\{0, \cos\beta_k^n\} - \sum\limits_m f_k^{mn} - \sum\limits_W f_k^{Wn}\right)\right]}, \tag{2}$$

where deviation parameter λ (≥ 0) represents the degree of deviation from the desired direction. A higher λ value indicates a smaller deviation, which means that the pedestrian is more likely to move in directions that are closer to the desired direction. The β_i^n is the angle between direction d_i^n and desired direction e_n, f_i^{mn} and f_i^{Wn} are the repulsive actions between pedestrian m and wall W, which affect the movement of pedestrian n in direction d_i^n according to the aforementioned rules.

The f_i^{mn} is given by

$$f_i^{mn} = \begin{cases} \dfrac{\tau(\max\{0, \cos\omega\} + \xi)(\max\{0, \cos\phi\} + \varepsilon)}{\max\{d_{mn} - r_m - r_n, \delta\}}, & \text{if } d_i^n \text{ is in the radial cone} \\ & \text{between radials } l_1 \text{ and } l_2, \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where τ (≥ 0), ξ (≥ 0) and ε (≥ 0) are the intensity parameters, ω is the angle between the desired direction of pedestrian n and the direct path to pedestrian m, ϕ is the angle between the desired direction of pedestrian m and the direct path to pedestrian n, d_{mn} denotes the distance between pedestrians m and n, and r_m and r_n are the radii of pedestrians m and n, respectively (see Fig. 2). Parameter δ is a positive number approaching zero to ensure that the denominator is larger than zero.

The f_i^{Wn} is given by

$$f_i^{Wn} = \begin{cases} \dfrac{\tau(\max\{0, \cos\omega\} + \xi)(1+\varepsilon)}{\max\{d_{Wn} - r_n, \delta\}}, & \text{if } d_i^n \text{ is in the radial cone} \\ & \text{between radials } l_1 \text{ and } l_2, \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

where ω is the angle between desired direction e_n of pedestrian n and the direct path to wall W, and d_{Wn} denotes the distance between pedestrian n and the wall (see Fig. 3). We then introduce a rule to determine the step size.

2.2. The determination of step size

Once the direction of movement has been determined, the pedestrian selects a step size. There are four possible choices of step size $s_n : \kappa_n \Delta t$, $2\kappa_n \Delta t/3$, $\kappa_n \Delta t/3$, and 0, where κ_n is the free velocity of pedestrian n. A longer step is the preferred choice among the four possibilities. It is necessary to guarantee that after each position update, the position of a pedestrian does not overlap the positions of the other pedestrians or the obstacles. We thus regulate the pedestrian selection of step size so that points c_i ($i = 1, 2, 3$) (see Fig. 4) do not overlap with areas occupied by the others. The angles between the direction from the centre of the dashed circle in Fig. 4 to points c_i ($i = 1, 2, 3$) and the movement direction are $\pi/4$, 0 and $-\pi/4$, respectively.

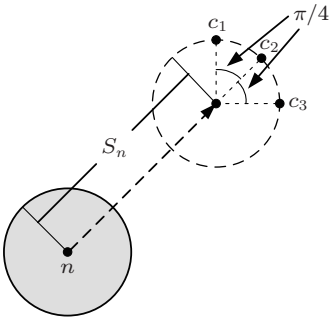


Fig. 4. Illustration of the movement of pedestrian n from one position to another for a given time step. The grey circle represents a pedestrian, and the dashed circle represents the possible movement of the pedestrian.

At each time step, the positions of all the pedestrians are updated synchronously. If, after the movement of pedestrians n and m, the distance between them is less than $r_n + r_m$, then we consider the two pedestrians to be targeting the same position, in which case their positions will overlap. We then assume that randomly one of them will move and the other will remain at his or her current position. In the following section, we propose a new rule for determining the step size of pedestrian movement.

