Dynamical localization effect in a coupled quantum dot array driven by an AC magnetic field

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We have studied the transport properties of a ring-coupled quantum dot array driven by an AC magnetic field, which is connected to two leads, and we give the response of the transport current to the dynamical localization. We found that when the ratio of the magnetic flux to the total quantum dots number is a root of the zeroth order Bessel function, dynamical localization and collapse of quasi-energy occurs and importantly, the transport current displays a dip which is the signal of dynamical localization. The dynamical localization effect is strengthened as a result of the increase of the quantum dot number, and it is weakened on account of the increase of the dots-lead hopping rate.

Keywords: dynamical localization, quantum dot ring, non-equilibrium Green's function

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1. Introduction

There has been growing interest in the response of the electron transport in mesoscopic systems to external fields over the past decade.[1−6] Especially for the response to an AC electric field, plentiful studies have been conducted.[2,3,7,8] It is theoretically known that a particle localized initially in a periodic system remains localized under the influence of a sinusoidal field if the ratio of the field magnitude to the frequency is a root of the zeroth order Bessel function, which is called the dynamical localization. The dynamical localization was first noted by Grossmann[9] and is a dramatic phenomenon which has attracted people to do a lot of follow-up work. Almost all the dynamical localization in the semiconductor superlattices is driven by an AC magnetic field[10,11] and the quantum dot arrays[12−15] are caused by an AC electric field. In addition, the dynamical behaviours in the mesoscopic metallic ring driven by an AC magnetic field[16−19] and in the quantum dot(QD) ring driven by a DC magnetic field[20−22] have also attracted much attention due to the progress in the nanofabrication technique. However, the dynamical localization in the QD ring induced by an AC magnetic field is rarely investigated. In this paper, we study the dynamical localization in a coupled QD ring induced by an AC magnetic field using the non-equilibrium Green's function technique. The model considered is a coupled QD ring threaded by an AC magnetic flux. We find that the average current nearly vanishs if the ratio of the magnetic flux to the total QD number is a root of the zeroth order Bessel function, which corresponds to the collapse of the quasi-energy spectrum of the system and makes a signal to the condition of dynamical localization of one initially localized electron. Thus, we can predict the dynamical localization by the fantastic quality of the average current of the system.

2. Physical model and formula

The system we consider is a coupled QD ring threaded by an AC magnetic flux. There are odd QDs in the ring. The first and the $\frac{N+1}{2}$th QD(N is the number of the quantum dots in the ring) are coupled to two metal leads with chemical potential $\mu_{\rm L}$ and $\mu_{\rm R}$, respectively. The total system under consideration can be modeled by the following Hamiltonian:

$$\begin{aligned} H = & \sum_{\alpha k} \epsilon_{\alpha k} a^{\dagger}_{\alpha k} a_{\alpha k} + \sum_{i=1}^{N} \epsilon_i d^{\dagger}_i d_i \\ & + \sum_{i=1}^{N-1} \Big[t_{\rm c} \exp\Big({\rm i} \frac{\Phi}{N\Phi_0} \sin \omega t \Big) d^{\dagger}_i d_{i+1} \\ & + t_{\rm c} \exp\Big(-{\rm i} \frac{\Phi}{N\Phi_0} \sin \omega t \Big) d^{\dagger}_{i+1} d_i \Big] \\ & + t_{\rm c} \exp\Big({\rm i} \frac{\Phi}{N\Phi_0} \sin \omega t \Big) d^{\dagger}_N d_1 \end{aligned}$$

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$$+ t_{\rm c}\exp\Big(-{\rm i}\frac{\Phi}{N\Phi_0}\sin\omega t\Big)d_1^{\dagger}d_N + \sum_k\Big[\Big(t_{\rm L}d_1^{\dagger}a_{{\rm L}k} + t_{\rm R}d^{\dagger}_{\frac{N+1}{2}}a_{{\rm R}k}\Big) + {\rm H.c.}\Big], \quad (1)$$

where $a^{\dagger}_{\alpha k}$ ($a_{\alpha k}$) is the creation (annihilation) operator of the electron in the α lead, and $d_i^{\dagger}$ (d_i) represents the creation (annihilation) operator of the electron in the ith QD ($i = 1, \ldots, N$). The interdot coupling $t_{\rm c}$ describes the electron tunneling between two nearest dots. The applied time-dependent magnetic field induces a time-dependent phase $\frac{\Phi}{N\Phi_0}\sin\omega t$ in the tunneling amplitude.[17] Here, Φ is the magnetic flux caused by the external field and ω is the frequency of the AC magnetic field. We choose a gauge such that the magnetic flux quantum $\Phi_0 = \hbar/e$. ϵ_i is the energy of an electron located in the dot i. The tunneling coupling $t_\alpha(\alpha = {\rm L}, {\rm R})$ describe the interaction between the leads and the QD ring.

