Rapid Communication

Spontaneous symmetry breaking of a Bose–Fermi mixture in a two-dimensional double-well potential*

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(Received 2 August 2011; revised manuscript received 24 August 2011)

We study the spontaneous symmetry breaking of a superfluid Bose–Fermi mixture in a two-dimensional double-well potential. The mixture is described by a set of coupled Gross–Pitaevskii equations. The symmetry breaking phenomenon is demonstrated in the two-dimensional double-well potential in the mixture. The results are summarized in the phase diagrams of the mixture particle numbers, which are divided into symmetric and asymmetric regions by the asymmetry ratios. The dynamical pictures of the spontaneous symmetry breaking induced by a gradual transformation of the single-well potential into a double-well one are also illustrated. The properties of the quantum degenerate mixture are explored using the realistic parameters for a ^{40}K–^{87}Rb system.

Keywords: spontaneous symmetry breaking, Bose–Fermi mixture, double-well potential

PACS: 03.75.Ss, 03.75.Hh **DOI:** 10.1088/1674-1056/21/1/010309

Using the sympathetic-cooling technique, degenerate Bose–Fermi (BF) mixtures of atoms were produced.[1] A series of successful experimental and theoretical investigations of the mixtures of the trapped degenerate Fermi gas (DFG) and the Bose–Einstein condensate (BEC) have been performed by different groups.[2−10] One topic of great interest of the BF mixtures is the spontaneous symmetry breaking (SSB) of nonlinear fields trapped in the double-well potentials (DWPs). SSB is a phenomenon in which symmetric states in symmetric potentials appear to be broken when the strength of the self-attractive nonlinearity exceeds a certain critical value.[11,12] In a coupled two-component Bose–Einstein condensate, both the mean-field and the full quantum dynamics of SSB have been studied. In particular, in various two-dimensional (2D) settings,[13−15] the matter-wave solitons and the solitary vortices were predicted under the combined action of the harmonic traps and the optical-lattice potentials.[16] SSB was also studied in a nonlinear DWP, which was induced by the spatial modulation of the nonlinearity coefficient rather than the use of the linear potential.[11] Several other settings have also been studied theoretically, including optical models with the cubic-quintic nonlinearity acting inside linearly coupled cores,[17] dipolar BECs,[18] BEC mixtures,[19] BF mixtures,[2] Fermi superfluids[20] and solitons in two-component linearly coupled nonlinear systems.[21]

Very recently, Adhikari *et al.*[22] have reported the SSB of the superfluid BF mixture in a one-dimensional (1D) DWP. They introduced a model, which considered the SSB in a semi-trapped BF mixture with the DWP acting on either the fermions or the bosons. The results demonstrated that the symmetry breaking was possible in the mixture with an attraction between the fermions and the bosons.

In this article, we study the SSB of the trapped BF mixture due to the BF attractive interaction in a 2D DWP and deal with the system with the confining potential acting on both the components. We take physical parameters corresponding to the mixture of the ultracold ^{40}K–^{87}Rb atoms. By means of numerical solutions of the Gross–Pitaevskii (GP) equations, the symmetry breaking phenomenon is demonstrated for the mixture in the 2D DWP. The results are summarized in the phase diagrams of the mixture particle numbers, which are divided into symmetric and asymmetric regions by the asymmetry ratios. The dynamical pictures of the SSB induced by a gradual transformation of the single-well potential into the DWP are also illustrated.

The action obtained from the density-functional theory[23,24] for the superfluid of the BF mixture is

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10974068 and 11174108).

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given by

$$S = \int (L_{\rm B} + L_{\rm F} + L_{\rm BF}) {\rm d}^3 r {\rm d}t, \tag{1}$$

where $L_{\rm B}$, $L_{\rm F}$, and $L_{\rm BF}$ are the Lagrangian densities of the ordinary Bose gas, the Fermi gas with two equally populated spin components and the interaction between the two components, respectively. Reference [22] gives a detailed account of their expressions. The confining potentials for the bosons and the fermions are $U_{\rm B}(\boldsymbol{r}) = \frac{1}{2} m_{\rm B} \omega_{\perp {\rm B}}^2 x^2 + V_{\rm B}(y,z)$ and $U_{\rm F}(\boldsymbol{r}) = \frac{1}{2} m_{\rm F} \omega_{\perp {\rm F}}^2 x^2 + V_{\rm F}(y,z)$, respectively, where $m_{\rm B}$ and $m_{\rm F}$ are the masses of the bosons and the fermions, respectively, $\omega_{\perp {\rm B}}$ and $\omega_{\perp {\rm F}}$ are the frequencies of the confining harmonic potentials in the x direction. The $V_{\rm B}(y,z)$ and $V_{\rm F}(y,z)$ are the potentials acting in the (y,z) plane,