2.3. New rule for determining step size

Here, a new rule for pedestrian position update, which is used to implement the excluded-volume effect, is proposed. At each time step, the positions of all pedestrians are updated synchronously. If pedestrian n determines his or her movement direction d_i^n, he or she will move a step size s_n in that direction, which is formulated as

$$s_n = \kappa_n \Delta t \exp\left[\theta\left(-\sum_m f_i^{mn} - \sum_W f_i^{Wn}\right)\right], \tag{5}$$

where θ (≥ 0) is an intense parameter. Equation (5) indicates that step size s_n is in inverse proportion to the repulsive actions, which are imposed by the other pedestrians and the obstacles affecting his or her movement in direction d_i^n at an exponential rate. At the same time, equation (5) also means that the intensity of the effect of the repulsive actions on step size is decided by parameter θ. When θ takes a relatively large value, step size becomes more sensitive to the variations of the repulsive actions.

To ensure that the excluded-volume effect is implemented and the pedestrians do not overlap with each other after the position update, the iterative time step should take a relatively small value. In this way, each pedestrian moves a small step, which is not more than a given value at each time step. When a pedestrian is very close to the other pedestrians or the obstacles in his or her movement direction, the repulsive

actions imposed by the others will be strong. As a result, the pedestrian only moves a very small step, even close to zero. Thus, their further closeness at the next time step is prevented, and any overlapping among them can be avoided.

The social force model[6,7] uses differential equation systems to describe the continuous movement of pedestrians in space and time. The group of equations must be solved by numerical methods. The iteration of position update also belongs to the second approach class, implementing the excluded-volume effect. However, the step size of the pedestrian is sensitive to the iterative time step, and a small time step may also lead to a very large step size, along with the occurrence of unreasonable pedestrian movement. In our model, the maximum movement distance of the pedestrians in each time step is limited, and hence the variation in the update time step in a proper range only affects the degree of overlap among the pedestrians.

3. Simulation results

The simulation scenario is as follows. A given number of pedestrians are to be evacuated from a room 20 ×15 m (see Fig. 5). The room has a 1.4 m wide exit in the north wall and the position coordinate on the left-hand side of the exit is (9.3, 15). The evacuation in the scenario is simulated by both the original and the modified models. The input data for both the models are identical: the free velocity $\kappa_n = 1.3$ m/s, the pedestrian radius $r_n = 0.2$ m, the deviation parameter $\lambda = 5$, and the intensity parameters $\tau = 0.1$, $\xi = 0.02$ and $\varepsilon = 0.2$.

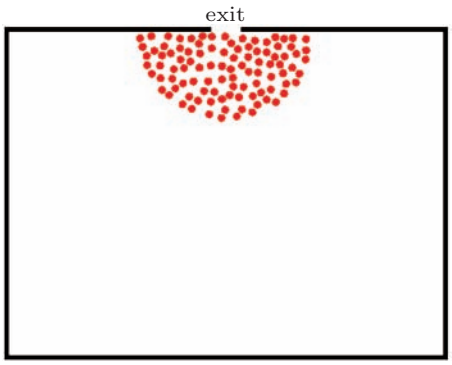


Fig. 5. The simulation scenario and a type of initial random pedestrian distribution in the semi-circular region with a radius of 4 m near the exit.

First, we investigate the influences of intensity parameter θ on the modified model, and the time step length in both models on the efficiency of pedestrian evacuation and the distribution of pedestrians in the space during the evacuation. The initial number of pedestrians in the room is 100, and they are randomly distributed in the semi-circular region with a radius of 4 m near the exit (see Fig. 5).

