Optimization criteria of a Bose Brayton heat engine*

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An irreversible cycle model of the quantum Bose Brayton engine is established, in which finite-time processes and irreversibilities in two adiabatic processes are taken into account. Based on the model, expressions for the power output and the efficiency are derived. By using a numerical computation, the optimal relationship between the power output and the efficiency of an irreversible Bose Brayton engine is obtained. The optimal regions of the power output and the efficiency are determined. It is found that the influences of the irreversibility and the quantum degeneracy on the main performance parameters of the Bose Brayton engine are remarkable. The results obtained in the present paper can provide some new theoretical information for the optimal design and the performance improvement of a real Brayton engine.

Keywords: irreversibility, finite time thermodynamic, Bose Brayton engine, optimum criterion

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1. Introduction

In recent years, quantum thermodynamic cycles working with spin systems,[1−4] harmonic oscillator systems,[5−8] and ideal quantum gases[9−11] have become one of the most interesting research subjects. The optimal analyses related to the performance characteristics of thermodynamic cycles have been extended from classical cycles[12−19] to quantum cycles.[20−26] Finite-time thermodynamics[27−29] is a powerful tool for the performance analysis and the optimization of the real engineering cycles.

The Brayton engine is one of the typical energy conversion devices. Much work has been done for the performance analysis and the optimization of either classical[30−35] or quantum[36−39] Brayton engines. Many authors have optimized the performance of the Brayton heat engines by using different objective functions, such as the work output, the power output, and the efficiency. A large number of significant results have been obtained.[40−44] However, the above mentioned investigations rarely deal with the influences of quantum degeneracy and the finite heat transfer rate between the working substance and the heat reservoirs on the optimal performance of the Brayton engine.

In the present paper, according to the quantum statistics, we derive general expressions for several important parameters, such as the power output and the efficiency of an irreversible Bose Brayton engine. Some important optimization criteria, which will be helpful for engineers to understand the Brayton engine more deeply, are obtained.

2. Thermodynamic properties of ideal quantum gas

Based on quantum statistics, the expressions for the pressure and the number density of an ideal Bose gas are given by[45]

$$p = \frac{gkTf_{5/2}(z)}{\lambda^3}, \tag{1}$$

$$n = \frac{N - N_0}{V} = \frac{1}{v} = \frac{gf_{3/2}(z)}{\lambda^3}, \tag{2}$$

where

$$f_l(z) = \frac{1}{\Gamma(l)} \int_0^\infty \frac{x^{l-1}}{z^{-1}\,\mathrm{e}^x - 1}\,\mathrm{d}x, \tag{3}$$

g is the number of possible spin orientations of a gas particle; k is the Boltzmann constant; T, N, N_0, and V are the gas temperature, the total number of the particles, the number of particles in the ground state, and the volume of the gas, respectively; v is the occupied volume of the particle; $z = \exp[\mu/kT]$ and $\lambda = h(2\pi mkT)^{-1/2}$ are the fugacity of the gas and

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the mean thermal wavelength of the particles, respectively; μ is the chemical potential of the gas; h is the Planck constant; m is the rest mass of a particle; and $\Gamma(l)$ is the Gamma function. Using Eqs. (1) and (2) yields

$$p = nkTF(z), \tag{4}$$

$$U = \frac{3}{2}NkT\frac{f_{5/2}(z)}{f_{3/2}(z)} = \frac{3}{2}NkTF(z), \tag{5}$$

$$C_v = Nk\left[\frac{15}{4}\frac{f_{5/2}(z)}{f_{3/2}(z)} - \frac{9}{4}\frac{f_{3/2}(z)}{f_{1/2}(z)}\right] = \frac{3}{2}Nk\frac{\mathrm{d}}{\mathrm{d}T}[TF(T,v)], \tag{6}$$

$$S = Nk\left[\frac{5}{2}\frac{f_{5/2}(z)}{f_{3/2}(z)} - z\right] = Nk\left[\frac{5}{2}F(z) - z\right]. \tag{7}$$

