# Integrating the environmental factor into the strategy updating rule to promote cooperation in evolutionary games*

Zhao Lin(赵 琳), Zhou Xin(周 鑫), Liang Zhi(梁 治)†, and Wu Jia-Rui(吴家睿)

*Hefei National Laboratory for Physical Sciences at the Microscale and School of Life Sciences, University of Science and Technology of China, Hefei* 230026, *China*

(Received 3 July 2011; revised manuscript received 13 July 2011)

Many previous studies have shown that the environment plays an important role for social individuals. In this paper, we integrate the environmental factor, which is defined as the average payoff of all a player's neighbours, with the standard Fermi updating rule by introducing a tunable parameter, $\omega$. It is found that the level of cooperation increases remarkably, and that the cooperators can better resist the invasion of defection with an increase in $\omega$. This interesting phenomenon is then explained from a microscopic view. In addition, the universality of this mechanism is also proved with the help of the small-world network and the random regular graph. This work may be helpful in understanding cooperation behaviour in species from unicellular organisms up to human beings.

**Keywords:** evolutionary game, cooperation, prisoner's dilemma game

**PACS:** 87.23.Kg, 87.23.Ge, 89.75.Fb, 02.50.Le **DOI:** 10.1088/1674-1056/21/1/018701

## 1. Introduction

Understanding ubiquitous cooperation among selfish individuals is still a major challenge in both natural and social sciences. According to Darwinian selection theory, any behaviour that brings benefits to others, but not directly to oneself, will soon disappear, which is inconsistent with observations in the real world. In order to resolve this puzzle, researchers have resorted to evolutionary game theory as a common framework, and the game models as metaphors for studying cooperative behaviour.[1−5] In the well known prisoner's dilemma game (PDG), two players can choose to cooperate or to defect simultaneously. They can receive a payoff $R$ ($P$) for mutual cooperation (defection), while a defector can get the payoff $T$ by exploiting the cooperator, and the other in turn will get the payoff $S$. Here, two conditions must be strictly satisfied: $T > R > P > S$ and $2R > T + S$. If the selfish players only want to maximize their own benefits, the better choice will always be defection, and the "tragedy of the commons" dilemma is inevitably induced.[6]

Over the past few decades, there have been many efforts to solve the cooperation dilemma. Nowak and May first studied the spatial PDG, in which players were assigned on a regular lattice and could only interact with their direct neighbours. Surprisingly, it was found that the simple spatial structure could induce steady cooperation clusters and amazing spatial chaos.[7] Along this line, a great number of fundamental works have subsequently been extended in theoretical and experimental studies.[8−13] In particular, with the small-world network model[14] and the scale-free network model[15] being put forward, researchers began to realize that most real systems are not regular lattices or random networks, but networks associated with scale-free and small-world properties,[16−19] which are called complex networks. Then the evolutionary spatial game is naturally extended to different variations of complex networks. Santos *et al.* found that scale-free topology has the capability of the mutual protection of hub cooperators to maintain cooperation.[20] Rong *et al.* further demonstrated that disassortative networks can better promote the cooperation level.[21] Ren *et al.* investigated the effect of topological randomness in a homogeneous small-world network, and the interesting resonance-type phenomenon was observed.[22] Recently, competitive behaviours among mobile agents has drawn more at-

*Project supported by the CAS/USTC Special Grant for Postgraduate Research, Innovation, and Practice.

†Corresponding author. E-mail: liangzhi@ustc.edu.cn

© 2012 Chinese Physical Society and IOP Publishing Ltd

tention, and several works have revealed that mobility is a critical issue in evolutionary games.[23−25] For reviews of structured games, one can refer to Refs. [26]–[28].

