Existence of periodic solutions for the nonlinear functional differential equation in the lossless transmission line model*

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We study a time delay equation for the lossless transmission line model. Under suitable conditions, by using the continuation theorem of the coincidence degree theory, the existence of the periodic solution for the nonlinear functional differential equation is obtained.

Keywords: periodic solution, time delay, nonlinearity, coincidence degree

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1. Introduction

The investigation of the solutions for nonlinear equations plays an important role in the study of the functional differential equation (FDE) theory.[1,2] Many natural sciences, such as chemistry, biology, mathematics and communication, and particularly almost all branches of physics including the lossless transmission line system, the electrodynamics problem, fluid dynamics and condensed matter physics, etc. are related to the functional differential equations. Many scholars have done a great deal of work on nonlinear differential equations. Using the differential inequalities and other methods, Mo *et al.* considered a class of singularly perturbed nonlinear boundary value problems for the ordinary differential equations,[3] the reaction–diffusion equations,[4−6] the shock layer and soliton solution of nonlinear equations,[7−10] and several problems in atmospheric physics.[11−13] Using the continuation theorem or the incremental harmonic balance method, many researchers including Lu,[14] Liu and Hiang,[15,16] Liu *et al.*,[17] and Zhang *et al.*,[18] also obtained the existence of periodic solutions for the ordinary differential equations or for the retarded functional differential equations. Recently, Wang and Ni[19] studied a kind of nonlinear singularly perturbed time delay equation by means of the boundary layer corrective method.

As we know, a realistic model must include some history of the system. In this paper, we consider the following nonlinear functional differential time delay equation:

$$\begin{aligned}&(x(t)-mx(t-\tau_0))'\\ =\ &f_1(t,x(t))+f_2(t,x(t-\tau(t)))\\ &+f_3(x(t-\tau(t)))+p(t),\end{aligned}\tag{1}$$

where $f_1, f_2 \in \mathrm{C}(\mathrm{R}^2,\mathrm{R})$ with $f_i(t,x)=f_i(t+T,x)$ $(i=1,2)$, $\forall x \in \mathrm{R}$; f_3 is a T-periodic function; $\tau, p \in \mathrm{C}(\mathrm{R},\mathrm{R})$ with $\tau(t+T)\equiv\tau(t)$ and $p(t+T)\equiv p(t)$; m, τ_0 are constants and $|m|\neq 1$, $T>0$.

Such a kind of FDE has been used for the study of distributed networks containing lossless transmission lines. We consider a system consisting of a long electrical cable of length l, one end of which is connected to a power source E with resistance R, while the other end is connected to an oscillating circuit formed by a condenser C_1 and a nonlinear element with the volt-ampere characteristic of $i=g(v)$. Let L and C denote the line inductance and the line capacitance of the cable, respectively, $x(t)=v(l,t)$. We assume that the cable is lossless. The processes in such a system are described by the following FDE:

$$\begin{aligned}&(x(t)-Kx(t-\tau))'\\ =\ &p(t)-\alpha x(t)-K\alpha x(t-\tau)\\ &-g(x(t))+Kg(x(t-\tau)),\end{aligned}\tag{2}$$

where $K=\frac{z-R}{z+R}$, $\tau=\frac{2l}{(LC)^{1/2}}$, and $p(t)=\frac{2E}{(z+R)C_1}$. If $R>0$, then $K<1$. It is easy to see that Eq. (2) is a

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special case of Eq. (1). The existence of periodic solutions for Eq. (1) is obtained by using the continuation theorem of the coincidence degree theory.

For ease of exposition, throughout this paper we will adopt the following notations:

$$|x|_\infty = \max_{t\in[0,T]} |x(t)|, \quad |x|_k = \left(\int_0^T |x(t)|^k \mathrm{d}t\right)^{1/k}.$$

Let $X = C_T = \{x | x \in \mathrm{C}(\mathrm{R},\mathrm{R}), x(t+T) = x(t)\}$ with the norm $||x||_X = |x|_\infty$. Clearly, X is a Banach space. We also define operators A and L in the following:

$$\begin{aligned} &\mathrm{A} : X \to X,\ (\mathrm{A}x)(t) = x(t) - mx(t-\tau_0),\\ &\mathrm{L} : D(\mathrm{L}) \subset X \to X,\ \mathrm{L}x = (\mathrm{A}x)', \end{aligned} \tag{3}$$

where $D(\mathrm{L}) = \{x | x \in \mathrm{C}^1(\mathrm{R},\mathrm{R}),\ x(t+T) \equiv x(t)\}$.