Figure 6 displays the evolutionary trend of the number of pedestrians staying in the room during

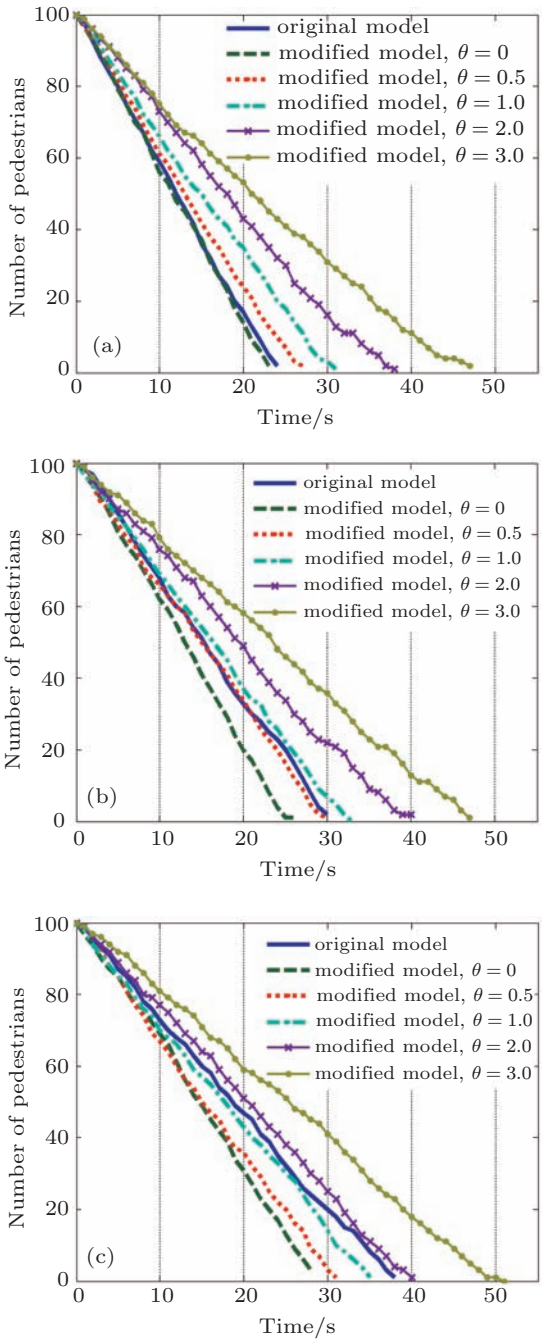


Fig. 6. (colour online) The evolutionary trends of the number of pedestrians staying in a room during evacuation when the time step takes the values of (a) 0.05 s, (b) 0.1 s and (c) 0.2 s in both models. Intensity parameter θ takes the values of 0, 0.5, 1, 2 and 3 in the modified model.

the evacuation when time step Δt takes the values of 0.05 s, 0.1 s and 0.2 s in both the models, respectively. Intensity parameter θ takes the values of 0, 0.5, 1, 2, and 3 in the modified model.

We can see that for the original model, as the time step increases the evacuation efficiency declines, and the pedestrians in the room need more time to leave. For the modified model with a certain θ value, as the time step increases the decline in evacuation efficiency can also be observed. However, the decline in the latter is not obvious when compared with that for the former. This can be understood by comparing the rules for determining the step size in the two models. In the original model, there are four possible choices of step size: 0, $\kappa_n \Delta t/3$, $2\kappa_n \Delta t/3$ and $\kappa_n \Delta t$, while in the modified model the step size is a value in interval $(0, \kappa_n \Delta t]$. During the evacuation, the pedestrians congregate near the exit, a dense crowd occurs, and the distances between the pedestrians become smaller. To avoid overlapping, the pedestrians have to move with relatively small step sizes. For the original model, as the time step increases, the optional non-zero step size of pedestrian movement at each time step also increases. Thus, they have to remain stationary, the movements of the pedestrians become more inflexible, and the evacuation becomes more inefficient. For the modified model, as the time step increases, the variation in the flexibility of pedestrian movement is not obvious. This indicates that the evacuation efficiency simulated by the modified model is less sensitive to the variation in the update time step than that simulated by the original model.

It can also be seen that for the modified model with a fixed time step, with the increase in parameter θ the evacuation efficiency diminishes. By adjusting the θ value, the evolutionary trend of the number of pedestrians obtained by the original model can be approximated by that obtained by the modified model. Therefore, the original model is extended, and the modified model can formulate the evolutionary trend of the number of pedestrians staying in the room during the evacuation simulated by the original model by adjusting the model parameters.