Many experimental works have shown that social individuals often exhibit more complex behaviours during cooperation, such as altruism[29−31] and inequality aversion.[32−34] Thus many researchers tend to explore a meaningful problem: the coevolution of cooperation and other behaviours, including preferential selection,[35−38] asymmetric cost,[39,40] time scale,[41−44] the expectation effect,[45−47] teaching activity,[48−50] the history effect[51−53] and self-adjusting[54], to name but a few. In particular, many experimental studies show that the environment plays an important role in real life, for example animals will find it easier to live in a forest with plenty of food and water than in a poorly resourced desert. Very recently, the environmental factor has been introduced into evolutionary spatial games by rescaling the payoff or the fitness of the players.[55,56] Inspired by their works and considering the crucial role of the strategy updating rule in spatial games,[3] we propose an environment-based strategy updating rule in the spatial PDG based on the commonly used Fermi function stochastic rule.[8] In the standard Fermi function stochastic rule, strategy transition probability is only determined by the payoff difference between the focal player and a neighbouring player if the noise denoting the irrational behaviours is fixed. In our updating rule, strategy transition probability is also affected by the environmental factor, which is defined as the average payoff of a player's neighbours. It is found that the cooperation level is remarkably enhanced. We then give a microscopic explanation of the observed phenomena.

This paper is organized as follows. In Section 2 we describe the spatial PDG model and the strategy updating rule. In Section 3 the simulation results and discussion are presented, and the paper is summarized in the last section.

## 2. Model

Let us simplify the standard PDG model by following the common practice as:[7] $T = b$, $R = 1$ and $P = S = 0$, where $b$ $(1 \leq b \leq 2)$ is the temptation to defect. Initially, cooperators and defectors are randomly distributed on a $100 \times 100$ square lattice.

At each time step, all pairs of directly linked nodes engage in a single round PDG and get payoffs accordingly. When player $i$ is updated, it will learn the strategy of a randomly selected neighbour $j$ with a probability

$$W_{i\to j} = \frac{A}{1 + \mathrm{e}^{(P_i - P_j)/\kappa}}, \tag{1}$$

with

$$A = \frac{1}{1 + \left(\dfrac{\bar{P} + \varepsilon}{P_i + \varepsilon}\right)^{\omega}}, \tag{2}$$

where $A$ is the environmental coefficient, $\bar{P}$ is the environment payoff and is equal to the average payoff of $i$'s neighbours, $\varepsilon$ is a tiny positive real number to avoid a possible zero denominator (we set $\varepsilon = 0.0001$ in all the simulations below), and $\kappa$ $(0 < \kappa < \infty)$ is the environmental noise characterizing the irrationality of the players (the effect of noise $\kappa$ has been well studied in Refs. [8] and [9]). In addition, $\omega$ is the newly introduced parameter to guide the environmental coefficient and the strategy transition intensity. In what follows, we will focus on the effect of $\omega$ on the behaviour of the system.

## 3. Simulation results and discussion

The most important quantity for characterizing cooperative behaviour is the cooperator frequency $f_{\mathrm{c}}$, which is defined as the fraction of cooperators within the whole system when the steady state is reached, $f_{\mathrm{c}} = N_{\mathrm{C}}/(N_{\mathrm{C}} + N_{\mathrm{D}})$, where $N_{\mathrm{C}}$ $(N_{\mathrm{D}})$ is the number of cooperators (defectors). This key quantity is obtained by averaging the last 2000 generations after a transient time of 20000 generations. To guarantee sufficient exactness, the data is obtained by averaging over 50 separate runs.

First we inspect the spatial patterns of the cooperators and the defectors for different values of $\omega$. Figure 1 shows the results obtained with $b = 1.3$ and $\kappa = 0.8$. For $\omega = 0$ (Fig. 1(a)), the system falls into the pure-defector state. However, for $\omega = 1$, 2, 3 (Fig. 1(b)–1(d)), a small fraction of cooperators can survive to resist the invasion of the defectors. When $\omega = 4$ is considered (Fig. 1(e)), the cooperators begin to outperform the defectors. For $\omega = 5$ (Fig. 1(f)), the cooperators will dominate the system. These results reveal that considering the environmental factor, i.e. integrating the neighbourhoods' payoff in the strategy updating process, can promote the evolution of cooperation remarkably.