Substituting Eqs. (1) and (2) into

$$C_p = C_v + T\left(\frac{\partial p}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_p, \tag{8}$$

we obtain

$$C_p = Nk\left[\frac{25}{4}\frac{f_{5/2}(z)}{f_{3/2}(z)} - \frac{15}{4}\frac{f_{3/2}(z)}{f_{1/2}(z)}\right] = \frac{5}{2}Nk\frac{\mathrm{d}}{\mathrm{d}T}[TF(T,p)], \tag{9}$$

where the correction factor $F(T,v)$ is a function of temperature T and volume v, the correction factor $F(T,p)$ is a function of temperature T and pressures p, and $F(z)$ can be called the correction factor for a Bose gas.

$$F(z) = \frac{f_{5/2}(z)}{f_{3/2}(z)}\left(\frac{T}{T_\mathrm{c}}\right)^{3/2} \times\left\{\left[\left(\frac{T_\mathrm{c}}{T}\right)^{3/2} - 1\right]\theta\left(T - T_\mathrm{c}\right) + 1\right\},$$

$$\theta(T - T_\mathrm{c}) = \begin{cases} 1, (T > T_\mathrm{c}), \\ 0, (T < T_\mathrm{c}), \end{cases} \tag{10}$$

where T_c is the Bose–Einstein condensation temperature defined as

$$T_\mathrm{c} = \frac{h^2}{2\pi mk}\left[\frac{N}{V\zeta(3/2)}\right]^{2/3} = \frac{h^2}{2\pi mk}\left[v\zeta(3/2)\right]^{-2/3}, \tag{11}$$

with $\zeta(3/2) = 2.612$.

3. An irreversible Bose Brayton engine model

A Brayton engine cycle using an ideal Bose gas as the working substance may be shortly called a Bose Brayton engine cycle, which is composed of two adiabatic and two isobaric processes. According to the expression of the entropy of an ideal gas, we can easily devise the schematic diagram of an irreversible Brayton engine cycle, as shown in Fig. 1, where p_H and p_L are the pressures of the high and the low isobaric processes, respectively; Q_H and Q_L are the amounts of heat exchange between the working substance and the heat reservoirs; T_L, T_2, $T_{2\mathrm{s}}$, T_H, $T_{4\mathrm{s}}$, and T_4 are the temperatures of the working substance in states 1, 2, 2s, 3, 4s, and 4, respectively; T_H and T_L are the temperatures of the heat and the cold reservoirs, respectively. The temperatures of the working substance obey the following relation: $T_\mathrm{L} < T_{4\mathrm{s}} < T_4 < T_{2\mathrm{s}} < T_2 < T_\mathrm{H}$.

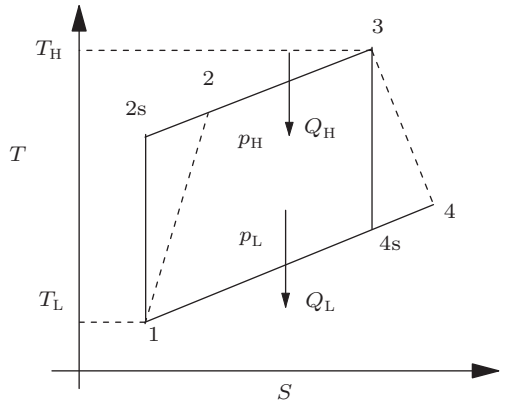


Fig. 1. Entropy–temperature diagram of quantum Brayton engine cycle.

The first stage of the cycle 1–2 is an irreversible adiabatic process, in which the pressure is increased from p_L to p_H and the temperature of the working substance is raised from T_L to T_2. In this process, the entropy of the working substance is increased. The process 1–2s is the corresponding reversible adiabatic process, in which the entropy of the working substance is unvaried. Using Eq. (7), we obtain

$$\frac{T_\mathrm{H}}{T_{2\mathrm{s}}} = \left(\frac{p_\mathrm{H}}{p_\mathrm{L}}\right)^{2/5} = r_p^{2/5}. \tag{12}$$