The snapshots at the third second in the simulations with different groups of parameters are displayed in Fig. 7. It can be seen that in the case where the time step equals 0.05, there is no overlapping of pedestrians in the original model. In this case, some neighbouring pedestrians overlap with each other, and the distances between them are less than the sum of their radii in the modified model if $\theta = 0$, and as parameter θ increases the distances between the neighbouring pedestrians widen. When parameter θ is fixed to be 1 in the modified model, as the time step increases the degree of overlap among the pedestrians becomes higher. This illustrates that the modified model can be used to formulate the degree of pedestrian overlap in the crowd by adjusting the model parameters.

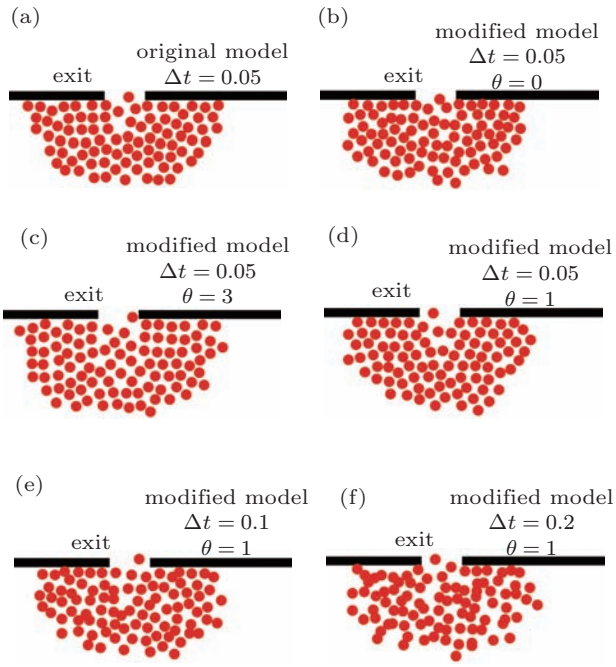


Fig. 7. Snapshots at the third second simulated by (a) the original model and (b)–(f) the modified model with parameters as shown in the plots.

Furthermore, the distances between each pedestrian and the one closest to him or her at each time step during the interval between 6 and 20 s in the simulations with different groups of parameters Δt and θ are recorded.

Table 1 shows the means and variances of these distances. It is found that for the original model and the modified model with a certain θ value, as the time step increases the means of the distances increase and decrease, respectively. For the original model, the means are larger than the pedestrian diameter. For the modified model, when Δt is smaller than a certain value, the means are larger than the pedestrian diameter, and when Δt is larger than that value, the means become smaller than that diameter. For both models,

with the time step increasing the variances of the distances also increase. For the modified model with a fixed time step, as parameter θ increases the means and the variances of the distances increase and decrease, respectively. Through observing the variances of the distances, we find that in the original model the distances between the pedestrians are almost uniform, however, in the modified model the variable range of the distances is larger. Therefore, compared with the original model, the modified model simulates the distribution of the pedestrians in the space in a very natural way.

Table 1. The means and variances of the distances between each pedestrian and the one closest to him or her at each time step during the interval between 6 and 20 s in the simulations with different groups of parameters. The two values in parentheses are the mean and variance, respectively, in metres.