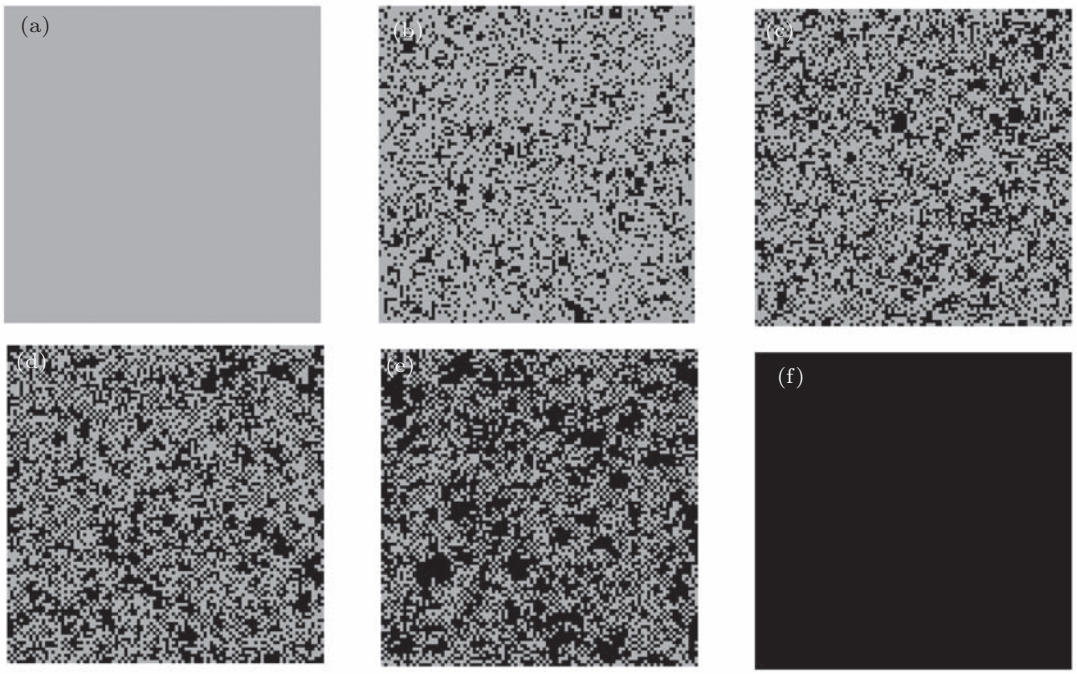



**Fig. 1.** Panels (a)–(f) show the snapshots for $\omega = 0, 1, 2, 3, 4and5$, respectively. Grey nodes denote defectors and dark nodes represent cooperators. The results are obtained on a regular lattice with the degree $k = 4$. Here $b = 1.3$ and $\kappa = 0.8$.

To quantify the effect of $\omega$ more accurately, we investigate the relationship between $f_c$ and $\omega$ for different values of the temptation to defect and different degrees of the regular lattices in Fig. 2. As can be observed in Fig. 2(a), it is evident that the cooperator frequency $f_c$ monotonously increases with the increment of $\omega$, irrespective of the value of $b$. In particular, when $\omega$ is large enough ($\omega = 4$), cooperation will become the dominant strategy, even when the advantage of defection is great, whereas in such a case the cooperators are extinct in the standard spatial PDG. We therefore argue that this environment-based strategy updating rule can conspicuously facilitate cooperation. Moreover, from Fig. 2(b) we find that the promotion of cooperation induced by $\omega$ is robust to different values of degree $k$, and a smaller $k$ is more beneficial for the emergence and persistence of the cooperation, which is consistent with the prediction of the mean-field approach. Since our main purpose is to study the effect of this new mechanism, we set $k = 4$ in the following simulations.

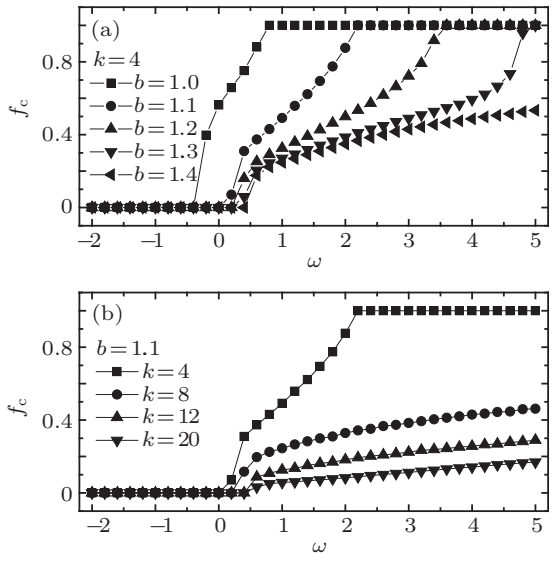



**Fig. 2.** (a) Cooperator frequency versus $\omega$ for different values of $b$ on a regular lattice with degree $k = 4$. (b) Cooperator frequency versus $\omega$ for $b = 1.1$ on regular lattices with different values of degree $k$. Here $\kappa = 0.8$.

