Entanglement transfer via two arrays of coupled resonator waveguides*

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We propose a scheme for transferring entanglement through two independent arrays of coupled resonator waveguides, where a three-level atom is embedded in each resonator. We investigate the entanglement dynamics of the transferred state. The influence of initial states and applied lasers on the entanglement sudden death phenomenon is also discussed. Furthermore, we study the dynamics of pairwise quantum correlations measured by the quantum discord.

Keywords: entanglement transfer, resonator waveguide

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1. Introduction

It is generally accepted that entanglement is the central physical resource in quantum information theory. It is used in various applications, such as quantum teleportation,[1−3] secret key distribution,[4] and superdense coding.[5,6] In order to understand how quantum entanglement is transferred between the parties, several investigations have been motivated in recent years.[7−9] For the case of long distance quantum communications optical fibres are adopted, through which photons transmit with encoded quantum information.[10,11] On the other hand, linking short distinct quantum processors or registers is a crucial part of scalable quantum computing technology. Studying the possibility of using spin chains as quantum wires for short distance quantum communication emerges as an area of significant activity.[12−15] Typically, the input state is placed at one end, and the output is detected by measuring the state at the other end without manipulating the other spins.

The individual spin is difficult to be controlled and addressed experimentally due to its small spatial scale. Therefore, artificial systems, in which many particle systems are effectively created, have been considered. Recently, a system consisting of coupled resonator waveguides has been studied extensively.[16−22] The advantage of such a system is that each resonator can be addressed independently by an optical laser and controlled individually. Its relatively long-lived atomic state for the embedded atom makes the coupled resonator waveguide suitable for implementation in the short distance quantum task.

In this paper, we propose a scheme of entanglement transfer based upon coupled resonator waveguides. Two independent arrays of coupled resonator waveguides are considered, which serve as the channel. The bipartite entanglement is placed at one end and is transferred to the other end. The transfer of the entanglement is due to the natural evolution of the system, which is characterized by the time evolution operator of the system. By applying a laser to each cavity, a controllable scheme is achieved. We studied the dynamics of the transferred entanglement analytically. It was found that the transferred entanglement varies periodically with time. In order to transfer the maximum amount of entanglement, we need to turn off the coupling at a suitable moment. Furthermore, a time period with no entanglement, which is known as entanglement sudden death, occurs in the system. We obtain the duration of the entanglement death period and show how it is related to the initial state and the applied lasers. Finally, the dynamics of the quantum discord, which measures the quantum correlations and has recently received much attention both in theory and in experiment,[23,24] is studied and compared with that of the entanglement.

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2. Model and process of entanglement transfer

We consider two arrays of coupled resonator waveguides A and B, each of which consists of N nodes, as sketched in Fig. 1. Each node of the arrays contains a trapped three-level atom, which has a level structure with two lower states $|0\rangle_j$ and $|1\rangle_j$ and an upper state $|e\rangle_j$, where j is the node index. The cavity mode is tuned far from the atomic transition $|0\rangle_j \leftrightarrow |e\rangle_j$ (transition frequency ω_0) with coupling constant g and detuning Δ. The transition $|1\rangle_j \leftrightarrow |e\rangle_j$ (transition frequency ω_1) in each atom is driven resonantly with lasers (frequency $\omega_{\mathrm{L}j} = \omega_1$) with Rabi frequency Ω_j.

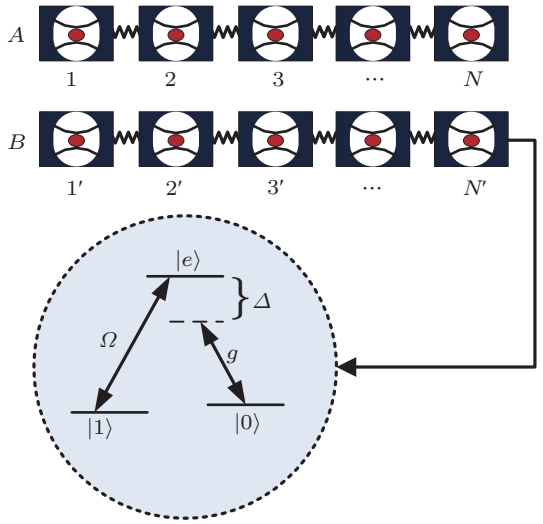


Fig. 1. (color online) Schematic configuration for two arrays of coupled resonator waveguides with a three-level atom embedded in each resonator.