Δt	Original model	Modified model with θ equaling				
		0	0.5	1	2	3
0.05	(0.41,0.0008)	(0.39,0.0016)	(0.40,0.0014)	(0.41,0.0012)	(0.42,0.0012)	(0.43,0.0012)
0.1	(0.41,0.0012)	(0.35,0.0045)	(0.37,0.0036)	(0.38,0.0032)	(0.40,0.0029)	(0.41,0.0026)
0.2	(0.42,0.0021)	(0.29,0.0127)	(0.31,0.0117)	(0.32,0.0100)	(0.35,0.0091)	(0.37,0.0076)
0.25	(0.43,0.0023)	(0.27,0.0167)	(0.29,0.0144)	(0.30,0.0148)	(0.33,0.0126)	(0.35,0.0119)

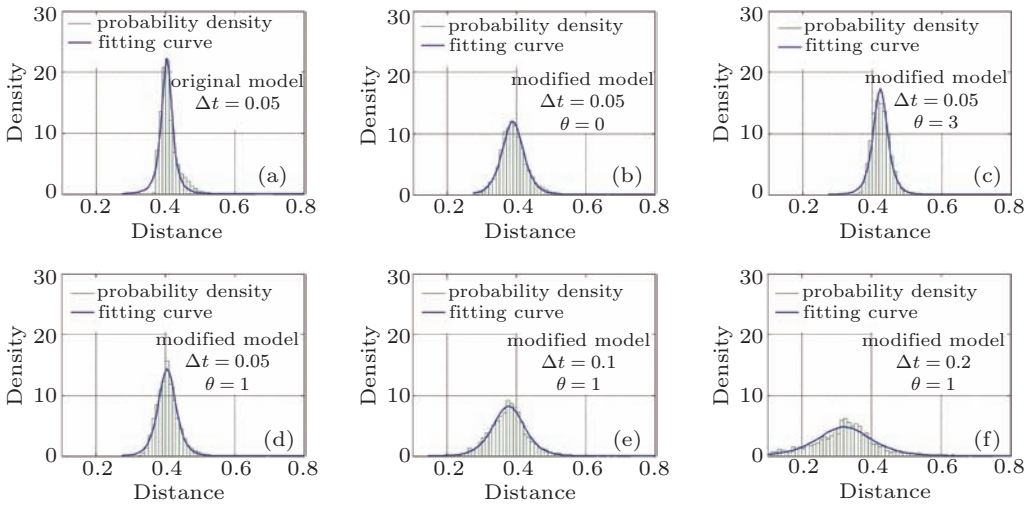


Fig. 8. (colour online) The probability densities and corresponding fitted curves of the distances between each pedestrian and the one closest to him or her at each time step during the interval between 6 and 20 s obtained in the simulations of (a) the original model and (b)–(f) the modified model with the parameters shown in the plots.

Figure 8 shows the probability densities and the corresponding fitting curves of the distance between each pedestrian and the one closest to him or her at each time step during the interval between 6 and 20 s in the simulations with different groups of parameters, Δt and θ. These probability densities are fitted by the t location-scale distribution, which has the density function

$$f(x) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sigma\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left[\frac{v + \left((x-\mu)^2/\sigma^2\right)^2}{v}\right]^{-\left(\frac{v+1}{2}\right)}, \tag{6}$$

where μ is the location parameter, $\sigma > 0$ is the scale parameter, $v > 0$ is the shape parameter and Γ is the Gamma function. The t location-scale distribution is useful for modeling data distributions with heavier tails (more prone to outliers). The means, variances and parameters of these fitting distributions can be seen in Table 2. Comparing the means and the variances of the distances in Tables 1 and 2, we find that these values approach each other very well for each group of parameters. This means that the distributions of the distances between each pedestrian and the one closest to him or her obtained by either the original model or the modified model can be fitted by the t location-scale distribution.

Table 2. The means, variances and parameters of fitting probability distributions of the distance between each pedestrian and the one closest to him or her at each time step during the interval between 6 and 20 s in the simulations with different groups of parameters. Data are fitted by the t location-scale distribution.

