Design of wideband graded-index antireflection coatings at oblique light incidence*

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We suggest a design method of graded-refractive-index (GRIN) antireflection (AR) coating for s-polarized or p-polarized light at off-normal incidence. The spectrum characteristic of the designed antireflection coating with a quintic effective refractive-index profile for a given state of polarization has been discussed. In addition, the genetic algorithm was used to optimize the refractive index profile of the GRIN antireflection for reducing the mean reflectance of s- and p-polarizations. The average reflectance loss was reduced to only 0.04% by applying optimized GRIN AR coatings onto BK7 glass over the wavelength range from 400 to 800 nm at the incident angle of $\theta_0 = 70°$.

Keywords: antireflection coatings, oblique incidence, graded-refractive-index profile, genetic algorithm

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1. Introduction

Antireflection (AR) coatings[1,2] are essential to reduce unwanted reflectance in optical devices. Conventional single layer AR coatings[3] provide excellent performance at a specific wavelength and normal incidence. These coatings should satisfy the following conditions: the optical thickness should be $\lambda/4$, where λ is the wavelength of the incident light; the refractive index of the coating is $n = \sqrt{n_0 n_1}$, where n_0 and n_1 are the refractive indices of the ambient medium and substrate, respectively. However, an effective reflectance is typically limited to a small range of incident angles and a narrow bandwidth. Optical coatings based on layers which are inhomogeneous along the stack axis[4] have optical and mechanical properties different from those of traditional coatings. The remarkable feature of graded-refractive-index (GRIN) coatings[5] with the broadband AR characteristic is that the refractive index varies gradually and monotonically along its thickness from the refractive index of the ambient medium to that of the substrate. Many specific gradient-index profiles have been studied previously, including linear,[6] sinusoid (cosinusoid), gaussian,[7] exponential-sine,[8,9] and quintic.[10,11] It has been shown that these profiles are effective in reducing reflection over a wide range of wavelengths, however most of them were designed for normal incidence. The performance of these GRIN AR coatings falls off at off-normal incidence, especially at large angles of incidence. Poitras *et al.*[12] suggested that to reduce the overall reflectance at high angles, one has to distort the thickness axis of traditional inhomogeneous transition-layer profiles. They introduced GRIN AR coatings with modified quintic refractive index profiles to reduce the reflectance at oblique incidence. The AR performance of modified profiles at high angles was significantly improved than that of quintic profiles, however there is relatively little literature on the design of GRIN AR coatings for p- or s-polarized radiation at oblique incidence angles.

In this study, a general method to design wideband GRIN AR coatings for a given state of polarization at oblique incidence is proposed. We designed GRIN AR coatings for s-polarization (or p-polarization) with the effective refractive index η_s (or η_p) varying gradually from that of the ambient medium to that of the substrate along its effective thickness. The quintic was adopted as the effective

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refractive-index profile in the paper. In order to obtain AR coating for both polarizations at off-normal incidence we optimized the refractive-index profile with the genetic algorithm. It is shown that the optimization method allows the realization of GRIN AR coatings with high performances.

2. Design of GRIN AR coatings for a given polarization

At oblique incidence, the optical characteristics of the film are different for s- and p-polarizations. For GRIN coatings, the effective refractive indices for s- and p-polarizations are given by[13]

$$\eta_{\rm s}(z) = n(z)\cos\theta(z), \tag{1a}$$

$$\eta_{\rm p}(z) = n(z)/\cos\theta(z), \tag{1b}$$

where z is the optical thickness and $\cos\theta(z)$ can be obtained from the solution of

$$\cos\theta(z) = \left[1 - \left(\frac{n_0\sin\theta_0}{n(z)}\right)^2\right]^{1/2}, \tag{2}$$

where θ_0 is the incident angle and $\theta(z)$ is the refractive angle in the coating. The effective optical thicknesses for s- and p-polarizations are both expressed by