We assume that there is no interaction between A and B. The Hamiltonian of one array system can be written as

$$H = H_{\mathrm{atom}} + H_{\mathrm{cav}} + H_{\mathrm{int}}. \tag{1}$$

The Hamiltonian of the atom is

$$H_{\mathrm{atom}} = \sum_j (\nu_e |e\rangle_j\langle e| + \nu_1 |1\rangle_j\langle 1| + \nu_0 |0\rangle_j\langle 0|), \tag{2}$$

where ν_e, ν_1, and ν_0 are the energies of atom levels $|e\rangle$, $|1\rangle$, and $|0\rangle$, respectively. The reduced Plank constant is set to 1 throughout the paper. By the definition of transition frequency, we have $\omega_1 = \nu_e - \nu_1$ and $\omega_0 = \nu_e - \nu_0$. The Hamiltonian describing the photons in the cavity modes is

$$H_{\mathrm{cav}} = \omega_{\mathrm{c}} \sum_j a_j^\dagger a_j + J_{\mathrm{c}} \sum_j (a_j^\dagger a_{j+1} + a_j a_{j+1}^\dagger), \tag{3}$$

where J_{c} is the hopping rate of the photons between neighbouring cavities, and a_j ($a_j^\dagger$) is the annihilation (creation) operator for the photon in the j-th cavity. Under the periodic boundary condition, H_{cav} can be diagonalized through the Fourier transformation as $H_{\mathrm{cav}} = \sum_k \omega_k a_k^\dagger a_k$, where $\omega_k = \omega_{\mathrm{c}} + 2J_{\mathrm{c}} \cos k$. The interaction Hamiltonian between the atoms and the cavity field and between the atoms and the lasers is

$$H_{\mathrm{int}} = \sum_j g(a_j^\dagger |0\rangle_j\langle e| + a_j |e\rangle_j\langle 0|) + \sum_j (\Omega_j \, \mathrm{e}^{-\mathrm{i}\,\omega_1 t} |e\rangle_j\langle 1| + \Omega_j^* \, \mathrm{e}^{\mathrm{i}\,\omega_1 t} |1\rangle_j\langle e|). \tag{4}$$

It is convenient to work in the interaction picture. The Hamiltonian in the interaction picture is given by

$$H_{\mathrm{I}} = \sum_j [\Omega_j |e\rangle_j\langle 1| + \Omega_j^* |1\rangle_j\langle e| + \frac{g}{\sqrt{N}} \sum_k (|0\rangle_j\langle e| a_k^\dagger \, \mathrm{e}^{\mathrm{i}(kj + \delta_k t)} + \mathrm{H.c.})], \tag{5}$$

where $\delta_k = \omega_k - \omega_0$. Assuming $\delta_k \gg g$ for all k, we can adiabatically eliminate the photons and obtain the effective Hamiltonian as

$$H_{\mathrm{eff}} = \sum_j [J_0 |e\rangle_j\langle e| + \Omega_j |e\rangle_j\langle 1| + \Omega_j^* |1\rangle_j\langle e| + \sum_l (J_l |0\rangle_j\langle e| \otimes |e\rangle_{j+1}\langle 0| + \mathrm{H.c.})], \tag{6}$$

where $J_0 = \sum_k [g^2/(N\delta_k)]$, and $J_l = \sum_k [g^2 \, \mathrm{e}^{\mathrm{i}\,kl}/(N\delta_k)]$. If we introduce state $|1_i\rangle = |00\cdots 1_i \cdots 00\rangle$, the effective Hamiltonian can be further simplified as

$$H_{\mathrm{eff}} = \Theta |\mathbf{1}_N\rangle\langle \mathbf{1}_1| + \Theta^* |\mathbf{1}_1\rangle\langle \mathbf{1}_N|, \tag{7}$$

where

$$\Theta = \sum_k \frac{\Omega_1 \Omega_N^* \, \mathrm{e}^{-\mathrm{i}\,k(N-1)}}{N[J_0 + \sum_l 2J_l \cos(kl)]}. \tag{8}$$

The above Hamiltonian describes direct Raman transitions between the first and the last nodes with transition rate Θ assisted by exchanging virtual photons.