Next we will analyse the underlying reason for this kind of cooperation facilitation from a microscopic viewpoint. Figure 3 shows three typical local patterns on square lattices. Pattern (i) is a defector in a sea of cooperators; pattern (ii) is a cooperator in a sea of defectors; and pattern (iii) has mixed cooperators and defectors. We then calculate the payoff of the circled defector $P_D$, the payoff of the circled cooperator $P_C$ and the environment payoff $\bar{P}$; $P_D > \bar{P} > P_C$ is available for all three patterns. According to Eq. (1), a player with high payoff ($P_i > \bar{P}$) is more active in imitating the strategy of other players as $\omega$ increases, whereas a player with low payoff (e.g. $P_i < \bar{P}$) is more likely to keep his own strategy. Figure 4 clearly demonstrates the transition possibilities of the circled

cooperator and the circled defector when changing $\omega$ for the three local patterns. We find that $W_{C\to D}$ (the transition possibility of the cooperator to become a defector) monotonously decreases with the increment in $\omega$, while $W_{D\to C}$ (the transition possibility of the defector to become a cooperator) monotonously increases, indicating that a defector is more likely to become a cooperator, yet the opposite situation seldom occurs.

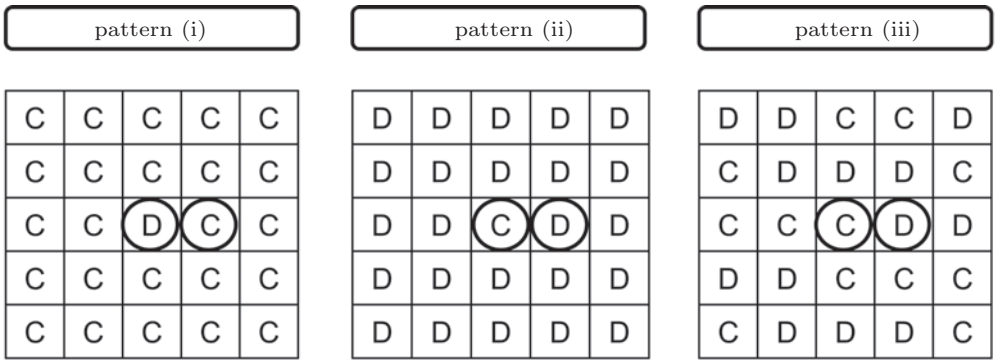



**Fig. 3.** Three typical local patterns in the evolution.

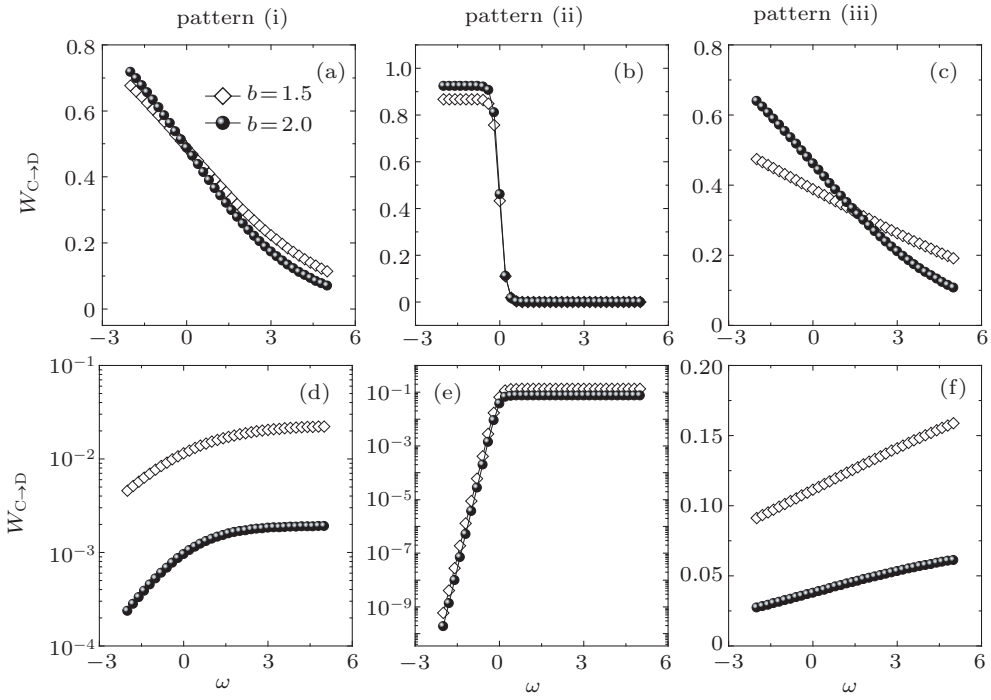



**Fig. 4.** The transition possibility between the circled cooperator and the circled defector in Fig. 3. Panels (a) and (d) correspond to pattern (i) of Fig. 3; panels (b) and (e) correspond to pattern (ii) of Fig. 3; and panels (c) and (f) correspond to pattern (iii) of Fig. 3. The results are obtained for $\kappa = 0.8$.