$$\mathrm{d}z_{\rm eff} = \mathrm{d}z\cos\theta(z). \tag{3}$$

Some studies have been carried out to find the refractive-index profiles of inhomogeneous layers that yield the lowest reflectance. These GRIN AR coatings can reduce the reflectance significantly at normal incidence. However, the reflectance increases dramatically for large incident angles. Different from normal incidence, two changes occur under the condition of oblique incidence. The first change is that s- and p-polarized lights split because of different effective refractive indices (Eq. (1)). The second is that the effective optical thickness (Eq. (3)) deforms the refractive-index profile. We hold the opinion that in order to obtain a GRIN AR coating for a given polarization at oblique incidence, one might design the film with the effective refractive index satisfying the quintic profile, which would have the best AR properties, along the effective thickness from the admittance of ambience to that of the substrate. As "seen" by the polarized light, the AR structure is similar to a coating with quintic profile designed at normal incidence. Thus, the GRIN AR coating can reduce the reflectance for the polarized light effectually. In the following design, the substrate is assumed to be BK7 glass ($n_1 = 1.52$) and the ambient medium is air ($n_0 = 1.0$). The total optical thicknesses of AR coatings corresponding to the given incident angles are all considered to be 1 μm.

2.1. Antireflection coatings for s-polarization

The proposed effective index profile for s-polarized light is expressed by

$$\eta_{\rm s}(z_{\rm eff}) = \eta_{\rm s0} + (\eta_{\rm s1} - \eta_{\rm s0})\left[10\left(\frac{z_{\rm eff}}{d_{\rm eff}}\right)^3 - 15\left(\frac{z_{\rm eff}}{d_{\rm eff}}\right)^4 + 6\left(\frac{z_{\rm eff}}{d_{\rm eff}}\right)^5\right], \tag{4}$$

where $d_{\rm eff}$ is the total effective optical thickness of the film; $\eta_{\rm s0}$ and $\eta_{\rm s1}$ are the effective refractive indices of ambient medium and substrate for s-polarization, respectively.

$$\eta_{\rm s0} = n_0\cos\theta_0, \tag{5a}$$

$$\eta_{\rm s1} = n_1\cos\theta_1, \tag{5b}$$

where θ_1 is the refractive angle in the substrate. We can obtain the following expressions based on Eqs. (1) and (2):

$$n = (\eta_{\rm s}^2 + n_0^2\sin^2\theta_0)^{1/2}. \tag{6}$$

The expression of $n(z)$ can be obtained by combining Eqs. (3)–(6).

Several wideband GRIN AR coating for s-polarization at oblique incidence are designed, using the method described above. Figure 1(a) shows the effective refractive indices for s-polarized light as a function of the effective optical thickness from the ambient medium. As can be seen, the effective thicknesses of AR coatings with equal optical thickness decrease with the increase of incident angles. The refractive-index profiles of the designed GRIN AR coatings are shown in Fig. 1(b). Note that the higher the angle of incidence, the smoother the profile at the incident ambient side. As shown in Fig. 2, the reflectances for s-polarized light over a wavelength range from 400 to 800 nm are all dramatically reduced. At an incident angle of 70°, the average reflectance is only 0.72%.

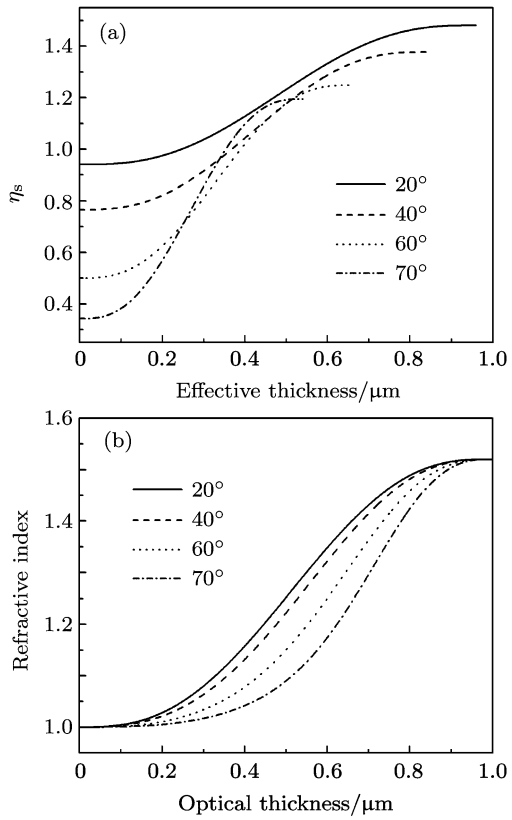


Fig. 1. The effective refractive-index profiles for s-polarization (a) and refractive-index profiles (b) of the four designed GRIN AR coatings for different incident angles.