The second stage 2–3 is an isobaric process, in which $p = p_\mathrm{H}$ and the working substance increases its temperature from T_2 to T_H by absorbing heat Q_H from the heat reservoir. Using Eq. (9), we obtain

$$Q_\mathrm{H} = Q_{23} = \int_{T_\mathrm{H}}^{T_2} C_p\mathrm{d}T = \frac{5}{2}Nk[T_2F(T_2,p_\mathrm{H}) - T_\mathrm{H}F(T_\mathrm{H},p_\mathrm{H})]. \tag{13}$$

The third stage 3–4 is an irreversible adiabatic process, in which the pressure is decreased from p_H

to $p_{\rm L}$ and the temperature of the working substance drops from $T_{\rm H}$ to T_4. The process 3–4s is the corresponding reversible adiabatic process, which satisfies the equation

$$\frac{T_{4s}}{T_{\rm L}} = \left(\frac{p_{\rm H}}{p_{\rm L}}\right)^{2/5} = r_p^{2/5}. \tag{14}$$

The fourth stage 4–1 is an isobaric process, in which $p = p_{\rm L}$ and the working substance reduces its temperature from T_4 to $T_{\rm L}$ by releasing heat $Q_{\rm L}$ to the cold reservoir. Using Eq. (9), we obtain

$$\begin{aligned} Q_{\rm H} = Q_{41} &= \int_{T_4}^{T_{\rm L}} C_p {\rm d}T \\ &= \frac{5}{2}Nk[T_{\rm L}F(T_{\rm L}, p_{\rm L}) - T_4 F(T_4, p_{\rm L})]. \end{aligned} \tag{15}$$

For the two adiabatic processes, the compression and the expansion efficiencies[3,13]

$$\eta_{\rm e} = \frac{T_4 - T_3}{T_{4\rm s} - T_3}, \tag{16}$$

and

$$\eta_{\rm c} = \frac{T_{2\rm s} - T_1}{T_2 - T_1} \tag{17}$$

may be used to describe the irreversibilities of these processes.

Using Eqs. (16) and (17) and adiabatic Eqs. (12) and (14) of an ideal Bose gas

$$\frac{T_{2\rm s}}{T_{\rm L}} = \frac{T_{\rm H}}{T_{4\rm s}} = \left(\frac{p_{\rm H}}{p_{\rm L}}\right)^{2/5} = r_p^{2/5}, \tag{18}$$

we obtain

$$T_2 = \alpha T_{\rm L} = \left(1 - \frac{1}{\eta_{\rm c}} + \frac{r_p^{2/5}}{\eta_{\rm c}}\right) T_{\rm L} \tag{19}$$

and

$$T_4 = \beta T_{\rm H} = \left(1 - \eta_{\rm e} + \frac{\eta_{\rm e}}{r_p^{2/5}}\right) T_{\rm H}. \tag{20}$$

By using Eqs. (13), (15), (19), and (20), according to quantum statistical mechanics and the thermodynamic properties of an ideal Bose gas, the amounts of heat exchange between the working substance and the heat reservoirs in the processes mentioned above are respectively given as

$$Q_{\rm H} = \frac{5}{2}NkT_{\rm H}[F(T_{\rm H}, p_{\rm H}) - \alpha\tau F(\alpha\tau T_{\rm H}, p_{\rm H})] \tag{21}$$

and

$$Q_{\rm L} = \frac{5}{2}NkT_{\rm H}[\beta F(\beta T_{\rm H}, p_{\rm L}) - \tau F(\tau T_{\rm H}, p_{\rm L})]. \tag{22}$$

Using Eqs. (21) and (22), we obtain the work output per cycle as

$$\begin{aligned} W = \frac{5}{2}NkT_{\rm H}\big\{&[F(T_{\rm H}, p_{\rm H}) - \alpha\tau F(\alpha\tau T_{\rm H}, p_{\rm H})] \\ &- [\beta F(\beta T_{\rm H}, p_{\rm L}) - \tau F(\tau T_{\rm H}, p_{\rm L})]\big\}. \end{aligned} \tag{23}$$