Model parameters	Original model	Modified model				
Δt	0.05	0.05	0.05	0.05	0.1	0.2
θ	–	0	1	3	1	1
Mean	0.40	0.39	0.40	0.42	0.38	0.32
Variance	0.0016	0.0016	0.0012	0.0010	0.0032	0.0106
μ	0.4031	0.3891	0.4043	0.4233	0.3770	0.3212
σ	0.0161	0.0316	0.0262	0.0216	0.0464	0.0787
ν	2.3906	5.2290	4.6023	3.7329	5.9379	4.8442

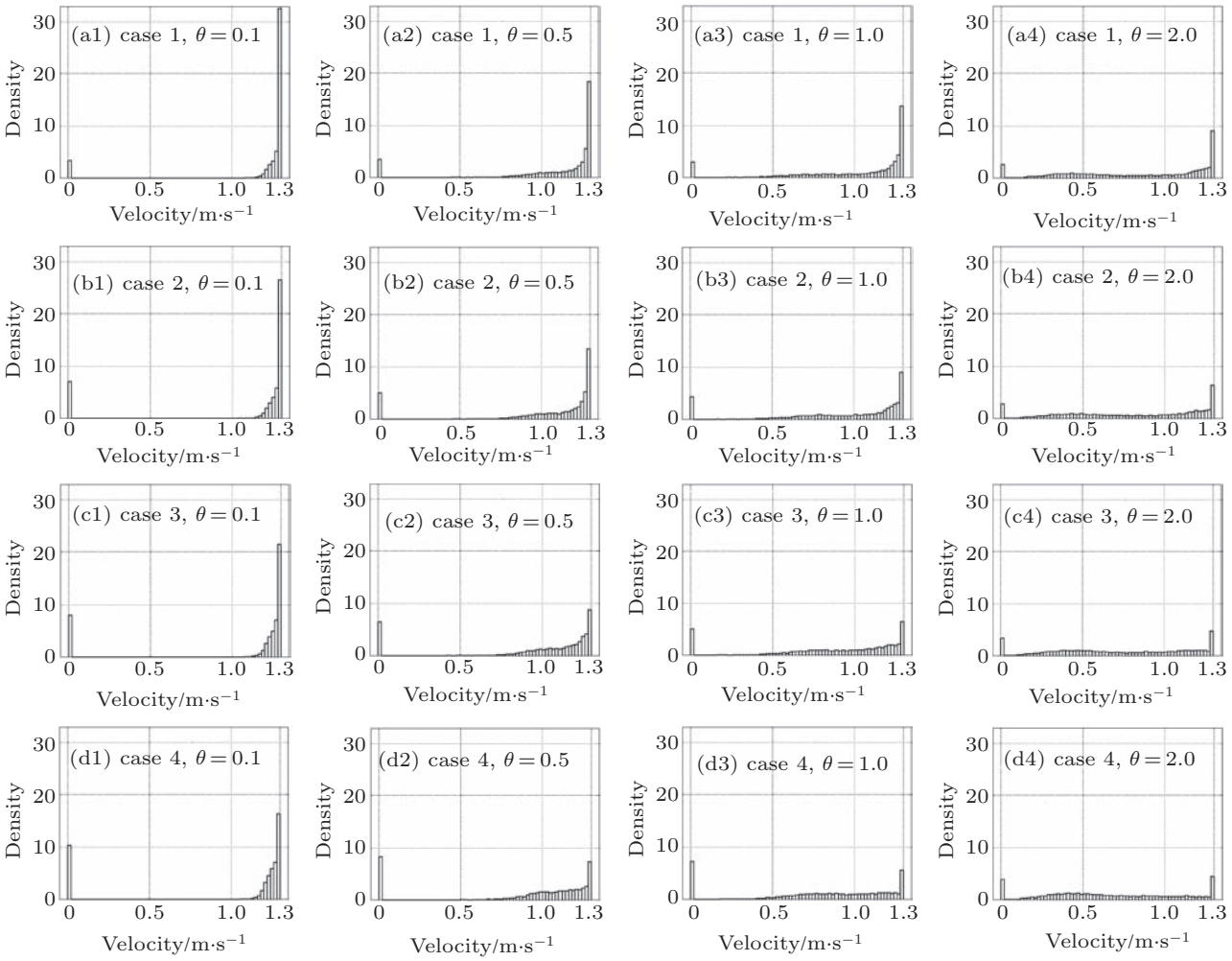


Fig. 9. The probability densities of pedestrian step velocity during evacuation. The results shown in panels (a)–(d) are obtained for four cases using the modified model with different parameters, respectively.

