Energy unidirectional transmission in an asymmetrically finite transmission line*

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The phenomenon of energy unidirectional transmission is numerically investigated by using a system of two coupled discrete nonlinear electrical transmission lines, each line of the network contains a finite number of cells and has different pass band structures, respectively. Using numerical simulations, we examine the frequency multiplication of the driving frequency and the lattice filtering effect in the line. These lead to the generation of energy unidirectional transmission. In the present work, energy is carried by the second harmonic wave in the pass band. In addition, we also study the dependence of the energy efficiency on the driving amplitude and other parameters of the model, such as the system size and the nonlinear coefficient, by calculation. Furthermore, after detailed numerical simulation, an experimental demonstration is realized. The experimental results agree with those in simulation qualitatively.

Keywords: energy, unidirectional, nonlinear transmission line, linear wave

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1. Introduction

In recent years, renewed attention has been paid to the study of the energy transport in a physical system, especially the energy unidirectional transmission in a nonlinear medium, namely, asymmetric energy flow. In this case, the system acts like a good conductor in one direction but a good insulator in the opposite direction when the position of the excitation is changed. This phenomenon is not like that of an electronic diode, which is closely related to the P–N junction. However, it relies mainly on nonlinear effects. In addition, many recent studies have already discovered the phenomenon of energy unidirectional transmission in different topic areas.[1−20]

In particular, a considerable amount of the effect is attributed to heat conduction, and a detail review can be found in Refs. [7]–[17]. Terraneo *et al.*[7] investigated the possibility of controlling the energy transport by coupling two nonlinear chains and noted that the nonlinear control of heat conduction opens a new possibility for a thermal rectifier. Li *et al.*[8−12] have also numerically studied the heat conduction in a 1–3D inhomogeneous nonlinear chain and presented a thermal rectifier model, which is related to the theory of spectrum match/mismatch. To improve the gain of the thermal diode, thermal rectifier effects have been also been studied in the F–K model,[13,14] Yukawa chains[15] and the fermionic model.[16] From a practical point of view, at the nanoscale, an asymmetric thermal energy flow can be also realized.[17]

In addition, by means of a solitary wave, Nesterenko *et al.*[18] have found the phenomenon of energy unidirectional transmission in granular media and named it the acoustic diode effect. By numerically inspecting the propagation of a longitudinal wave, Liang and coworkers have observed the significant rectifying effect on acoustic flux in a nonlinear system[19] and built the rectifying model, which is called the acoustic rectifier in experiment.[20] In the optical fields, through laminating two different periodic photonic media, the rectifying effect has been studied numerically[1−4] and an optical diode model has been presented in experiment.[5,6]

Hence, motivated by the significant rectifying phenomenon of the above topics, we suppose that a

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similar phenomenon could also exist in a nonlinear electric transmission line containing a linear inductor L and a nonlinear capacitor C, in a manner similar to those in Refs. [5], [19], and [20]. To date, only a single experiment has been reported in such a system.[21] In order to study this phenomenon further in the transmission line, in this paper, we mainly numerically investigate this interesting phenomenon using a coupled LC nonlinear transmission line. Namely, the possibility to control the electrical energy transmission inside a 1D nonlinear electric network.

The outline of the paper is as follows. In Section 2, we present the model of our nonlinear electrical network cell. We show that general nonlinear equations govern the physics of the network. In Section 3, we describe the numerical simulations and present the results concerning the energy unidirectional transmission. The influence of many parameters is then investigated in Section 4. In the final section, we give the concluding remarks.