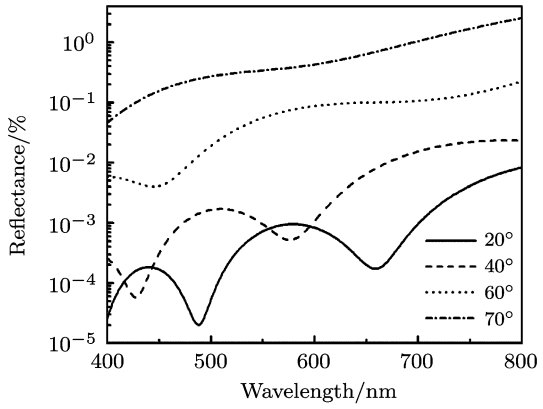


Fig. 2. Reflectance curves of the corresponding four AR coatings for s-polarized light at different incident angles.

2.2. Antireflection coatings for p-polarization

Similarly, the effective index profile for p-polarization is also adapted as quintic function of the form

$$\eta_{\rm p}(z_{\rm eff}) = \eta_{\rm p0} + (\eta_{\rm p1} - \eta_{\rm p0}) \left[10 \left(\frac{z_{\rm eff}}{d_{\rm eff}} \right)^3 - 15 \left(\frac{z_{\rm eff}}{d_{\rm eff}} \right)^4 + 6 \left(\frac{z_{\rm eff}}{d_{\rm eff}} \right)^5 \right], \quad (7)$$

where $\eta_{\rm p0}$ and $\eta_{\rm p1}$ are the effective refractive indices of ambient medium and substrate for p-polarized light, which are defined as follows:

$$\eta_{\rm p0} = n_0 / \cos\theta_0, \quad (8a)$$

$$\eta_{\rm p1} = n_1 / \cos\theta_1. \quad (8b)$$

The relation between the normal incidence refractive index and the effective refractive index for p-polarization is given by[14]

$$n = \left(\frac{\eta_{\rm p}^2 + (\eta_{\rm p}^4 - 4\eta_{\rm p}^2 \sin^2\theta_0)^{1/2}}{2} \right)^{1/2}. \quad (9)$$

The effective refractive indices of ambient medium and substrate both increase with the increase of incident angles, as shown in Fig. 3(a). The growth of the

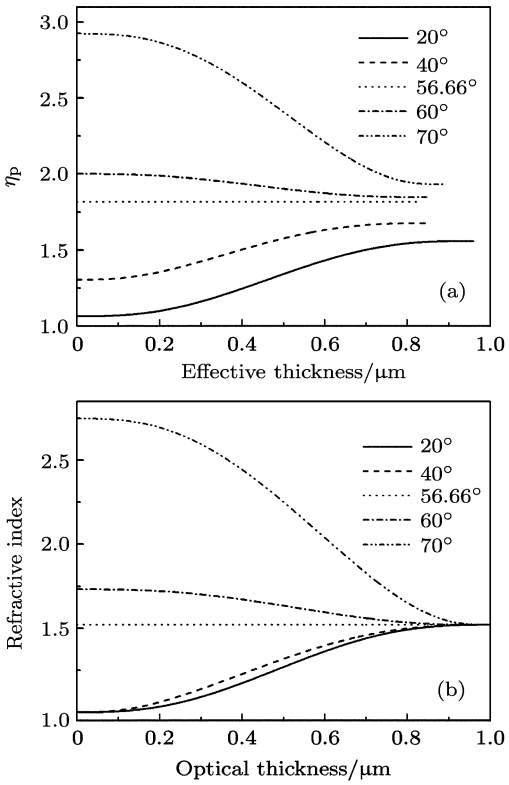


Fig. 3. The effective refractive-index profiles for p-polarization (a) and refractive-index profiles (b) of the designed several GRIN AR coatings for different angles of incidence.