4. Expressions for several important parameters

The power output and the efficiency are two important parameters of heat engines, which are often considered in the optimal design and the theoretical analysis of heat engines. In order to optimize the power output and the efficiency of the Bose Brayton engine cycle, we need to calculate the cycle period. It is often assumed that the time spent on a process is proportional to the temperature difference of the process.[46–47] Consequently, we can calculate the time spent on the various processes as

$$\begin{aligned} t_{12} &= k_1(T_2 - T_1) = k_1(T_2 - T_{\rm L}) \\ &= k_1 T_{\rm H}\tau(\alpha - 1), \end{aligned} \tag{24}$$

$$\begin{aligned} t_{23} &= k_2(T_3 - T_2) = k_2(T_{\rm H} - T_2) \\ &= k_2 T_{\rm H}(1 - \alpha\tau), \end{aligned} \tag{25}$$

$$\begin{aligned} t_{34} &= k_3(T_{\rm H} - T_4) = k_3(T_{\rm H} - \beta T_{\rm H}) \\ &= k_3 T_{\rm H}(1 - \beta), \end{aligned} \tag{26}$$

$$\begin{aligned} t_{41} &= k_4(T_4 - T_1) = k_4(T_4 - T_{\rm L}) \\ &= k_4 T_{\rm H}(\beta - \tau), \end{aligned} \tag{27}$$

where k_1, k_2, k_3, and k_4 are four proportionality coefficients. Thus, the cycle period is given by

$$\begin{aligned} t &= t_{12} + t_{23} + t_{34} + t_{41} \\ &= k_1 T_{\rm H}\Big[\tau(\alpha - 1) + \frac{k_2}{k_1}(1 - \alpha\tau) + \frac{k_3}{k_1}(1 - \beta) \\ &\quad + \frac{k_4}{k_1}(\beta - \tau)\Big]. \end{aligned} \tag{28}$$

By using Eqs. (21), (22), (23), and (28), we can derive the expressions for the power output and the power efficiency. They may be written as

$$P = \frac{W}{t} = \frac{5}{2}Nk\frac{[F(T_{\rm H}, p_{\rm H}) - \alpha\tau F(\alpha\tau T_{\rm H}, p_{\rm H})] - [\beta F(\beta T_{\rm H}, p_{\rm L}) - \tau F(\tau T_{\rm H}, p_{\rm L})]}{k_1\tau(\alpha - 1) + k_2(1 - \alpha\tau) + k_3(1 - \beta) + k_4(\beta - \tau)}, \tag{29}$$

$$\eta = 1 - \frac{Q_{\rm L}}{Q_{\rm H}} = 1 - \frac{\beta F(\beta\tau T_{\rm L}, p_{\rm L}) - \tau F(T_{\rm L}, p_{\rm L})}{F(\tau T_{\rm L}, r_p p_{\rm L}) - \alpha\tau F(\alpha T_{\rm L}, r_p p_{\rm L})}. \tag{30}$$

5. Important performance bounds and the optimal operating region

It is clearly seen from Eqs. (29) and (30) that there are different performance characteristics for the Bose Brayton engine cycles operated under different temperatures, pressures, and other parameters. Some special cases are discussed here.

5.1. Weak gas degeneracy

Under the high-temperature or the low-density condition, i.e., the condition of weak gas degeneracy, function $F(T,P)$ and fugacity z in the weak gas degeneracy case may be extended to the first approximation as

$$F(T,p)=1-\frac{Cp}{T^{5/2}}. \tag{31}$$

By using Eq. (31), Eqs. (29) and (30) can be respectively expressed as

$$P=\frac{W}{t}=\frac{5Nk}{2k_1}\frac{1-\alpha\tau-\beta+\tau+\left[\left(\alpha^{-3/2}-\tau^{3/2}\right)r_p-\left(1-\tau^{3/2}\beta^{-3/2}\right)\right]\tau Cp_{\rm L}/T_{\rm L}^{5/2}}{\tau(\alpha-1)+\dfrac{k_2}{k_1}(1-\alpha\tau)+\dfrac{k_3}{k_1}(1-\beta)+\dfrac{k_4}{k_1}(\beta-\tau)}, \tag{32}$$

$$\eta=1-\frac{Q_{\rm L}}{Q_{\rm H}}=1-\frac{\beta-\tau+\tau\left(1-\tau^{3/2}\beta^{-3/2}\right)Cp_{\rm L}/T_{\rm L}^{5/2}}{1-\alpha\tau+\tau r_p\left(\alpha^{-3/2}-\tau^{3/2}\right)Cp_{\rm L}/T_{\rm L}^{5/2}}. \tag{33}$$