2. Model description

We consider a coupled LC transmission line composed of two lossless nonlinear electrical lattice systems (media I and II) made of a size N of identical cells, respectively. The medium I is connected by medium II in a sequence. Each cell[22] in the whole coupled line contains a series linear inductor L_i and a linear inductor L_{si} in parallel with a nonlinear capacitor $C(V_n)$, whose identical structure is illustrated in Fig. 1, except the different component values in different media. In medium I, $L_{\rm I}$ and $L_{\rm sI}$ denote the values of the series and parallel inductors, respectively. In a similar way, the values of the series and parallel inductors in medium II are designated by $L_{\rm II}$ and $L_{\rm sII}$, respectively. Both media are coupled at the $(N+1)$-th cell by using a linear inductor $L_{\rm c} = 10.94$ mH. In the two media, the nonlinear properties are both provided by the nonlinear capacitor, which consists of a reversed biased diode (BB112, Philips Electronics Ltd.). The capacitance depends on the voltage V_n as[23]

$$C(V_n) = C_0(1 - 2\alpha V_n + 3\beta V_n^2), \tag{1}$$

where $C_0 = 320$ pF is the linear capacitance at the biased voltage 2V, and $\alpha = 0.21$ V^{-1} and $\beta = 0.0197$ V^{-2} are the nonlinear coefficients.

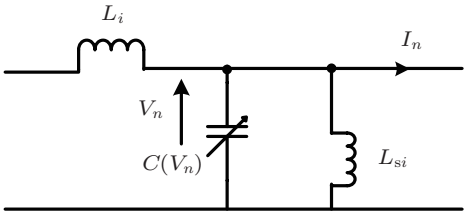


Fig. 1. The structure of each cell in the whole nonlinear dispersive transmission line (i=I or II). The nonlinear capacitance $C(V_n)$ consists of a varicap diode BB112 biased by a constant voltage of 2 V. V_n represents the voltage across the n-th capacitor, while I_n depicts the current through the inductor of the n-th cell with n being the cell number.

From Kirchhoff's laws, we can find that a set of equations governing the wave propagation are given by

$$\frac{{\rm d}^2V_n}{{\rm d}t^2} + \omega_{0i}^2 V_n - u_{0i}^2(V_{n+1} - 2V_n + V_{n-1}) = \alpha\frac{{\rm d}^2V_n^2}{{\rm d}t^2} - \beta\frac{{\rm d}^2V_n^3}{{\rm d}t^2}, \tag{2}$$

where $u_{0i} = 1/\sqrt{L_iC_0}$ and $\omega_{0i} = 1/\sqrt{L_{si}C_0}$ (i=I, II; $n = 1, 2, 3, \ldots$).

$$\frac{{\rm d}^2V_n}{{\rm d}t^2} + \omega_{0{\rm I}}^2 V_n + u_{0{\rm I}}^2(V_n - V_{n-1}) + \frac{V_n - V_{n+1}}{L_{\rm c}C_0} = \alpha\frac{{\rm d}^2V_n^2}{{\rm d}t^2} - \beta\frac{{\rm d}^2V_n^3}{{\rm d}t^2}, \tag{3}$$

with $n = N$.

$$\frac{{\rm d}^2V_n}{{\rm d}t^2} + \omega_{0{\rm II}}^2 V_n + u_{0{\rm II}}^2(V_n - V_{n+1}) + \frac{V_n - V_{n-1}}{L_{\rm c}C_0} = \alpha\frac{{\rm d}^2V_n^2}{{\rm d}t^2} - \beta\frac{{\rm d}^2V_n^3}{{\rm d}t^2}, \tag{4}$$

with $n = N + 1$.

In the present study, we assume that the voltage is a function of time in the form of

$$V(t) = A\cos(2\pi ft), \tag{5}$$

where f and A are the driving frequency and amplitude, respectively.

The numerical values of the components of the network are: $L_{\rm I} = L_{\rm II} = 314$ μH, $L_{\rm sI} = 200$ μH, and $L_{\rm sII} = 44$ μH. For our designed LC transmission line, the corresponding spectra of media I and II are illustrated in Figs. 2(a) and 2(b), respectively. Evidently, the pass band of medium II lies in the double of the pass band of medium I. In a later investigation, the driving frequency f (see Eq. (5)) is selected in the shadow area (see Fig. 2(a)) which lies in the pass band of medium I and its double in the medium II.