former is more pronounced. The two values are equal for p-polarization, $n_0/\cos\theta_0 = n_1/\cos\theta_1$, at the incident angle of $\theta_0 = 56.66°$. Under this condition, the reflectances are zero (see Fig. 4) because no variation in refractive index can be "seen" by the p-polarized light which is incident from air to BK7 glass. The corresponding incident angle of 56.66° is Brewster angle. At this incident angle, AR coating is not necessary to be added onto the surface of substrate to reduce

the reflectance for p-polarization. Figure 3(b) shows the refractive-index profile of designed AR coatings, which are applied at different incident angles. Different from traditional GRIN AR coatings, the profiles do not change from the refractive index of air, $n_0 = 1.0$ at high angles of incidence. As can be seen, the reflectance for p-polarization remains below 0.025% over a wavelength range from 400 to 800 nm at the incident angle of 70°.

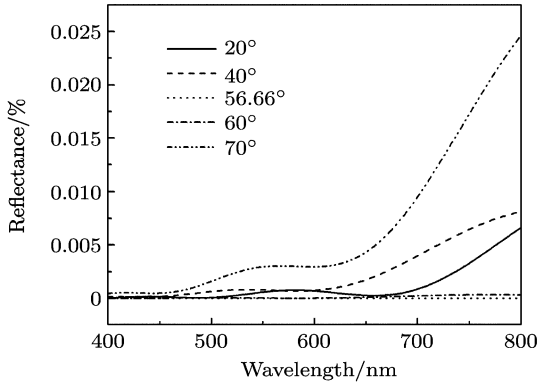


Fig. 4. Spectral reflectances of the corresponding five AR coatings for p-polarized light.

3. Optimization design of GRIN AR coatings for both the polarizations

The relation between the effective refractive index for s- and p-polarizations is given by

$$\eta_{\rm p} = \eta_{\rm s} + \frac{n_{\rm min}^2 \sin^2 \theta_0}{\eta_{\rm s}}. \tag{10}$$

As can be seen from the above equation, at a given incident angle, the effective refractive indices for the two polarizations are interdependent. By calculating the derivation of Eq. (10), we come to the conclusion that the effective refractive indices for s- and p-polarizations cannot be simultaneously monotonic over all the total effective optical distance in the case of $\theta_0 > 45°$ for $n_0 = 1.0$, $n_1 = 1.52$. In order to design GRIN AR coatings for both the polarized light at oblique incidence, we optimized the profile of the refractive index using the iterative genetic algorithm computational method.[15–19] In the process, the refractive-index profile, which varies from the refractive index of air to that of substrate, was optimized. The effective refractive-index profiles for s- and p-polarizations were determined by Eqs. (3), (6), and (9).

The inhomogeneous layer was modeled by a 1000-layer homogeneous layer system whose refractive-index profile is shown in Fig. 5. All the differences of the refractive index of any two adjacent layers (Δn_i) were optimized. Thus, the profile of refractive index was optimized indirectly. The optical thickness of each sublayer (ΔZ_i) is equivalent.

The merit function ($f_{\rm MF}$) in the research process is given by

$$f_{\rm MF} = \left\{ \frac{1}{m} \sum_{j=1}^{m} \left[\frac{R_{\rm s}(\lambda_j) + R_{\rm p}(\lambda_j)}{2} \right]^2 \right\}^{1/2}, \tag{11}$$

with the same weighting over 80 equal wavelength positions in the 400–800 nm spectral regions, where $R_{\rm s}$ and $R_{\rm p}$ are the wavelength-dependent reflectance coefficients for s- and p-polarizations, respectively.

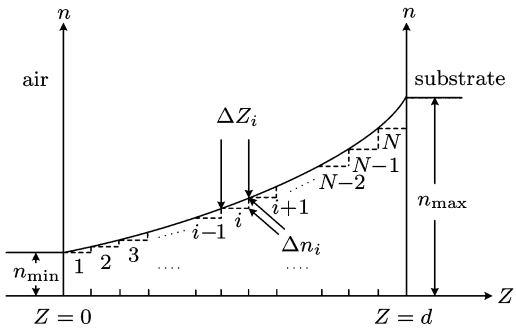


Fig. 5. Schematic representation of inhomogeneous coating modeled by sublayer system.