Now we consider the process of entanglement transfer based on two arrays of coupled resonator waveguides. The total Hamiltonian can be written as

$$H' = \sum_{i=A,B} \Theta |\mathbf{1}_N\rangle_{ii}\langle \mathbf{1}_1| + \Theta^* |\mathbf{1}_1\rangle_{ii}\langle \mathbf{1}_N|. \tag{9}$$

Initially the entanglement state to be transferred is placed at sites $11'$. We study the entanglement dynamics of the transferred entanglement state at sites

NN'. Two different types of initial entanglement are considered, which correspond to the Bell states

$$|\psi\rangle = \alpha|01\rangle + \beta|10\rangle,$$
$$|\phi\rangle = \alpha|00\rangle + \beta|11\rangle. \quad (10)$$

We first consider the initial state $|\psi\rangle$, where only a single excitation presents. The total state vector at the initial time is

$$|\Psi_0\rangle = \alpha|\mathbf{0}\rangle_A \otimes |\mathbf{1}_1\rangle_B + \beta|\mathbf{1}_1\rangle_A \otimes |\mathbf{0}\rangle_B, \quad (11)$$

where $|\mathbf{0}\rangle = |00\cdots00\rangle$ represents that every atom stays in the $|0\rangle$ state. At any moment of the following time, the state vector is obtained as

$$\begin{aligned}&|\Psi(t)\rangle\\ &= \alpha|\mathbf{0}\rangle_A[\cos(\Theta t)|\mathbf{1}_1\rangle_B - \mathrm{i}\sin(\Theta t)|\mathbf{1}_N\rangle_B]\\ &\quad + \beta[\cos(\Theta t)|\mathbf{1}_1\rangle_A - \mathrm{i}\sin(\Theta t)|\mathbf{1}_N\rangle_A]|\mathbf{0}\rangle_B. \quad (12)\end{aligned}$$

The density matrix $\rho_{NN'}(t)$ of sites NN' at time t is obtained as

$$\begin{aligned}&\rho^{\Psi}_{NN'}(t)\\ &=|\alpha|^2\cos^2(\Theta t)|00\rangle\langle 00| + |\alpha|^2\sin^2(\Theta t)|01\rangle\langle 01|\\ &\quad + |\beta|^2\sin^2(\Theta t)|10\rangle\langle 10| + \alpha\beta^*\sin^2(\Theta t)|01\rangle\langle 10|\\ &\quad + \beta\alpha^*\sin^2(\Theta t)|10\rangle\langle 01|. \quad (13)\end{aligned}$$

Next, we consider $|\phi\rangle$ as the initial entangled state. The total state vector is

$$|\Phi_0\rangle = \alpha|\mathbf{0}\rangle_A \otimes |\mathbf{0}\rangle_B + \beta|\mathbf{1}_1\rangle_A \otimes |\mathbf{1}_1\rangle_B. \quad (14)$$

The state vector at time t is obtained as

$$\begin{aligned}|\Phi(t)\rangle =& \alpha|\mathbf{0}\rangle_A|\mathbf{0}\rangle_B + \beta[\cos(\Theta t)|\mathbf{1}_1\rangle_A - \mathrm{i}\sin(\Theta t)|\mathbf{1}_N\rangle_A]\\ &\otimes[\cos(\Theta t)|\mathbf{1}_1\rangle_B - \mathrm{i}\sin(\Theta t)|\mathbf{1}_N\rangle_B]. \quad (15)\end{aligned}$$

The density matrix of the transferred state at sites NN' is

$$\begin{aligned}\rho^{\Phi}_{NN'}(t) =&[|\alpha|^2 + |\beta|^2\cos^4(\Theta t)]|00\rangle\langle 00|\\ &- \alpha\beta^*\sin^2(\Theta t)|00\rangle\langle 11|\\ &+ |\beta|^2\sin^2(\Theta t)\cos^2(\Theta t)|01\rangle\langle 01|\\ &+ |\beta|^2\sin^2(\Theta t)\cos^2(\Theta t)|10\rangle\langle 10|\\ &- \beta\alpha^*\sin^2(\Theta t)|11\rangle\langle 00|\\ &+ |\beta|^2\sin^4(\Theta t)|11\rangle\langle 11|. \quad (16)\end{aligned}$$