Previous research revealed that the survival approach of the cooperators is to form clusters to resist the invasion of the defectors. In standard spatial PDG, many low-payoff cooperators will soon be infected by the high-payoff defectors, and the lucky surviving cooperators will form compact cooperator clusters to expand cooperation and compete with the defectors. Thus the cooperators and the defectors usually co-exist competitively in the form of clusters. Chen *et al.* further found that this pattern formation can affect the payoff collection of the players: the average payoff of the cooperators is larger than that of the defectors, not only with respect to the whole system, but also for players along the boundaries.[57] Under our mechanism, the results are quite different. As Fig. 5 shows, $\bar{P}_{\text{D-all}}$ (the average payoff of all defectors) is apparently larger than $\bar{P}_{\text{C-all}}$ (the average payoff of all cooperators), and $\bar{P}_{\text{D-bound}}$ (the

average payoff of the defectors along the boundaries) is larger than $\bar{P}_{\text{C-bound}}$ (the average payoff of the cooperators along the boundaries). Compared with the traditional model, we find that the low-payoff cooperators themselves can probably resist the invasion of the defectors without the support of other cooperators, and thus the cooperation will be robust even without the formation of cooperator clusters (see Fig. 1(b)–(e)). Since the cooperators and the defectors are always well blended, the defectors can still obtain high payoff through the neighbouring cooperators, and the cooperators can maintain cooperation with a lower payoff.

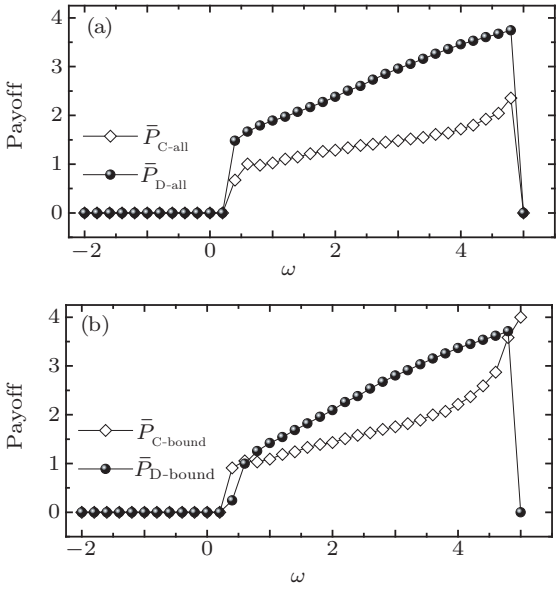



**Fig. 5.** (a) The average payoffs of cooperators and defectors in the whole population, and (b) the average payoffs of cooperators and defectors along the boundary. Here $b = 1.30$ and $\kappa = 0.8$.