$$\begin{aligned} V_{\rm B}(y,z) &= \frac{1}{2} m_{\rm B} \omega_{\rm B}^2 \left(y^2 + z^2\right) \\ &\quad + A \exp(-y^2/2\sigma_0^2), \end{aligned} \tag{2}$$

$$\begin{aligned} V_{\rm F}(y,z) &= \frac{1}{2} m_{\rm F} \omega_{\rm F}^2 \left(y^2 + z^2\right) \\ &\quad + A \exp(-y^2/2\sigma_0^2), \end{aligned} \tag{3}$$

where $\omega_{\rm B}$ and $\omega_{\rm F}$ are the frequencies of the confining harmonic potentials in the y and the z directions, respectively, and A and σ_0 are the height and the width of a Gaussian profile modeling the barrier between the two potential wells along the z direction. For the numerical results presented below, we set $\omega_{\perp {\rm B}}/\omega_{\rm B} = \omega_{\perp {\rm F}}/\omega_{\rm F} = 10$, $A = A_0 \hbar \omega_{\rm F}, A_0 = 5$, $\sigma_0 = 0.2 a_{h_0}$, where $\hbar$ is the reduced Planck constant, and $a_{h_0} = \sqrt{\hbar/(2 m_{\rm F} \omega_{\rm F})}$ is the oscillator length in the y and the z directions. The harmonic frequency for K is $\omega_{\rm F} = 2\pi \times 40$ Hz, and $\omega_{\rm B} = \omega_{\rm F}/\sqrt{m_{\rm B}/m_{\rm F}}$.[1] The system is quasi-2D, that is, the BF mixture is disk shaped in the (y,z) plane. The variational wave functions for the macroscopic BEC and the Fermi gas at unitary can be written as

$$\psi_{\rm B}(\boldsymbol{r},t) = \sqrt{N_{\rm B}} \frac{{\rm e}^{-x^2/\left(2a_{\perp {\rm B}}^2\right)}}{\left(\pi^{1/2} a_{\perp {\rm B}}\right)^{1/2}} \varphi_{\rm B}\left(y,z,t\right), \tag{4}$$

$$\psi_{\rm F}(\boldsymbol{r},t) = \sqrt{N_{\rm F}} \frac{{\rm e}^{-x^2/\left(2a_{\perp {\rm F}}^2\right)}}{\left(\pi^{1/2} a_{\perp {\rm F}}\right)^{1/2}} \varphi_{\rm F}\left(y,z,t\right), \tag{5}$$

where $N_{\rm B}$ and $N_{\rm F}$ are the particle numbers of the bosons and the fermions, $a_{\perp {\rm B}} = \sqrt{\hbar/(m_{\rm B}\omega_{\perp {\rm B}})}$ and $a_{\perp {\rm F}} = \sqrt{\hbar/(2 m_{\rm F}\omega_{\perp {\rm F}})}$ are the characteristic harmonic lengths of the oscillators in the x direction. By inserting expressions (4) and (5) into Eq. (1), we can write the action as

$$S = S^{(2{\rm D})} - \hbar \omega_{\perp {\rm B}} N_{\rm B} t/2 - \hbar \omega_{\perp {\rm F}} N_{\rm F} t/4, \tag{6}$$

with the effective 2D action

$$\begin{aligned} S^{(2{\rm D})} &= \int {\rm d}t \int_{-\infty}^{+\infty} {\rm d}y \int_{-\infty}^{+\infty} {\rm d}z \\ &\quad \times (L_{\rm B}^{(2{\rm D})} + L_{\rm F}^{(2{\rm D})} + L_{\rm BF}^{(2{\rm D})}), \end{aligned} \tag{7}$$