Next we define a nonlinear operator $\mathrm{N} : X \to X$ by setting

$$\begin{aligned} \mathrm{N}x =\ & f_1(t, x(t)) + f_2(t, x(t-\tau(t)))\\ & + f_3(x(t-\tau(t))) + p(t). \end{aligned} \tag{4}$$

According to Hale's terminology,[20] a solution $x(t)$ of Eq. (1) should have $x \in \mathrm{C}(\mathrm{R},\mathrm{R})$, $\mathrm{A}x \in \mathrm{C}^1(\mathrm{R},\mathrm{R})$, and Eq. (1) satisfied on R. In general, $x \notin \mathrm{C}^1(\mathrm{R},\mathrm{R})$. However from lemma 1 of Ref. [14], it is easy to see that $(\mathrm{A}x)' = \mathrm{A}x'$, $\mathrm{KerL} = \mathrm{R}$, and $\mathrm{ImL} = \{x | x \in X, \int_0^T x(s)\mathrm{d}s = 0\}$. So a T-periodic solution $x(t)$ of Eq. (1) must have $x \in \mathrm{C}^1(\mathrm{R},\mathrm{R})$. Therefore, operator L is a Fredholm operator with index zero.[21] We define the continuous projectors $\mathrm{P} : X \to \mathrm{KerL}$ and $\mathrm{Q} : X \to X/\mathrm{ImL}$ by setting

$$\mathrm{P}x = \frac{1}{T}\int_0^T x(s)\mathrm{d}s, \quad \mathrm{Q}x = \frac{1}{T}\int_0^T x(s)\mathrm{d}s.$$

Hence, $\mathrm{ImP} = \mathrm{KerL}$ and $\mathrm{KerQ} = \mathrm{ImL}$. Set $\mathrm{L_P} = \mathrm{L}|_{\mathrm{D(L)}\bigcap \mathrm{KerP}}$, then $\mathrm{L_P}$ has continuous inverse $\mathrm{L_P^{-1}}$ defined by

$$\mathrm{L_P^{-1}}y(t) = (\mathrm{A^{-1}F}y)(t), \tag{5}$$

where

$$(\mathrm{F}y)(t) = \frac{1}{T}\int_0^T sy(s)\mathrm{d}s + \int_0^T y(s)\mathrm{d}s.$$

Therefore, it is easy to see that N is L-compact on $\bar{\Omega}$, where Ω is an open bounded set in X.

We need the following hypotheses:

Hypothesis 1 One of the following conditions holds:

(a) $(f_i(t,v_1) - f_i(t,v_2))(v_1 - v_2) > 0$, $(f_3(v_1) - f_3(v_2))(v_1 - v_2) > 0$, for $i = 1, 2$, $v_i \in \mathrm{R}$, $\forall t \in \mathrm{R}$ and $v_1 \neq v_2$,

(b) $(f_i(t,v_1) - f_i(t,v_2))(v_1 - v_2) < 0$, $(f_3(v_1) - f_3(v_2))(v_1 - v_2) < 0$, for $i = 1, 2$, $v_i \in \mathrm{R}$, $\forall t \in \mathrm{R}$ and $v_1 \neq v_2$.

Hypothesis 2 One of the following conditions holds:

(a) There exist three positive constants L_i $(i = 1, 2, 3)$ such that $|m| + \frac{L_1 T}{\pi} + (L_2 + L_3)T < 1$,

$$\begin{aligned} &|f_i(t,v_1) - f_i(t,v_2)| \le L_i|v_1 - v_2|,\\ &\text{for}\ \ i = 1,2,\ v_i \in \mathrm{R},\ \forall t \in \mathrm{R}, \end{aligned}$$

and

$$\begin{aligned} &|f_3(v_1) - f_3(v_2)| \le L_3|v_1 - v_2|,\\ &\text{for}\ \ v_{1,2} \in R,\ \forall t \in R; \end{aligned}$$