Figure 6 illustrates the profiles of the refractive index and effective refractive indices for both polarizations of the optimized AR coating, which is designed for the incident angle of 70°. The effective refractive index for p-polarization decreases slightly and then increases rapidly along the effective thickness of the optimized AR coating. Figure 7 shows the reflectance characteristics of the optimized GRIN AR coatings as a function of wavelength. It is shown that the average reflectance is only 0.04% over the design wavelength range with the maximum value of 0.125%. The results demonstrate that the GRIN AR coatings optimized by the genetic algorithm can reduce the reflectance significantly at oblique incidence. The method of optimizing the refractive-index profile can be used as a criterion in the design of gradient-index AR coatings.

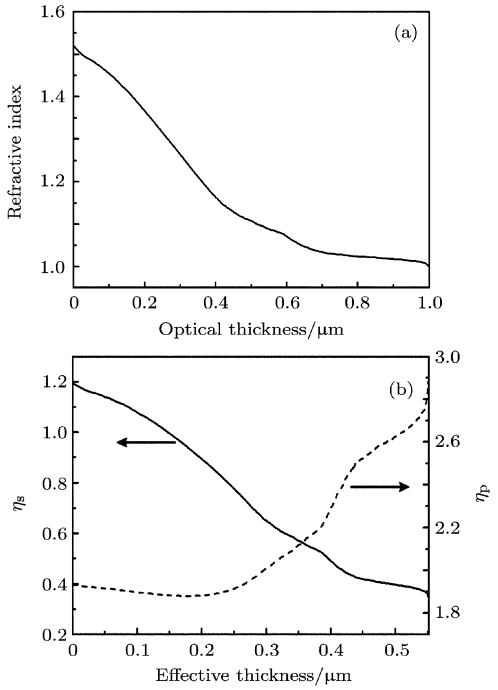


Fig. 6. Optimized refractive-index profile as a function of optical thickness for $\theta_0 = 70°$ incident angle (a) and the admittances for s- and p-polarized light varying with effective optical thickness (b).

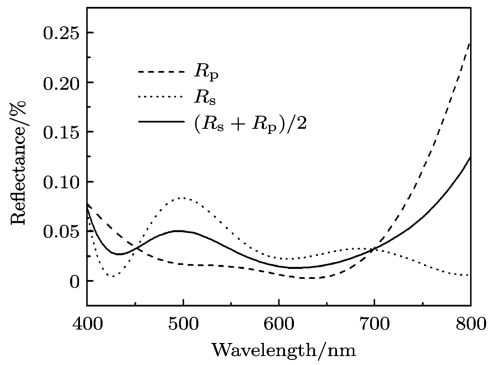


Fig. 7. Reflectance of optimized GRIN AR coating for s- and p-polarizations.

It is necessary to note that the fabrication of the GRIN AR coatings is very challenging, because thin film materials with very low refractive indices close to 1.0 are unavailable. Recently, a low refractive index of 1.05 was reported for SiO_2 grown by glancing angle deposition technique using electron-beam evaporation.[11] With the development of the deposition techniques, the graded index AR coatings would be realized accurately.

4. Conclusions

A plain design method was applied to achieve a broadband GRIN AR for s- or p-polarization at a given angle of incidence. The designed AR coatings with the quintic effective refractive-index profiles show good performance in reducing the reflectance for the given state of polarization at off-normal incidence. The reflectance for p-polarization is zero, when the effective refractive index of the ambient medium is equal to the value of substrate. Under such conditions, the AR coating is unnecessary. We find that the effective refractive-index profiles for s- and p-polarized light are not simultaneously monotonic over the total effective thickness when $\theta_0 > 45°$ for $n_0 = 1.0$, $n_1 = 1.52$. Therefore, the antireflective condition is not satisfactory at the same time for both the polarizations at high angles. In addition, the method of optimizing the refractive-index profile using the genetic algorithm is applied to the design of GRIN AR coatings for both s- and p-polarizations. It has been demonstrated that the optimized GRIN AR coatings can reduce the reflectance remarkably at oblique incidence. The average visible reflectance is only 0.04% at the incident angle of $\theta_0 = 70°$.

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