where the usual 2D GP Lagrangian densities are

$$\begin{aligned} L_{\rm B}^{(2{\rm D})} &= N_{\rm B} \Bigg[\frac{{\rm i}\hbar}{2} \left(\varphi_{\rm B}^* \frac{\partial \varphi_{\rm B}}{\partial t} - \varphi_{\rm B} \frac{\partial \varphi_{\rm B}^*}{\partial t} \right) \\ &\quad - \frac{\hbar^2}{2 m_{\rm B}} \left(\left| \frac{\partial \varphi_{\rm B}}{\partial y} \right|^2 + \left| \frac{\partial \varphi_{\rm B}}{\partial z} \right|^2 \right) \\ &\quad - V_{\rm B} \left|\varphi_{\rm B}\right|^2 - G_{\rm B} \left|\varphi_{\rm B}\right|^4 \Bigg], \end{aligned} \tag{8}$$

$$\begin{aligned} L_{\rm F}^{(2{\rm D})} &= N_{\rm F} \Bigg[\frac{{\rm i}\hbar}{4} \left(\varphi_{\rm F}^* \frac{\partial \varphi_{\rm F}}{\partial t} - \varphi_{\rm F} \frac{\partial \varphi_{\rm F}^*}{\partial t} \right) \\ &\quad - \frac{\hbar^2}{8 m_{\rm F}} \left(\left| \frac{\partial \varphi_{\rm F}}{\partial y} \right|^2 + \left| \frac{\partial \varphi_{\rm F}}{\partial z} \right|^2 \right) \\ &\quad - V_{\rm F} \left|\varphi_{\rm F}\right|^2 - G_{\rm F} \left|\varphi_{\rm F}\right|^{10/3} \Bigg], \end{aligned} \tag{9}$$

$$L_{\rm BF}^{(2{\rm D})} = -G_{\rm BF} N_{\rm B} N_{\rm F} \left|\varphi_{\rm B}\right|^2 \left|\varphi_{\rm F}\right|^2. \tag{10}$$

The coupling constants $G_{\rm B}$, $G_{\rm F}$ and $G_{\rm BF}$ are the intra- and inter-species atom–atom interaction strengths, respectively. These constants are given by

$$G_{\rm B} = (2\pi)^{1/2} N_{\rm B} \hbar^2 a_{\rm B}/(m_{\rm B} a_{\perp {\rm B}}), \tag{11}$$

$$G_{\rm F} = 3\pi\xi(3/5)^{1/2} (3N_{\rm F})^{2/3} \hbar^2 / \left(10 m_{\rm F} a_{\perp {\rm F}}^{2/3}\right), \tag{12}$$

$$G_{\rm BF} = 2\hbar^2 a_{\rm BF} \left[\pi/\left(a_{\perp {\rm B}}^2 + a_{\perp {\rm F}}^2\right)\right]^{1/2}/m_{\rm R}, \tag{13}$$

where $m_{\rm R} = m_{\rm B} m_{\rm F}/(m_{\rm B} + m_{\rm F})$ is the reduced mass. In the mixture, ^{40}K is in two equally populated hyperfine states $|F = 9/2,\ M_F = -9/2\rangle$ and $|F = 9/2,\ M_F = -7/2\rangle$, and ^{87}Rb is in the $|F = 2,\ M_F = 2\rangle$ state. The s wave corresponds to an attractive BF interaction $a_{\rm BF} \approx -284 a_0$, where a_0 is the Bohr radius.[5] The boson–boson scattering parameter is $a_{\rm B} = 108 a_0$.[5] We introduce units of length and frequency as $a = 1$ μm and $\omega = 2\pi \times 100$ Hz, respectively.

Using the Euler–Lagrange equations with respect to $\varphi_{\rm B}^*$ and $\varphi_{\rm F}^*$ to effective action (7), we obtain a set of coupled 2D GP equations

$$\begin{aligned} {\rm i} \frac{\partial}{\partial \tau} \Phi_{\rm B} &= \Bigg[-\frac{\lambda_{\rm B}}{2} \left(\frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} \right) + W_{\rm B} \\ &\quad + g_{\rm B} \left|\Phi_{\rm B}\right|^2 + g_{\rm BF} N_{\rm F} \left|\Phi_{\rm F}\right|^2 \Bigg] \Phi_{\rm B}, \end{aligned} \tag{14}$$

$$\frac{\rm i}{2} \frac{\partial}{\partial \tau} \Phi_{\rm F} = \Bigg[-\frac{\lambda_{\rm F}}{4} \left(\frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} \right) + W_{\rm F}$$