(b) there exists three positive constants L_i $(i = 1, 2, 3)$ such that $1 + \frac{L_1 T}{\pi} + (L_2 + L_3)T < |m|$,

$$\begin{aligned} &|f_i(t,v_1) - f_i(t,v_2)| \le L_i|v_1 - v_2|,\\ &\text{for}\ \ v_{1,2} \in \mathrm{R},\ \forall t \in \mathrm{R}, \end{aligned}$$

and

$$\begin{aligned} &|f_3(v_1) - f_3(v_2)| \le L_3|v_1 - v_2|,\\ &\text{for}\ \ v_{1,2} \in \mathrm{R},\ \forall t \in \mathrm{R}. \end{aligned}$$

Hypothesis 3 there exists a constant $\rho > 0$ such that one of the following conditions holds:

(a) $x(f_1(t,x) + f_2(t,x) + f_3(x) + p(t)) > 0$, $\forall t \in \mathrm{R}$, $|x| \ge \rho$,

(b) $x(f_1(t,x) + f_2(t,x) + f_3(x) + p(t)) < 0$, $\forall t \in \mathrm{R}$, $|x| \ge \rho$.

Similar to Ref. [22], throughout this paper we assume that $\tau'(t) < 1$, $\forall t \in [0, T]$.

2. Periodic solution of a nonlinear problem

With Eqs. (3) and (4), the operator equation

$$\mathrm{L}x = \lambda \mathrm{N}x$$

is equivalent to the following equation:

$$\begin{aligned} &x'(t) - mx'(t-\tau_0)\\ &= \lambda(f_1(t, x(t)) + f_2(t, x(t-\tau(t)))\\ &\quad + f_3(x(t-\tau(t))) + p(t)), \end{aligned} \tag{6}$$

where $\lambda \in (0, 1)$.

The following lemma gives a more accurate inequality about the deviating argument τ than that given by lemma 2 in Ref. [14].

Lemma 1 Let $\tau \in (-\frac{T}{2}, 0)\bigcup(0, \frac{T}{2})$ be a constant. Then for $\forall x \in \mathrm{C}^1(\mathrm{R}, \mathrm{R})\bigcap C_T$, we have

$$\int_0^T |x(t) - x(t-\tau)|^2 \mathrm{d}t \leq \tau^2\left(1 + \frac{|\tau|}{2T}\right)\int_0^T |x'(t)|^2 \mathrm{d}t. \tag{7}$$

Proof (i) If $\tau \in (0, \frac{T}{2})$, then for $\forall r \in [0, T]$, we have

$$\begin{aligned}&\int_0^T |x(t) - x(t-\tau)|^2 \mathrm{d}t \\ \leq\ & \tau\int_{r-\tau}^{r+T}\int_{\sigma}^{\sigma+\tau} |x'(\sigma)|^2 \mathrm{d}t\mathrm{d}\sigma \\ & - \tau\int_{r-\tau}^{r}\int_{\sigma}^{r} |x'(\sigma)|^2 \mathrm{d}t\mathrm{d}\sigma \\ & - \tau\int_{r+T-\tau}^{r+T}\int_{r+T}^{\sigma+\tau} |x'(\sigma)|^2 \mathrm{d}t\mathrm{d}\sigma. \end{aligned} \tag{8}$$

Since

$$\begin{aligned}&\tau\int_{r-\tau}^{r+T}\int_{\sigma}^{\sigma+\tau} |x'(\sigma)|^2 \mathrm{d}t\mathrm{d}\sigma \\ =\ & \tau^2\int_{r-\tau}^{r} |x'(\sigma)|^2 d\sigma + \tau^2\int_0^T |x'(\sigma)|^2 \mathrm{d}\sigma,\end{aligned}$$

$$\tau\int_{r-\tau}^{r}\int_{\sigma}^{r} |x'(\sigma)|^2 \mathrm{d}t\mathrm{d}\sigma \geq \frac{\tau^2}{2}\int_{r-\tau}^{r-\frac{\tau}{2}} |x'(\sigma)|^2 \mathrm{d}\sigma,$$

and

$$\tau\int_{r+T-\tau}^{r+T}\int_{r+T}^{\sigma+\tau} |x'(\sigma)|^2 \mathrm{d}t\mathrm{d}\sigma \geq \frac{\tau^2}{2}\int_{r-\frac{\tau}{2}}^{r} |x'(\sigma)|^2 \mathrm{d}\sigma,$$

together with inequality (8), we obtain

$$\begin{aligned}&\int_0^T |x(t) - x(t-\tau)|^2 \mathrm{d}t \\ \leq\ & \tau^2\int_0^T |x'(\sigma)|^2 \mathrm{d}\sigma + \frac{\tau^2}{2}\int_{r-\tau}^{r} |x'(\sigma)|^2 \mathrm{d}\sigma. \end{aligned} \tag{9}$$