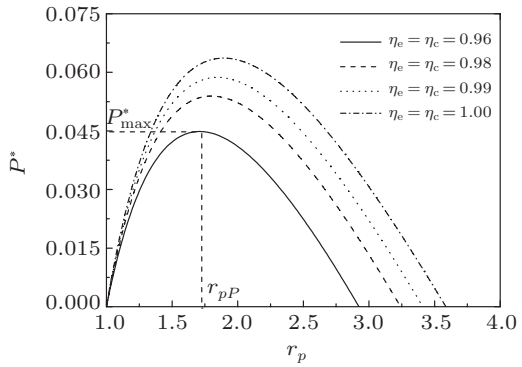


Fig. 2. The P^*–r_p curves of an irreversible quantum Brayton heat engine with parameters of $p_{\rm L}=2\times10^5$ Pa, $T_{\rm L}=15$ K, $\tau=0.6$, and $k_1=k_2=k_3=k_4$.

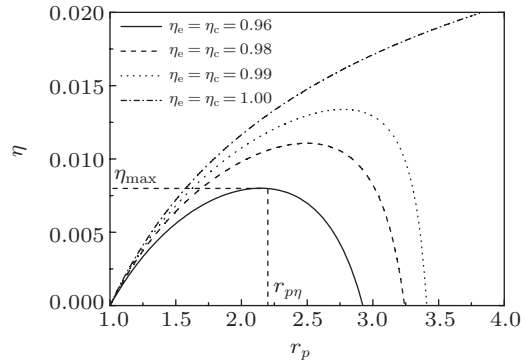


Fig. 3. The η–r_p curves of an irreversible quantum Brayton heat engine with parameters of $p_{\rm L}=2\times10^5$ Pa, $T_{\rm L}=15$ K, $\tau=0.6$, and $k_1=k_2=k_3=k_4$.

Using Eqs. (32) and (33), we can plot the curves of the power output and the power efficiency varying with the pressure ratio, as shown in Figs. 2 and 3, where the parameters are chosen as $p_{\rm L}=2\times10^5$ Pa, $T_{\rm L}=15$ K, $\tau=0.6$ and $k_1=k_2=k_3=k_4$. The $P^*=\frac{2k_1}{5Nk}P$ is the dimensionless power output, r_{pP} and $r_{p\eta}$ are the pressure ratios at the maximum dimensionless power output $P^*_{\rm max}$ and the maximum efficiency $\eta_{\rm max}$, respectively. The curves in Figs. 2 and 3 show clearly that for general cases, the compression and the expansion efficiencies are smaller than one, and there always exist a maximum power output and a maximum efficiency.

According to Eqs. (32) and (33) and their extremal conditions, we can prove that when $\partial P/\partial r_p=0$ and $\partial\eta/\partial r_p=0$, the power output and the efficiency attain their maxima, respectively. However the corresponding pressure ratios r_{pP} and $r_{p\eta}$ are too complicated to yield simple analytical solutions. By using the numerical computation, we obtain $P^*_{\rm max}$, $\eta_{\rm max}$, r_{pP}, and $r_{p\eta}$ shown in Figs. 2 and 3.

Using Eqs. (32) and (33), we can further obtain the characteristic curves of the dimensionless power output versus the efficiency for the irreversible quantum Bose Brayton engine, as shown in Figs. 4 and 5, where $\eta_{\rm m}$ and $P^*_{\rm m}$ are the efficiency at the maximum dimensionless power output and the dimensionless power output at the maximum efficiency, respectively. Figures 4 and 5 show that when the irreversible quantum Bose Brayton engine is operating in those parts of the P^*–η curve, each of which has a positive slope, the dimensionless power output will decrease as the efficiency decreases. Those regions are not the optimal operating regions of the engine. The conclusion can also be explained by using Figs. 2 and 3. In the region of $r_p\geq r_{pP}$, both the dimensionless power

output and the efficiency will decrease as the pressure ratio is increased, while in the region of $r_p \leq r_{p\eta}$, both the dimensionless power output and the efficiency will decrease as the pressure ratio is decreased. It is obvious that the regions of $r_p \geq r_{pP}$ and $r_p \leq r_{p\eta}$ are not optimal, although the Bose Brayton engine can work in these regions.