$$+ g_{\rm F} |\Phi_{\rm F}|^{4/3} + g_{\rm BF} N_{\rm B} |\Phi_{\rm B}|^2 \Big] \Phi_{\rm F}, \quad (15)$$

where $Y = y/a$, $Z = z/a$, $\tau = \omega t$, $\Phi_{\rm B} = a\varphi_{\rm B}$, $\Phi_{\rm F} = a\varphi_{\rm F}$, $W_{\rm B} = V_{\rm B}/(\hbar\omega)$, $W_{\rm F} = V_{\rm F}/(\hbar\omega)$, $\lambda_{\rm B} = a_{\perp{\rm B}}^2 \omega_{\perp{\rm B}}/(a^2\omega)$, $\lambda_{\rm F} = a_{\perp{\rm F}}^2 \omega_{\perp{\rm F}}/(a^2\omega)$, and the rescaled coupling constants derived from expressions (11)–(13) are

$$g_{\rm B} = 2(2\pi)^{1/2} N_{\rm B} a_{\rm B} a_{\perp{\rm B}}/a^2, \quad (16)$$

$$g_{\rm BF} = 2\hbar a_{\rm BF} \left[\pi/\left(a_{\perp{\rm B}}^2 + a_{\perp{\rm F}}^2\right)\right]^{1/2}/\left(m_{\rm R}\omega a^2\right), \quad (17)$$

$$g_{\rm F} = (3/5)^{1/2} \pi\hbar\xi (3N_{\rm F})^{2/3}/\left(2m_{\rm F}\omega a_{\perp{\rm F}}^{2/3} a^{4/3}\right). \quad (18)$$

In our numerical simulations, we adopt the Crank–Nicolson finite-difference scheme in imaginary time and use the FORTRAN codes provided in Ref. [25]. The discretization used in our calculations is $\Delta\tau = 0.002$ in time and $\Delta Y = \Delta Z = 0.045$ in space. The calculation is performed in an area that is large enough to trap the mixture.

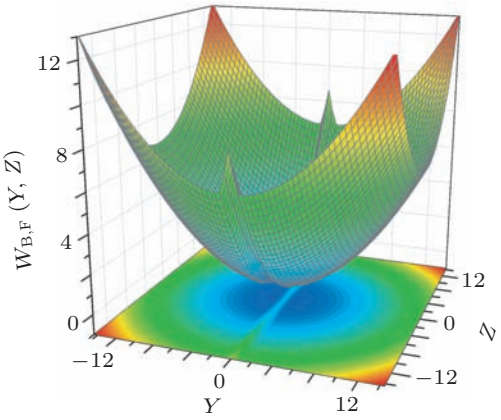


Fig. 1. (colour online) The 2D double-well potential.