As r is arbitrary in $[0, T]$, it follows that

$$\begin{aligned}&\int_0^T |x(t) - x(t-\tau)|^2 \mathrm{d}t \\ \leq\ & \tau^2\int_0^T |x'(\sigma)|^2 \mathrm{d}\sigma + \frac{\tau^2}{2}\min_{r\in[0,T]}\int_{r-\tau}^{r} |x'(\sigma)|^2 \mathrm{d}\sigma. \end{aligned}$$

Since

$$\min_{r\in[0,T]}\frac{\int_{r-\tau}^{r} |x'(\sigma)|^2 \mathrm{d}\sigma}{\tau} \leq \frac{\int_0^T |x'(\sigma)|^2 \mathrm{d}\sigma}{T},$$

we have

$$\int_{r-\tau}^{r} |x'(\sigma)|^2 \mathrm{d}\sigma \leq \frac{\tau}{T}\int_0^T |x'(\sigma)|^2 \mathrm{d}\sigma.$$

Therefore, we obtain

$$\begin{aligned}&\int_0^T |x(t) - x(t-\tau)|^2 \mathrm{d}t \\ &\leq \tau^2\left(1 + \frac{\tau}{2T}\right)\int_0^T |x'(\sigma)|^2 \mathrm{d}\sigma. \end{aligned} \tag{10}$$

(ii) If $\tau \in (-\frac{T}{2}, 0)$, then

$$\begin{aligned}&\int_0^T |x(t) - x(t-\tau)|^2 \mathrm{d}t \\ =\ & \int_{-\tau}^{T-\tau} |x(s) - x(s+\tau)|^2 \mathrm{d}s \\ =\ & \int_0^T |x(s) - x(s-|\tau|)|^2 \mathrm{d}s. \end{aligned} \tag{11}$$

Using the methods similar to those used in the previous case, we can show that inequality (7) holds.

Theorem 1 If Hypotheses 1 and 2 hold, either condition (a) or (b) in Hypothesis 3 is satisfied, and the following inequality holds:

$$\begin{aligned}&(1-|m|)^2 - \frac{T}{\pi}\left(1 + |m|\right) \\ &\quad\times \left[\left|\frac{1}{1-\tau'}\right|_{\infty}^{1/2}(L_2 + \sqrt{2}L_3) + L_1\right] > 0,\end{aligned}$$

then Eq. (1) has a unique T-periodic solution.

Proof From Hypotheses 1 and 2, it is easy to see that equation (1) has at most one T-periodic solution. Thus, to prove Theorem 1, it is sufficient to show that Eq. (1) has at least one T-periodic solution. Firstly, we claim that all possible T-periodic solutions of Eq. (6) are bounded.

Let $x(t)$ be a T-periodic solution of Eq. (6). If Hypothesis 2 holds, multiplying both sides of Eq. (6) by $(x(t) - mx(t-\tau_0))'$ and then integrating them from 0 to T, we have

$$\begin{aligned}&\int_0^T |(Ax')(t)|^2 \mathrm{d}t \\ \leq\ & \Big[\max_{t\in[0,T]}|f_1(t,0)|\sqrt{T}(1+|m|) \\ & + \max_{t\in[0,T]}|f_2(t,0)|\sqrt{T}(1+|m|) \\ & + |p|_2(1+|m|)\Big]|x'|_2 + L_1(1+|m|)|x|_2|x'|_2 \\ & + L_2(1+|m|)\left(\int_0^T |x(t-\tau(t))|^2\right)^{1/2}|x'|_2 \\ & + (1+|m|)\left(\int_0^T |f_3(x(t-\tau(t)))|^2 \mathrm{d}t\right)^{1/2}|x'|_2. \end{aligned} \tag{12}$$