In order to quantify the degree of entanglement, several measurements[25] of entanglement have been introduced for both pure and mixed quantum states. In this paper, we adopt the concurrence to calculate the entanglement between all bipartite partitions in qubits and bus systems, which varies from $C = 0$ for a separable state to $C = 1$ for a maximally entangled state. The concurrence related to the density operator ρ of a mixed state is defined as[26]

$$C(\rho) = \max\left\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\}, \quad (17)$$

where λ_i are the eigenvalues in decreasing order of the matrix

$$\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y), \quad (18)$$

where ρ^* denotes the complex conjugation of ρ, and σ_y is the Pauli matrix expressed in the same basis

$$\sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}. \quad (19)$$

If ρ can be written in the form of X state

$$\rho^{\mathrm{X}} = \begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}, \quad (20)$$

which arises in a wide variety of physical situations, the concurrence can be easily derived to be

$$C(\rho^{\mathrm{X}}) = 2\max\left\{0, |z| - \sqrt{ad}, |w| - \sqrt{bc}\right\}. \quad (21)$$

Then, the concurrences of Eqs. (13) and (16) are

$$C^{\Psi}_{NN'} = 2\max\left\{0, |\alpha\beta^*|\sin^2(\Theta t)\right\} \quad (22)$$

and

$$\begin{aligned}&C^{\Phi}_{NN'}\\ &=2\max\left\{0, \sin^2(\Theta t)[|\alpha\beta^*| - |\beta|^2\cos^2(\Theta t)]\right\}, \quad (23)\end{aligned}$$

respectively.

In Fig. 2, we plot $C^{\Psi}_{NN'}$ and $C^{\Phi}_{NN'}$ as functions of Θt for different values of α. It is shown that for a given initial state, the maximum value of the entanglement at sites NN' can be obtained at $\Theta t = (n+1/2)\pi$ with n being an integer. The amount of entanglement depends on the initial state, while the frequency of the entanglement evolution depends on Θ, which can be controlled by adjusting the lasers. Unlike Fig. 2(a), we can see in Fig. 2(b) that the entanglement can fall abruptly to zero, remain zero for a period of time, and then recover. The condition for the occurrence of the entanglement sudden death is $|\alpha| < 1/\sqrt{2}$. The duration of the entanglement death in one period can be obtained as

$$\Delta t = \left[\arccos\left(2\left|\frac{\alpha}{\beta}\right| - 1\right)\right]\Big/\Theta, \quad (24)$$

which is dependent on the degree of the entanglement of the initial state and the applied lasers. The greater the entanglement of the initial state the shorter the transferred state stays in the separable state.

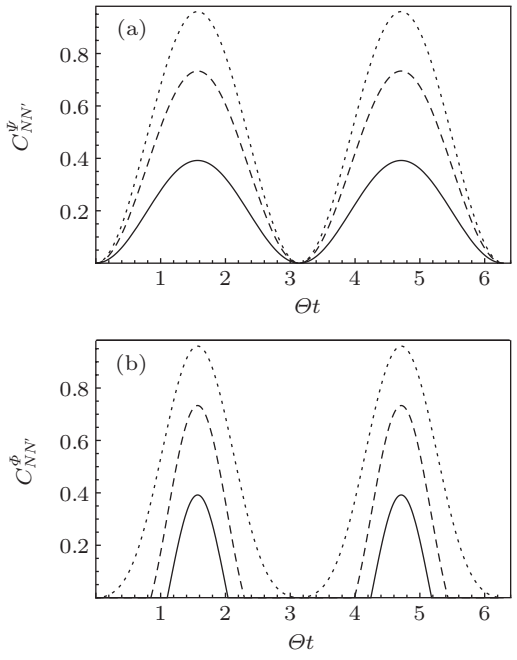


Fig. 2. The (a) $C^{\Psi}_{NN'}$ and (b) $C^{\Phi}_{NN'}$ each are plotted as a function of Θt for $\alpha = 0.2$ (solid line), $\alpha = 0.4$ (dashed line), and $\alpha = 0.8$ (dotted line).