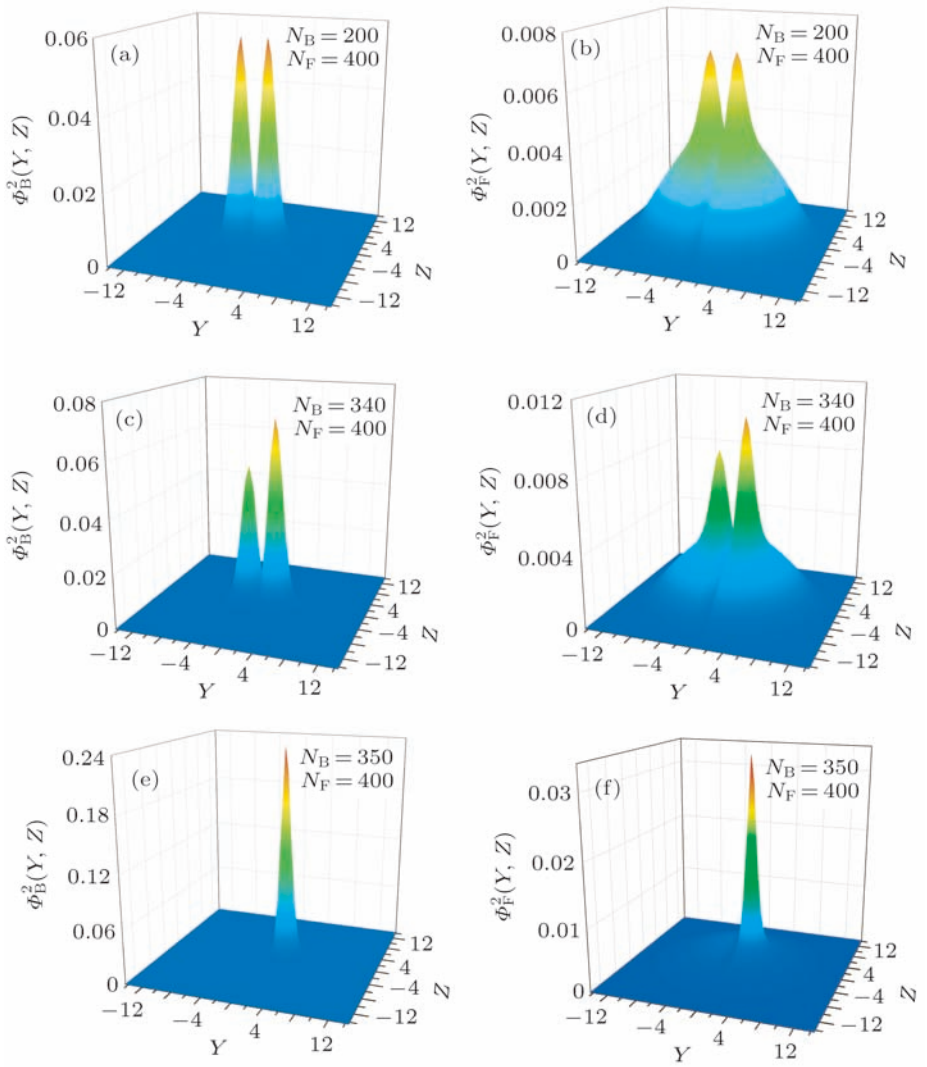


Fig. 2. (colour online) The 2D ground-state density profiles of the ^{40}K and ^{87}Rb mixture. The numbers of bosons $N_{\rm B}$ are 200 for panels (a) and (b), 340 for panels (c) and (d), and 350 for panels (e) and (f). The number of fermions $N_{\rm F}$ is fixed at 400.

We consider the 2D DWP (Eqs. (2) and (3)) acting on the two components. The 2D DWP is depicted in Fig. 1. We demonstrate that the BF interaction can substantially affect the symmetry of the degenerate mixture by varying the values of $N_{\rm B}$ and $N_{\rm F}$. In Fig. 2, we plot the 2D density profiles of the ^{40}K and ^{87}Rb mixture in the ground state obtained by evolving Eqs. (14) and (15) in imaginary time with different values of $N_{\rm B}$ and a fixed number of fermions ($N_{\rm F} = 400$). We can clearly see that the wave functions of both components are symmetric and equally distributed in the two wells in Fig. 2(a) and 2(b) for $N_{\rm B} = 200$ and $N_{\rm F} = 400$. The repulsive interactions between boson and boson and between fermion and fermion make the density distributions of both components very broad. When we increase the number of the bosons, the profile of the ground state is not symmetric, as shown in Figs. 2(c) and 2(d) for $N_{\rm B} = 340$ and $N_{\rm F} = 400$, that is to say, an attractive interaction can lead to SSB and narrower distributions for a sufficiently large number of bosons. For $N_{\rm B} = 350$ and $N_{\rm F} = 400$, Figs. 2(e) and 2(f) show that the density profile of the asymmetric states evolves from the double-peak pattern into an asymmetric single-peak one and concentrtes nearly in one of the wells. SSB happens when $N_{\rm B}$ is in the range from 200 and 340 with a fixed number of fermions ($N_{\rm F} = 400$).

Similar to that shown in Fig. 2, the symmetric ground state is also replaced by an asymmetric one with the increase of $N_{\rm F}$ at fixed $N_{\rm B}$. An example of the transition from the symmetric ground state of the BF mixture to an asymmetric one is displayed in Fig. 3. When the number of the fermions increases from 200 to 390 at $N_{\rm B} = 350$, the SSB phenomenon appears.