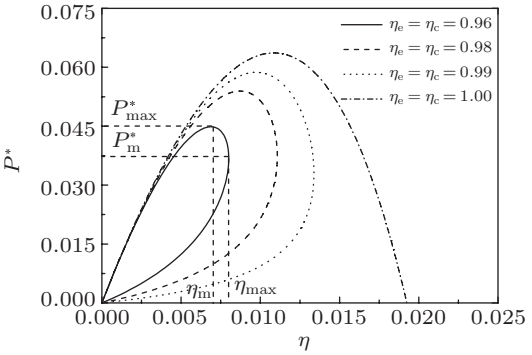


Fig. 4. The P^*–η curves of irreversible quantum Brayton heat engine with parameters of $p_{\rm L} = 2 \times 10^5$ Pa, $T_{\rm L} = 15$ K, $\tau = 0.6$, and $k_1 = k_2 = k_3 = k_4$.

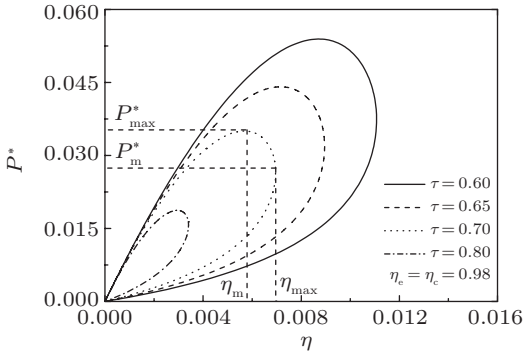


Fig. 5. The P^*–η curves of irreversible quantum Brayton heat engine with parameters of $p_{\rm L} = 2 \times 10^5$ Pa, $T_{\rm L} = 15$ K, $\eta_{\rm e} = \eta_{\rm e} = 0.98$, and $k_1 = k_2 = k_3 = k_4$.

Obviously, the optimal operating region of the quantum Bose Brayton engine should be situated in the part of the P^*–η curve that has a negative slope. When the quantum Bose Brayton engine is operated in this region, the dimensionless power output will increase as the efficiency is decreased, and vice versa. Thus, the optimal ranges of the dimensionless power output and the efficiency should be

$$P^*_{\rm m} \leq P^* \leq P^*_{\max} \tag{34}$$

and

$$\eta_{\max} \geq \eta \geq \eta_{\rm m}. \tag{35}$$

The above results show that $P^*_{\max}$, $\eta_{\max}$, $P^*_{\rm m}$, and $\eta_{\rm m}$ are four important parameters of the Bose Brayton engine. The $P^*_{\max}$ and $\eta_{\max}$ determine the upper bounds of the dimensionless power output and the efficiency, while $P^*_{\rm m}$ and $\eta_{\rm m}$ determine the allowable values of the lower bounds of the dimensionless power output and the efficiency. Moreover, it is also clearly seen in Figs. 4 and 5 that the optimal ranges of the power output and the efficiency decrease with the decreases of the compression and the expansion efficiency.

According to Eqs. (34) and (35), we can further determine the optimal region of pressure ratio r_{pP} for the irreversible Brayton heat engine as

$$r_{p\eta} \leq r_p \leq r_{pP}. \tag{36}$$

Thus, it is clear that r_{pP} and $r_{p\eta}$ are two important parameters of the quantum Bose Brayton engine, which determine the upper and the lower bounds of the optimized pressure ratio, respectively. As shown in Fig. 5, both dimensionless power output $P^*_{\max}$ and efficiency $\eta_{\max}$ decrease with the increases of temperature ratio τ and cold reservoir temperature $T_{\rm L}$ for given parameters of $p_{\rm L}$, $\eta_{\rm e}$, $\eta_{\rm c}$, k_1, k_2, k_3, and k_4. This indicates that the larger the Bose gas degeneracy is, the lower the power output and the efficiency of the Bose Brayton engine cycle are.