With Hypothesis 2, we obtain

$$\begin{aligned}\lim_{|x|\to+\infty}\left|\frac{f_3(x)}{x}\right| &= \lim_{|x|\to+\infty}\left|\frac{f_3(x)-f_3(0)+f_3(0)}{x}\right| \\ &\le \lim_{|x|\to+\infty}\left(L_3\left|\frac{x-0}{x}\right|+\left|\frac{f_3(0)}{x}\right|\right) \\ &= L_3.\end{aligned}$$

Therefore, $\lim_{|x|\to+\infty}|f_3(x)/x| \le L_3$, there exists a positive constant ρ^* such that $|f_3(u)| \le L_3|u|$ ($|u| > \rho^*$). Let $f_\rho = \max_{|u|\le\rho^*}|f_3(u)|$ ($|u| \le \rho^*$), we have

$$|f_3(u)| \le L_3|u| + f_\rho, \forall u \in \mathrm{R}. \tag{13}$$

Using inequality $(|a|+|b|)^p \le 2^{p-1}(|a|^p+|b|^p)$ ($p \ge 1$), from inequality (13), we obtain

$$|f_3(u)|^2 \le (L_3|u|+f_\rho)^2 \le 2L_3^2|u|^2 + 2f_\rho^2.$$

Then

$$\begin{aligned}&\int_0^T |f_3(x(t-\tau(t)))|^2\,\mathrm{d}t \\ &\le 2L_3^2\int_0^T |x(t-\tau(t))|^2\,\mathrm{d}t + 2Tf_\rho^2.\end{aligned}$$

Since

$$\begin{aligned}\int_0^T |x(t-\tau(t))|^2\,\mathrm{d}t &= \int_{-\tau(0)}^{T-\tau(T)} \frac{|x(s)|^2}{1-\tau'(\mu(s))}\,\mathrm{d}s \\ &\le \left|\frac{1}{1-\tau'}\right|_\infty \int_0^T |x(s)|^2\,\mathrm{d}s, \end{aligned}\tag{14}$$

we have

$$\begin{aligned}&\int_0^T |f_3(x(t-\tau(t)))|^2\,\mathrm{d}t \\ &\le 2L_3^2\left|\frac{1}{1-\tau'}\right|_\infty \int_0^T |x(t)|^2\,\mathrm{d}t + 2Tf_\rho^2.\end{aligned}\tag{15}$$

Using inequality

$$\begin{aligned}&(a+b)^r \le a^r + b^r \\ &\quad \text{for } a,b \ge 0 \text{ and } 0 \le r \le 1,\end{aligned}\tag{16}$$

from inequality (15), we obtain

$$\begin{aligned}&\left(\int_0^T |f_3(x(t-\tau(t)))|^2\,\mathrm{d}t\right)^{1/2} \\ &\le \sqrt{2}L_3\left|\frac{1}{1-\tau'}\right|_\infty^{1/2}\left(\int_0^T |x(t)|^2\,\mathrm{d}t\right)^{1/2} + \sqrt{2T}f_\rho.\end{aligned}\tag{17}$$

Substituting inequalities (14) and (16) into inequality (12) and using lemma 2.2 in Ref. [15], we have

$$\int_0^T |(Ax')(t)|^2\,\mathrm{d}t \le \sigma_1|x'|_2 + \sigma_2|x'|_2^2, \tag{18}$$

where

$$\begin{aligned}\sigma_1 &= \max_{t\in[0,T]}|f_1(t,0)|\sqrt{T}(1+|m|) \\ &\quad + \max_{t\in[0,T]}|f_2(t,0)|\sqrt{T}(1+|m|) \\ &\quad + |p|_2(1+|m|) + L_1\sqrt{T}\rho(1+|m|) \\ &\quad + \sqrt{2T}f_\rho(1+|m|) \\ &\quad + \sqrt{T}\rho(1+|m|)\left|\frac{1}{1-\tau'}\right|_\infty^{1/2}(L_2+\sqrt{2}L_3), \\ \sigma_2 &= \frac{T}{\pi}(1+|m|)\left[\left|\frac{1}{1-\tau'}\right|_\infty^{1/2}(L_2+\sqrt{2}L_3)+L_1\right].\end{aligned}$$