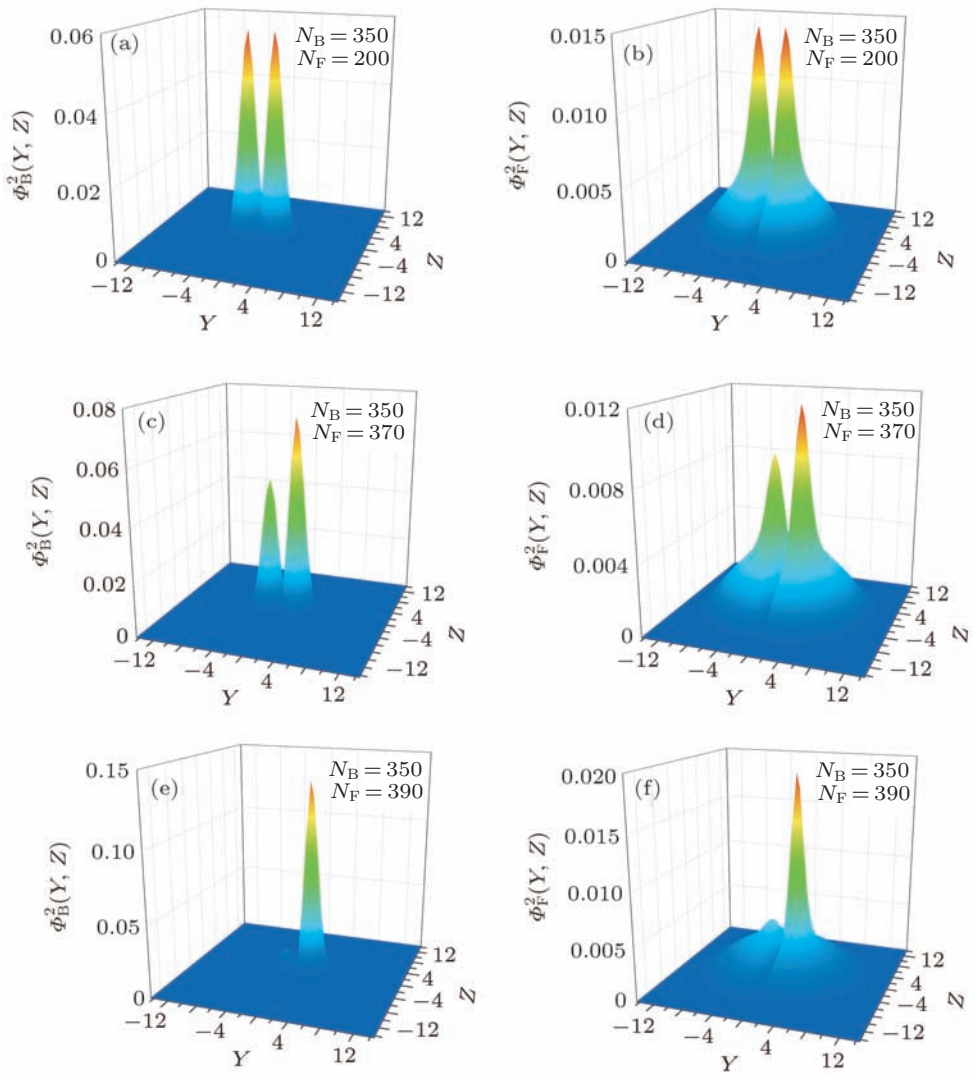


Fig. 3. (colour online) The 2D ground-state density profiles of the ^{40}K and ^{87}Rb mixture. The numbers of fermions $N_{\rm F}$ are 200 for panels (a) and (b), 370 for panels (c) and (d), and 390 for panels (e) and (d). The number of bosons $N_{\rm B}$ is fixed at 350.

The SSB can be further illustrated by using the phase diagram of the mixture in the $(N_{\rm B}, N_{\rm F})$ plane. As the potential barrier divides the well into two portions along the Y axis, we analyze the Y-symmetric and asymmetric ground states of the system. The asymmetry of the Fermi and the Bose components is characterized by the asymmetry ratio, which is defined as

$$\theta_{\rm B,F} = \frac{\left|\int_0^{+\infty} {\rm d}Y \int_{-\infty}^{+\infty} {\rm d}Z \left|\Phi_{\rm B,F}\left(Y,Z\right)\right|^2 - \int_{-\infty}^{0} {\rm d}Y \int_{-\infty}^{+\infty} {\rm d}Z \left|\Phi_{\rm B,F}\left(Y,Z\right)\right|^2\right|}{\int_{-\infty}^{+\infty} {\rm d}Y \int_{-\infty}^{+\infty} {\rm d}Z \left|\Phi_{\rm B,F}\left(Y,Z\right)\right|^2}. \tag{19}$$

The denominator in the above expression is actually 1 for both species. The 3D profiles of the asymmetry ratios of the bosons and the fermions are displayed in Fig. 4. Figures 4(a) and 4(b) show similar transitions from the symmetric ground state to the asymmetric one when the numbers of bosons and fermions are increased, that is to say, both the boson–boson and the BF interactions are increased. We can see that each figure can be divided into a symmetric region and an SSB region by the critical values of the numbers of the bosons and the fermions. In the symmetric region, the bosons and the fermions are trapped in the symmetric state with respect to the DWP. In the SSB region, the mixture is trapped in the asymmetric ground state.