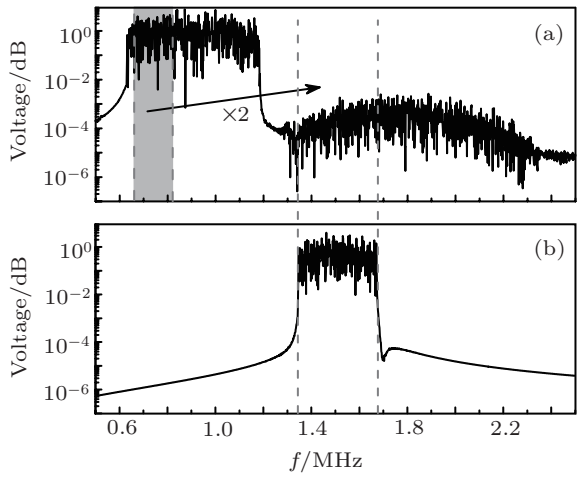


Fig. 2. The frequency spectra of media I and II: (a) medium I, (b) medium II. The shadow area in the plot (a) is the driving frequency band designed to form the asymmetric energy flow.

3. Numerical results

In our numerical study, the driving amplitude and frequency are $A = 0.30$ V and $f = 0.70$ MHz. The total simulation time is set to be 10 ms. The whole coupled system size is set to be $2N + 1 = 501$ cells, where each medium consists of $N = 250$ cells. The two media are coupled at the 251st cell by the coupling inductance L_c. The propagation equations are integrated by means of a fourth-order Runge–Kutta algorithm and the external driving amplitude A is given gradually from zero to its maximum value so as to avoid initial shock.

The computed energy distributions are shown in Fig. 3. We can see that the energy can penetrate the coupling interface and flow in the medium II when the driving voltage is launched at the left end of the coupled system (see Fig. 3(a)). However, as the driving voltage is forced at the right end, the energy cannot transmit in medium I (see Fig. 3(b)). The inset of Fig. 3(b) illustrates that the energy undergoes exponential attenuation rapidly with the wave propagation. In fact, in this case, the wave cannot propagate at all in medium II due to its frequency location in the gap band of medium II. Therefore, by means of numerical simulation, we can observe the phenomenon of asymmetric energy flow. The result qualitatively agrees with the experimental result.[21]

To well understand the physical mechanism, in Fig. 4, we plot the numerical fast Fourier transforms (FFTs) of the voltages obtained in media I and II. It is clear that the frequency spectrum has plentiful harmonics, such as the second and third harmonics labeled by 2 and 3, respectively, at the end of medium I (see Fig. 4(a)). However, the fundamental frequency f has been filtered when the wave passes through medium II (see Fig. 4(b)). The frequency $2f$ can propagate in medium II, of course, its harmonics at $4f$ are generated in medium II(see the label 4 in Fig. 4(b)).