5.2. At high temperatures

When the temperature of the working substance is high enough and its density is low enough, the fugacity of the gas, z, is much smaller than 1. In such a case, $F(z) = 1$, and the ideal Bose gas becomes the ideal classical gas. Equations (29) and (30) can be respectively simplified into

$$P = \frac{W}{t} = \frac{5Nk}{2k_1} \frac{1 - \beta + \tau - \alpha\tau}{\tau(\alpha - 1) + \frac{k_2}{k_1}(1 - \alpha\tau) + \frac{k_3}{k_1}(1 - \beta) + \frac{k_4}{k_1}(\beta - \tau)} \tag{37}$$

and

$$\eta = 1 - \frac{Q_{\rm L}}{Q_{\rm H}} = 1 - \frac{\beta - \tau}{1 - \alpha\tau}, \tag{38}$$

which are just the results of a classical Brayton power cycle. This shows clearly that at high temperatures, the Bose Brayton engine becomes a classical one.

Finally, according to the numerical computation,

we calculate the corresponding dimensionless power inputs $P^*_{\max}$ and $P^*_{\rm m}$, efficiencies $\eta_{\max}$ and $\eta_{\rm m}$, and pressure ratios r_{pP} and $r_{p\eta}$ of the Bose Brayton heat engine cycle for the parameters of $p_{\rm L} = 2 \times 10^5$ Pa, $T_{\rm L} = 15$ K, and $k_1 = k_2 = k_3 = k_4$. The results are listed in Table 1.

Table 1. Corresponding dimensionless power outputs $P^*_{\max}$ and $P^*_{\rm m}$, efficiencies $\eta_{\max}$ and $\eta_{\rm m}$, pressure ratios r_{pP} and $r_{p\eta}$ of the Bose Brayton engine cycle with parameters of $p_{\rm L} = 2 \times 10^5$ Pa, $T_{\rm L} = 15$ K, and $k_1 = k_2 = k_3 = k_4$.

τ	$\eta_{\rm c}$ $(= \eta_{\rm e})$	$r_{p\eta}$	$\eta_{\max}$	$P^*_{\rm m}$	r_{pP}	$P^*_{\max}$	$\eta_{\rm m}$
$\tau = 0.60$	0.96	2.148	0.0080	0.0368	1.1714	0.0448	0.0069
	0.98	2.498	0.0111	0.0373	1.798	0.0539	0.0087
	0.99	2.778	0.0134	0.0328	1.854	0.0587	0.0098
	1	–	–	–	1.896	0.0636	0.01086
$\tau = 0.70$	0.96	1.588	0.0044	0.0233	1.406	0.0265	0.0039
	0.98	1.812	0.0070	0.0262	1.49	0.0350	0.0057
	0.99	1.966	0.0090	0.0251	1.518	0.0397	0.0067
	1	–	–	–	1.56	0.0446	0.0078
$\tau = 0.80$	0.96	1.252	0.0016	0.0104	1.21	0.0112	0.0016
	0.98	1.378	0.0034	0.0157	1.252	0.0188	0.0029
	0.99	1.476	0.0050	0.0165	1.294	0.0231	0.0040

6. Conclusion

A thermodynamic model for an irreversible quantum Brayton engine operating with the ideal Bose gas is established. The influences of both the quantum degeneracy and the finite time thermodynamic processes on the performance of the Brayton engine are investigated. Based on the theory of finite time thermodynamic processes, the expressions for the power output and the efficiency are derived. By using the numerical computation, the optimal relationship between the power output and the efficiency of an irreversible Bose Brayton engine is obtained. The optimal regions of the power output and the efficiency are determined. It is found that the influences of the irreversibility and the quantum degeneracy on the main performance parameters of the Bose Brayton engine are remarkable. The results obtained in the present paper can provide some new theoretical information for the optimal design and the performance improvement of a real Brayton engine.

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