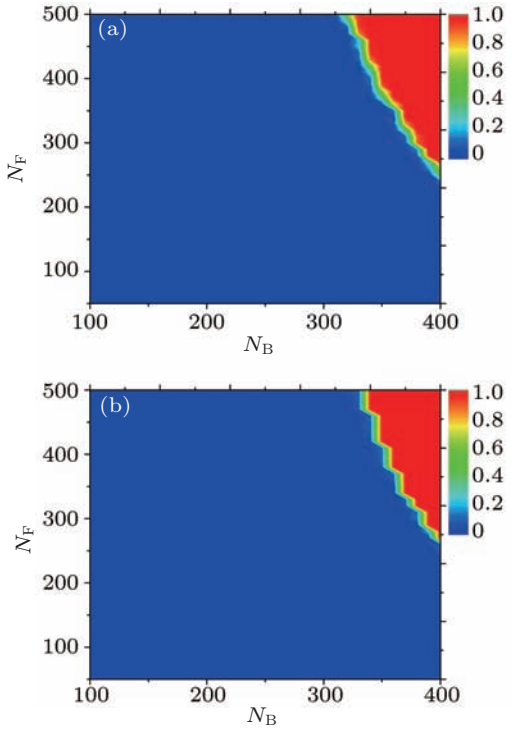


Fig. 4. (colour online) Phase diagrams of the ^{40}K and ^{87}Rb mixture in the $(N_{\rm B}, N_{\rm F})$ plane for the asymmetry ratios of bosons (a) and fermions (b).

The dynamical property of the system is demonstrated. The mixture in the ground state is initially prepared with $A_0 = 0$. Then we linearly increase A_0 to 5 in the time interval of $0 < t < 5$. In this process, the single potential well is split into two wells. Figures 5(a) and 5(b) show the boson and the fermion populations in the ground states. At $t = 0$, the single potential well acts on the mixture, the densities of the bosons and the fermions are equally distributed in a very narrow area due to their mutual attraction. Then the distributions in Figs. 5(a) and 5(b) show a leap from narrow to broad at $t \approx 2.3$. The bifurcations of the populations indicate the appearance of new stationary states induced by the increased potential barrier and the mutual attraction. At $t > 4.8$, both components are mainly localized in the same well due to the mutual attraction, which means the breaking of the left-right symmetry of the DWP.

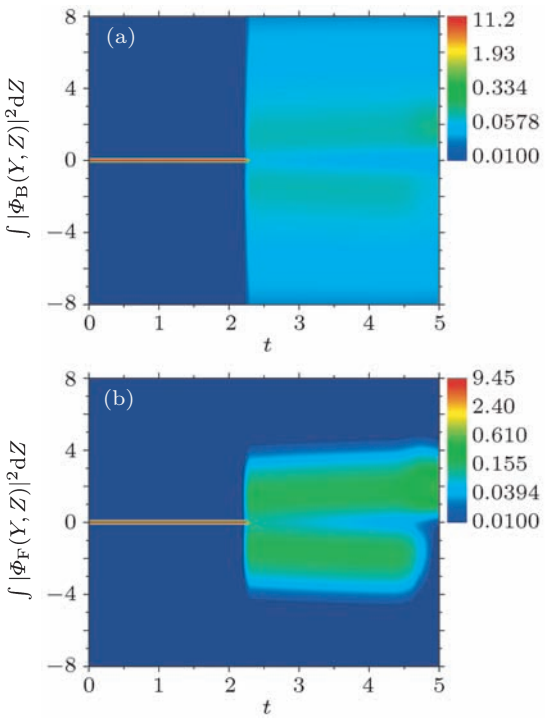


Fig. 5. (colour online) Contour plots for the evolution of densities $\int_{-\infty}^{+\infty} |\Phi_{\rm B}\left(Y,Z\right)|^2 \, {\rm d}Z$ and $\int_{-\infty}^{+\infty} |\Phi_{\rm F}\left(Y,Z\right)|^2 \, {\rm d}Z$ of (a) bosons and (b) fermions in the ^{87}Rb and ^{40}Kb mixture.