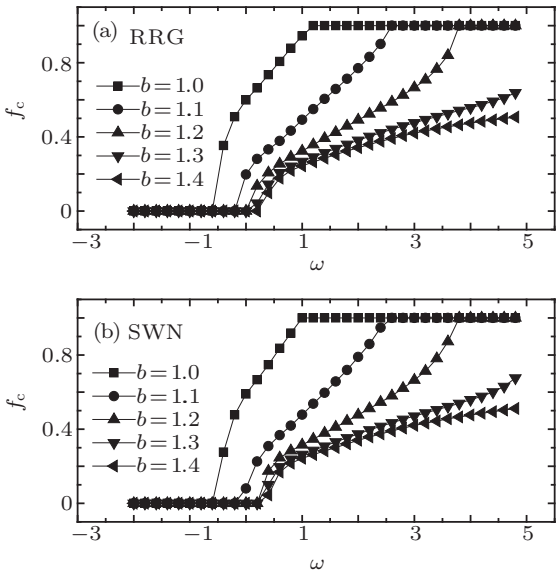



**Fig. 6.** The frequency of the cooperators versus $\omega$ for different values of $b$ on (a) a random regular graph (RRG) and (b) a small world network (SWN). Here $\kappa = 0.8$.

Finally, it remains interesting to explore the universality of cooperation promotion for such a new mechanism. We investigate the cooperative behaviours on different networks in Fig. 6. Similar to Fig. 2, it can be observed that the cooperation level can be remarkably advanced with an increase in $\omega$ on the random regular graph and the small-world network, which implies that the effect of the environment-based strategy updating rule is quite robust.

## 4. Conclusion

In summary, we investigated the effect of the environment on cooperation behaviour in the framework of evolutionary networked games. In our model, the environmental factor (the average payoff of all a player's neighbours) is integrated into the strategy updating rule. It is found that the cooperation level is remarkably promoted by this mechanism, especially when the high-payoff players are more active in imitating the strategy of others. By studying three typical patterns, we demonstrate that the cooperators can resist the invasion of the defectors, even without the formation of clusters. We also examine the cooperation behaviour for different average degrees and different network models, and find that the promotion of cooperation is robust. Our work reveals that the environment may play a crucial role in the emergence and persistence of cooperation in nature and society.

## Acknowledgment

We thank Du Wen-Bo and Yang Han-Xin for their useful discussions.

## References

[1] Von Neumann J and Morgenstern O 1953 *Theory of Games and Economic Behaviour* (Princeton: Princeton University Press)
[2] Smith J M 1982 *Evolution and the Theory of Games* (Cambridge: Cambridge University Press)
[3] Axelrod R 1984 *The Evolution of Cooperation* (New York: Basic Books)
[4] Hofbauer J and Sigmund K 1998 *Evolutionary Games and Population Dynamics* (Cambridge: Cambridge University Press)
[5] Ginits H 2000 *Game Theory Evolving* (Princeton: Princeton University Press)