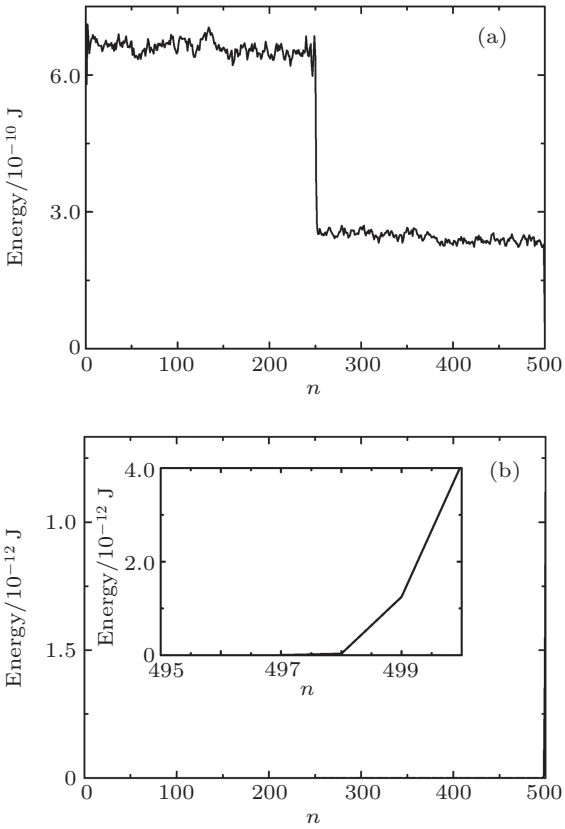


Fig. 3. The energy distributions along the whole transmission line as the excitation is (a) on the left side and (b) on the right side.

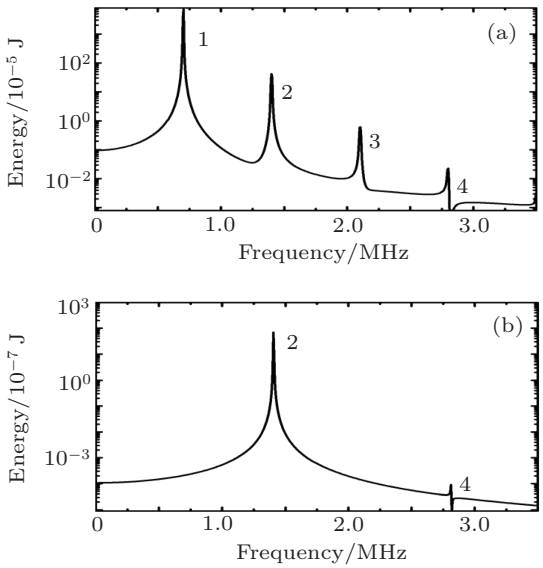


Fig. 4. The numerical FFTs of the voltages for media (a) I and (b) II.

As a matter of fact, the asymmetric energy flow is closely related to the frequency multiplication and lattice effect. In other words, the second harmonic wave plays a key role in the production of asymmetric energy flow.[21]

4. Dependence of parameters

In this section, we will restrict our attention to analyze the relations between the asymmetric energy flow change and the parameters. The phenomenon of asymmetric energy flow is closely related to the effects of the frequency multiplication and the pass band of the LC transmission line. In addition, factors, such as driving frequency f, nonlinear coefficient α, and the system size $2N$, have effect on the intensity of asymmetric energy flow. Therefore, these relations are also investigated by systematic numerical simulation here.

The numerical frequency response of the whole coupled LC transmission line is investigated by measuring the energy at the 300th cell. The results are qualitatively consistent with the one in the previous work[21] except for different driving amplitudes. It clearly reveals that, to the coupled system, the frequency pass band is about [0.67 0.82] MHz, which agrees with the design in Fig. 2(a) (see the shadow area). This phenomenon can demonstrate that the second harmonic is the main factor in the generation of asymmetric energy flow.

By means of measuring the energy of the cell in medium II, Fig. 5 illustrates the dependence of energy flow on the whole system size $2N + 1$ when the driving is forced at the left end. By changing the number of cells, we can find that the intensity of the energy decreases slowly with the increasing system size because the finite energy transmitted into medium II is distributed in more cells in the same simulation time.

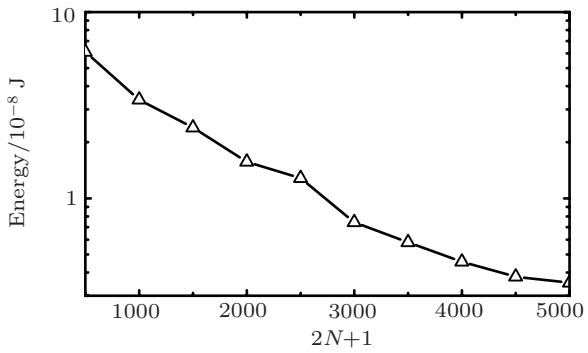


Fig. 5. The relation between energy and the size of the total LC transmission line.