As ${x'}^2(t) = |\mathrm{A}^{-1}\mathrm{A}x'(t)|^2 \le \frac{1}{(1-|m|)^2}|(\mathrm{A}x')(t)|^2$, from inequality (18), we obtain

$$|x'|_2^2 \le \frac{\sigma_1}{(1-|m|)^2-\sigma_2}. \tag{19}$$

Since $\sigma_1 > 0$ and $(1-|m|)^2 - \sigma_2 > 0$, there exists a positive constant M_2^* such that

$$|x'|_2^2 \le M_2^*, \quad |x|_\infty \le \rho + \sqrt{TM_2^*} := M_2. \tag{20}$$

If $x \in \Omega_1 = \{x : x \in \mathrm{KerL} \cap X \text{ and } \mathrm{N}x \in \mathrm{ImL}\}$, then there exists a constant M_1 such that

$$x(t) \equiv M_1$$

and

$$\int_0^T [f_1(t,M_1)+f_2(t,M_1)+f_3(M_1)+p(t)]\,\mathrm{d}t = 0. \tag{21}$$

Thus,

$$|x(t)| \equiv |M_1| < \rho, \quad \forall x(t) \in \Omega_1. \tag{22}$$

Let $M = M_2 + 1$ and $\Omega = \{x \in X : \ ||x||_X \le M\}$. It is easy to see that N is L-compact on $\bar{\Omega}$. Furthermore, we define continuous functions $H_1(x,\mu)$ and $H_2(x,\mu)$ by setting

$$\begin{aligned}H_1(x,\mu) &= (1-\mu)x + \frac{\mu}{T}\int_0^T [f_1(t,x)+f_2(t,x) \\ &\quad + f_3(x)+p(t)]\,\mathrm{d}t, \ \ \mu \in [0,1]; \\ H_2(x,\mu) &= -(1-\mu)x + \frac{\mu}{T}\int_0^T [f_1(t,x)+f_2(t,x) \\ &\quad + f_3(x)+p(t)]\,\mathrm{d}t, \ \ \mu \in [0,1].\end{aligned}$$

If condition (a) in Hypothesis 3 holds, then

$$xH_1(x,\mu) \ne 0, \quad \forall x \in \partial\Omega \cap \mathrm{KerL}.$$

Correspondingly, we have

$$\begin{aligned}&\deg\{\mathrm{QN}, \ \Omega \cap \mathrm{KerL}, \ 0\} \\ &= \deg\{x, \ \Omega \cap \mathrm{KerL}, \ 0\} \ne 0.\end{aligned}$$

If condition (b) in Hypothesis 3 holds, then

$$xH_2(x,\mu) \neq 0, \quad \forall x \in \partial\Omega \cap \mathrm{KerL}.$$

Hence, using the homotopy invariance theorem, we have

$$\begin{aligned} &\deg\{\mathrm{QN},\ \Omega \cap \mathrm{KerL},\ 0\} \\ &= \deg\{-x,\ \Omega \cap \mathrm{KerL},\ 0\} \neq 0. \end{aligned}$$

The proof is completed.

Theorem 2 If Hypotheses 1 and 2 hold, either condition (a) or (b) in Hypothesis 3 is satisfied, and the following inequality holds:

$$|m| - 1 - L_1|\tau_0|\sqrt{1 + \frac{|\tau_0|}{2T}} - T(L_2 + L_3) > 0,$$

then equation (1) has a unique T-periodic solution.

Proof If condition (a) in Hypothesis 2 holds, then we multiply the two sides of Eq. (6) by $x'(t)$. If Hypothesis 2 (b) holds, then we multiply the two sides of Eq. (6) by $x'(t-\tau_0)$. The rest of the proof is similar to that of Theorem 1 and is omitted.

Remark If the right function in Eq. (1) is of the form $f_1(t, x(t), x(t-\tau(t))) + f_2(x(t-\tau(t))) + p(t)$, we can also apply the same method to obtain similar results as Theorems 1 and 2.

3. Conclusion and discussion

From Theorems 1 and 2, we can see that the results are related to the deviating arguments τ_0 and $\tau(t)$, which are different from the theorems in Refs. [1], [2] and [14]–[17] and the references therein. This implies that the results of this paper are essentially new and are more consist with the actual requirement.

The continuation theorem of the coincidence degree theory is applicable to other nonlinear differential equations with small parameters for obtaining periodic solutions and it is interesting to explore a general principle for nonlinear differential equations with similar results.

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