[6] Hardin G 1968 *Science* **162** 1243
[7] Nowak M A and May R M 1992 *Nature* **359** 826
[8] Szabó G and Töke C 1998 *Phys. Rev.* E **58** 69
[9] Szabó G and Hauert C 2002 *Phys. Rev.* E **66** 062903
[10] Rong Z H and Wu Z X 2009 *Europhys. Lett.* **87** 30001
[11] Cao X B, Du W B and Rong Z H 2010 *Physica* A **389** 1273
[12] Wang Z, Xu Z J and Zhang L Z 2010 *Chin. Phys.* B **19** 110201
[13] Wang Z, Xu Z J, Huang J H and Zhang L Z 2010 *Chin. Phys.* B **19** 100204
[14] Watts D J and Strogatz S H 1998 *Nature* **393** 440
[15] Barabási A L and Albert R 1999 *Science* **286** 509
[16] Guimerà R, Mossa S, Turtschi A and Amaral L A N 2005 *Proc. Natl. Acad. Sci.* USA **102** 7794
[17] Zhang J, Cao X B, Du W B and Cai K Q 2010 *Physica* A **389** 3922
[18] Albert R and Barabási A L 2000 *Phys. Rev. Lett.* **85** 5234
[19] Barabási A L, Jeong H, Ravasz E, Néda Z, Schubert A and Vicsek T 2002 *Physica* A **311** 3
[20] Santos F C and Pacheco J M 2005 *Phys. Rev. Lett.* **95** 098104
[21] Rong Z H, Li X and Wang X F 2007 *Phys. Rev.* E **76** 027101
[22] Ren J, Wang W X and Qi F 2007 *Phys. Rev.* E **75** 045101(R)
[23] Meloni S, Buscarino A, Fortuna L, Frasca M, Gómez-Gardenẽs J, Latora V and Moreno Y 2009 *Phys. Rev.* E **79** 067101
[24] Zhang J, Wang W Y, Du W B and Cao X B 2011 *Physica* A **390** 2251
[25] Guan J Y, Wu Z X and Wang Y H 2007 *Chin. Phys.* **16** 3566
[26] Szabó G and Fáth G 2007 *Phys. Rep.* **446** 97
[27] Perc M and Szolnoki A 2010 *BioSystems* **99** 109
[28] Roca C P, Cuesta J A and Sánchez A 2009 *Phys. Lif. Rev.* **6** 208
[29] Choi J K and Bowles S 2007 *Science* **318** 636
[30] Boyd R, Gintis H, Bowles S and Richerson P 2003 *Proc. Natl. Acad. Sci.* USA **100** 3531
[31] Fehr E and Fischbacher U 2003 *Nature* **425** 785
[32] S Gavrilets, Duenez-Guzman E A and Vose M D 2008 *PLoS ONE* **3** 3293
[33] Dawes C T, Fowler J H, Johnson T, McElreath R and Smirnov O 2007 *Nature* **446** 794
[34] Fehr E, Bernhard H and Rockenbach B 2008 *Nature* **454** 1079
[35] Guan J Y, Wu Z X, Huang Z G, Xu X J and Wang Y H 2006 *Europhys. Lett.* **76** 1214
[36] Du W B, Cao X B, Zhao L and Hu M B 2009 *Physica* A **388** 4509
[37] Yang H X, Wang W X, Wu Z X, Lai Y C and Wang B H 2009 *Phys. Rev.* E **79** 056107
[38] Perc M and Wang Z 2010 *PLoS ONE* **5** e15117
[39] Du W B, Cao X B and Hu M B 2009 *Physica* A **388** 5005
[40] Du W B, Cao X B, Yang H X and Hu M B 2010 *Chin. Phys.* B **19** 010204
[41] Szolnoki A, Perc M, Szabó G and Stark H U 2009 *Phys. Rev.* E **80** 021901
[42] Wu Z X, Rong Z H and Holme P 2009 *Phys. Rev.* E **80** 036106
[43] Roca C P, Cuesta J A and Sánchez A 2006 *Phys. Rev. Lett.* **97** 158701
[44] Rong Z H, Wu Z X and Wang W X 2010 *Phys. Rev.* E **82** 026101
[45] Chen X J and Wang L 2008 *Phys. Rev.* E **77** 017103
[46] Du W B, Cao X B, Hu M B, Yang H X and Zhou H 2009 *Physica* A **388** 2215
[47] Zhang J, Fang Y P, Du W B and Cao X B 2011 *Physica* A **390** 2258
[48] Szolnoki A and Perc M 2008 *New J. Phys.* **10** 043036
[49] Liu R R, Rong Z H, Jia C X and Wang B H 2010 *Europhys. Lett.* **91** 20002
[50] Zhang H F, Small M, Yang H X and Wang B H 2010 *Physica* A **389** 4734
[51] Wang W X, Ren J, Chen G and Wang B H 2006 *Phys. Rev.* E **74** 056113
[52] Liu R R, Jia C X and Wang B H 2010 *Physica* A **389** 5719
[53] Du W B, Cao X B, Liu R R and Jia C X 2010 *Int. J. Mod. Phys.* C **21** 1433
[54] Xu Z J, Wang Z, Song H P and Zhang L Z 2010 *Europhys. Lett.* **90** 20001
[55] Wang Z, Du W B, Cao X B and Zhang L Z 2011 *Physica* A **390** 1234
[56] Wang Z, Murks A, Du W B, Rong Z H and Perc M 2011 *J. Theor. Biol.* **277** 19
[57] Chen X J, Fu F and Wang L 2008 *Phys. Rev.* E **78** 051120