3. Dynamics of quantum discord

Quantum discord is a measure of the difference between two classical identical expressions for the mutual information in quantum systems.[23] It has been regarded as a measure of nonclassical correlations, which may include the entanglement but is an independent measurement.[27,28] The total correlation of bipartite quantum systems A and B is calculated by the quantum mutual information

$$\mathcal{I}(\rho_{A:B}) = S(\rho_A) - S(\rho_{A|B}), \tag{25}$$

where $S(\rho_{A|B}) = S(\rho_{AB}) - S(\rho_B)$, $S(\rho) = -\mathrm{Tr}\rho\log_2\rho$ is the von Neumann entropy, and ρ_A (ρ_B) is the reduced density operator of partition A (B). The classical correlation presented in composite state ρ_{AB} is

$$\mathcal{C}(\rho_{AB}) = \min_{\{\Pi_j\}} S_{\{\Pi_j\}}(\rho_{A|B}), \tag{26}$$

where $S_{\{\Pi_j\}}(\rho_{A|B})$ is the conditional entropy of the system after the measurement, and $\{\Pi_j\}$ defines a complete set of projective measurements on partition B. It is shown that the discord is more robust than the entanglement under the Markovian environments[29] and may become a more powerful resource in quantum tasks.

The quantum discord, which quantifies the nonclassical correlation presented in ρ_{AB}, is defined as

$$\mathcal{Q}(\rho_{AB}) = \mathcal{I}(\rho_{AB}) - \mathcal{C}(\rho_{AB}). \tag{27}$$

Both quantum and classical correlations are equal to the entanglement entropy for pure states, but the entanglement is only a part of the quantum correlation for mixed states.

In order to compare the dynamics of the transferred entanglement and the discord, we plot them in Fig. 3(a) for $\rho^{\Phi}_{NN'}$ as functions of Θt. Interestingly, although the entanglement is only a part of the quantum correlation for mixed states, its value can be less than, equal to, or even greater than the quantum discord. This is due to the fact that the measurements adopted by the quantum entanglement and the quantum discord are different. Moreover, the quantum discord vanishes only in the asymptotic limit, in which case the entanglement suddenly disappears. In Fig. 3(b), we plot the discord as a function of Θt for different values of α. It is shown that the dynamics of the discord is dependent on the initial state and the transferred quantum discord varies with time periodically.

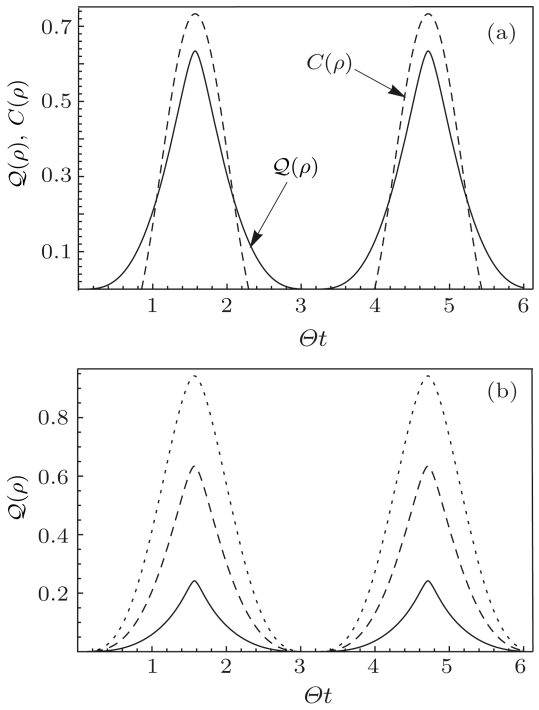


Fig. 3. (a) Quantum discord $\mathcal{Q}(\rho)$ (solid line) and concurrence $C(\rho)$ (dashed line) of C^{Φ} versus Θt for $\alpha = 0.4$. (b) Quantum discord is plotted as a function of Θt for $\alpha = 0.2$ (solid line), $\alpha = 0.4$ (dashed line), and $\alpha = 0.8$ (dotted line).

4. Summary

A scheme of entanglement transfer via two arrays of coupled resonator waveguides is proposed. Each node consists of a trapped three-level atom and an applied laser. The entanglement evolution of the transferred state has been studied analytically. The time for achieving the maximum amount of entanglement is found, which is independent of the initial states. It is shown that the entanglement sudden death phenomenon appears and its duration depends on the amount of the initial entanglement as well as the applied laser. Moreover, the relation between the quantum discord and the quantum entanglement in such a system is also studied.

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