In this work, we have introduced a physical model that gives rise to the SSB of the 2D DWP in a BF

system. Previously, this effect was studied in a 1D setting. Our model is based on a 2D set of coupled GP equations derived from the density functional in the unitarity limit. We study the SSB in the trapped BF mixture with the DWP acting on both species. We have demonstrated that the SSB is possible by varying the values of $N_{\rm B}$ and $N_{\rm F}$ in the 2D DWP for ^{87}Rb and ^{40}K atoms due to the attraction between them. The 3D phase diagrams of the asymmetry ratio of the bosons and the fermions in the $(N_{\rm B}, N_{\rm F})$ plane are calculated by means of numerical methods. They clearly show the transition from the symmetric ground state to the asymmetric one. Finally, we study the dynamical process of the SSB induced by a gradual transformation of the single-well potential into the DWP. The populations of the fermions and the bosons are mainly trapped in the same well instead of equally distributed in the two wells with the increased potential barrier.

References

[1] Modugno G, Ferrari G, Roati G, Brecha R J, Simoni A and Inguscio M 2001 *Science* **294** 1320

[2] Caballero-Benítez S F, Ostrovskaya E A, Gulácsí M and Kivshar Y S 2009 *J. Phys. B* **42** 215308

[3] Goldwin J, Papp S B, DeMarco B and Jin D S 2002 *Phys. Rev.* A **65** 021402(R)

[4] Adhikari S K and Salasnich L 2008 *Phys. Rev.* A **78** 043616

[5] Hadzibabic Z, Stan C A, Dieckmann K, Gupta S, Zwierlein M W, Görlitz A and Ketterle W 2002 *Phys. Rev. Lett.* **88** 160401

[6] Roati G, Riboli F, Modugno G and Inguscio M 2002 *Phys. Rev. Lett.* **89** 150403

[7] Wang P J, Chen H X, Xiong D Z, Yu X D, Gao F and Zhang J 2008 *Acta Phys. Sin.* **57** 4840 (in Chinese)

[8] Hao Y J 2011 *Chin. Phys. Lett.* **28** 010302

[9] Chen H J, Li G Q and Xue J K 2011 *Acta Phys. Sin.* **60** 040305 (in Chinese)

[10] Meng S Y, Wu W, Chen X H, Zhang J and Fu L B 2011 *Chin. Phys.* B **20** 080309

[11] Mayteevarunyoo T, Malomed B A and Dong G 2008 *Phys. Rev.* A **78** 053601

[12] Nambu Y 2009 *Rev. Mod. Phys.* **81** 1015

[13] Gubeskys A and Malomed B A 2007 *Phys. Rev.* A **76** 043623

[14] Matuszewski M, Malomed B A and Trippenbach M 2007 *Phys. Rev.* A **75** 063621

[15] Trippenbach M, Infeld E, Gocałek J, Matuszewski M, Oberthaler M and Malomed B A 2008 *Phys. Rev.* A **78** 013603

[16] Mayteevarunyoo T and Malomed B A 2009 *J. Opt.* A: *Pure Appl. Opt.* **11** 094015

[17] Dror N and Malomed B A 2011 *Physica* D **240** 526

[18] Xiong B, Gong J, Pu H, Bao W and Li B 2009 *Phys. Rev.* A **79** 013626

[19] Wang C, Kevrekidis P G, Whitaker N and Malomed B A 2008 *Physica* D **237** 2922

[20] Adhikari S K, Lu H and Pu H 2009 *Phys. Rev.* A **80** 063607

[21] Sakaguchi H and Malomed B A 2011 *Phys. Rev.* E **83** 036608

[22] Adhikari S K, Malomed B A, Salasnich L and Toigo F 2010 *Phys. Rev.* A **81** 053630

[23] Salasnich L, Ancilotto F and Toigo F 2010 *Laser Phys. Lett.* **7** 78

[24] Adhikari S K and Salasnich L 2009 *New J. Phys.* **11** 023011

[25] Muruganandam P and Adhikari S K 2009 *Comput. Phys. Commun.* **180** 1888