In Fig. 6, we show how the energy measured at the 300th cell changes with the driving amplitude as the driving incidents from the left side. As in the small amplitude case, there exists a small energy flow closely equal to zero. However, away from 0.2 V, the energy flow increases with the amplitude increasing. This property can be also observed in the experiment of Ref. [21]. It indicates that the efficiency of the asymmetric energy flow is closely related to the driving amplitude A.

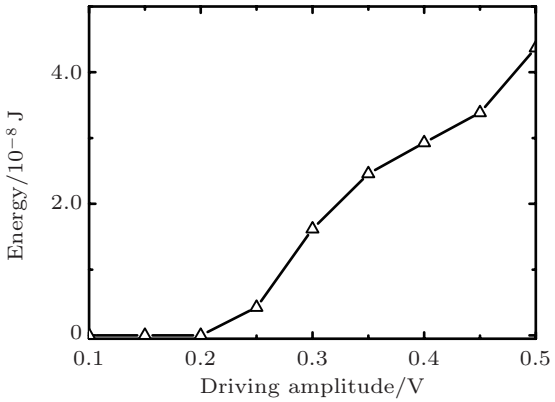


Fig. 6. The relation between energy and the driving amplitude forced at the left end.

By analysis, the asymmetric energy flow has a close relation with the coupling intensity $L_{\rm c}$, as shown in Fig. 7. The energy is measured at the 300th cell. As we all know, the large $L_{\rm c}$ results in the weak coupling intensity. Namely, the larger $L_{\rm c}$ is, the higher the resistance is. From Fig. 7, we can see that the energy decreases with the increasing $L_{\rm c}$. This result qualitatively coincides with the one in experiment.[21]

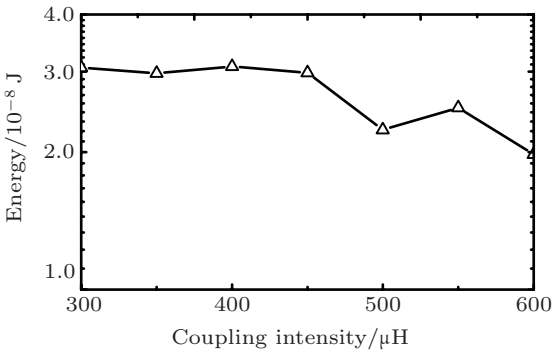


Fig. 7. The relation between energy and the coupling intensity.

Finally, the dependence of energy flow on the nonlinear coefficient α is also investigated. The measured energy at the 300th cell serves as the description of the energy transmitted into medium II. It is clear that the energy transmitted into medium II increases with

α increasing, because the stronger nonlinearity can result in the increasing of energy carried by the second harmonic wave. So it can be also assumed that the nonlinear coefficient α plays an important role in the generation of effective asymmetric energy flow. The results are qualitatively in good agreement with the one in the previous work[21] except for different driving amplitudes.

5. Conclusion

In this paper, we have reported the interesting phenomenon of energy unidirectional transmission by calculation. By using an asymmetric nonlinear transmission line which has two different pass bands, the energy can flow in a special direction, but not vice versa. The model generating the energy unidirectional transmission is very simple. Note that the mechanism in this work has a close relation with the frequency multiplication and filtering effect of the lattices. In our system, the second harmonic wave plays a key role in the generation of energy unidirectional transmission. Namely, the carrier of energy is the linear wave in the pass band. Furthermore, we also have numerically investigated the parameter dependence of efficiency of the energy unidirectional transmission, such as the driving amplitude, nonlinear coefficient α, and the coupling intensity.

It should be stressed that the numerical results are qualitatively in good agreement with the ones in experiment,[21] despite the existence of the actual component impedance which strongly affects the energy transmission. Namely, the numerical results further demonstrate that the experimental results are valid.

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