The step velocity of a pedestrian is usually described as a stochastic variable with a certain distribution. The distribution plays an important role in formulating pedestrian flow models. Insights into distribution are also important from the viewpoint of the design of facilities and public transport timetables. In Ref. [29], the probability densities of pedestrian free step velocities were fitted by the lognormal distribution. Henderson[33] fitted the distributions of pedestrian velocities in several crowds with sufficiently small pedestrian densities by the Maxwell–Boltzmann distribution. Fruin[34] observed pedestrian walking

velocities within transportation terminals under the free-flow conditions, and found that the distribution of pedestrian velocity is bell-shaped with a peak at a certain value. Young[35] found that the walking velocities of pedestrian movements within airport terminal corridors satisfy the normal distribution.

We investigated how the initial pedestrian distribution in a room affects the distribution of pedestrian step velocity using the modified model. Here, the time step is $\Delta t = 0.05$ s and the initial number of pedestrians in the room is 50. Four pedestrian distribution cases are considered, that is, 50 pedestrians are randomly distributed in a semi-circular region with radii of 10, 8, 6 and 4 m near the exit in cases 1, 2, 3 and 4, respectively. Figure 9 shows the probability densities of pedestrian step velocity during an evacuation, when the values of intensity parameter θ take 0.1, 0.5, 1 and 2 for the four cases. We can see that the distributions of step velocity are somewhat bimodal, with a portion of the pedestrians traveling with step velocities near the free velocity and a portion of the pedestrians with a step velocity of zero. For each of the four cases, with an increase in the θ value, the proportion of the former portion ascends and that of the latter portion descends. The peak of the former portion also becomes lower and the tail of the former portion becomes heavier. For each fixed θ value, as the radius of the semi-circular region decreases, that is, the pedestrians become denser, the proportion of the former portion descends and that of the latter portion ascends.

4. Conclusions

In this paper, a pedestrian position update rule is proposed to formulate the excluded-volume effect in the microscopic pedestrian simulation. The rule is compared to that given in Ref. [29] by simulating an evacuation process of pedestrians in a closed area. The simulation results indicate that the evacuation efficiency simulated by the modified model is less sensitive to the variation in update time step than that simulated by the original model. The modified model can formulate the evolutionary trend of the number of pedestrians staying in the room during the evacuation simulated by the original model by adjusting the model parameters. The modified model can also be used to formulate the degree of pedestrian overlap in a crowd by adjusting the model parameters. The distributions of the distances between each pedestrian and the one closest to him or her, obtained by either the original model or the modified model, can be fitted by the t location-scale distribution. The distribution of pedestrian step velocity in a relatively dense crowd is somewhat bimodal, with a portion of the pedestrians walking with step velocities near the free velocity and a portion with a step velocity of zero.

The modified model is superior to the original model in several aspects. First, there are four possible choices of step size in the original model, while in the modified model the step size is a value in an interval. Therefore, the pedestrian velocity formulated by the modified model is more consistent with that in found reality. Due to the inflexible movement of pedestrians in the original model, it may also not be suitable for simulating pedestrian flow through a narrow exit, which can be simulated by the modified model. Second, the modified model can formulate the evolutionary trend of the number of pedestrians staying in a room during the evacuation simulated by the original model, and extends the original model. Third, the modified model can be used to formulate the degree of pedestrian overlap in a crowd, and fourth, compared with the original model, the modified model simulates the distribution of pedestrians in a space in a very natural way.

We believe that the idea of formulating the step size can also be applied to other microscopic pedestrian models based on the dynamic system formulating the relation between pedestrian displacement and velocity due to repulsive actions among the pedestrians, e.g. those in Refs. [6]–[8] and [30]–[32]. This problem can be considered in further works.

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