The current from L leading to the QD ring can be calculated from standard non-equilibrium Green's function (NGF) techniques and expressed in terms of the dots' Green's function as[22–25]

$$I_{\rm L} = \frac{2e}{\hbar}{\rm Re}\Big\{\int {\rm d}t_1\Big[G^{\rm r}_{11}(t,t_1)\,\Sigma^{<}_{{\rm L},11}(t_1,t) + G^{<}_{11}(t,t_1)\Sigma^{\rm a}_{{\rm L},11}(t_1,t),\Big]\Big\}, \quad (2)$$

where the Green's function $\boldsymbol{G}^{\rm r,<}$ and the self-energy $\boldsymbol{\Sigma}^{\rm a,<}$ are N-dimensional matrices for the ring-coupled quantum dot array. The bold-faced letters are used to denote matrices. The retarded and the lesser Green's function are defined as $\boldsymbol{G}^{\rm r}(t,t') = -{\rm i}\theta(t-t')\Big\langle\Big\{\boldsymbol{\Psi}(t),\boldsymbol{\Psi}^{\dagger}(t')\Big\}\Big\rangle$ and $\boldsymbol{G}^{<}(t,t') = {\rm i}\Big\langle\boldsymbol{\Psi}^{\dagger}(t')\boldsymbol{\Psi}(t)\Big\rangle$, respectively, with $\boldsymbol{\Psi}(t) = (d_1^{\dagger},\ldots,d_N^{\dagger})^{\dagger}$. The retarded self-energy $\boldsymbol{\Sigma}^{\rm r} = \sum_\alpha\boldsymbol{\Sigma}^{\rm r}_\alpha + \boldsymbol{\Sigma}^{\rm r}_{\rm c}$, where $\boldsymbol{\Sigma}^{\rm r}_\alpha$ and $\boldsymbol{\Sigma}^{\rm r}_{\rm c}$ are caused by the α lead and the interdot coupling, respectively. The retarded Green's functions can be calculated by using the Dyson equation $\boldsymbol{G}^{\rm r}(t,t') = \boldsymbol{g}^{\rm r}(t,t') + \int{\rm d}t_1\int{\rm d}t_2\boldsymbol{G}^{\rm r}(t,t_1)\boldsymbol{\Sigma}^{\rm r}(t_1,t_2)\boldsymbol{g}^{\rm r}(t_2,t')$, where $\boldsymbol{g}^{\rm r}(t,t')$ is the retarded Green's function for the N separate QDs without interdot coupling and couplings to the leads, and its matrix element is expressed as

$$\boldsymbol{g}^{\rm r}_{ij}(t,t') = -{\rm i}\delta_{ij}\theta(t-t')e^{-{\rm i}\epsilon_i(t-t')}. \quad (3)$$

The lesser Green's function may be related to the retarded Green's function via the Keldysh equation $\boldsymbol{G}^{<}(t,t') = \int{\rm d}t_1\int{\rm d}t_2\boldsymbol{G}^{\rm r}(t,t_1)\boldsymbol{\Sigma}^{<}(t_1,t_2)\boldsymbol{G}^{\rm a}(t_2,t')$, with $\boldsymbol{G}^{\rm a} = (\boldsymbol{G}^{\rm r})^{\dagger}$ and $\boldsymbol{\Sigma}^{<} = \sum_\alpha\boldsymbol{\Sigma}^{<}_\alpha + \boldsymbol{\Sigma}^{<}_{\rm c}$.

In order to obtain the expression for the current, we can take the Fourier expansion of the two-time Green's function as $\boldsymbol{G}(t,t') = \sum_l {\rm e}^{{\rm i}l\omega t}\int\frac{{\rm d}\epsilon}{2\pi}{\rm e}^{-{\rm i}\epsilon(t-t')}\tilde{\boldsymbol{G}}_l(\epsilon)$,[26,27] with the definition $\boldsymbol{G}_{mn}(\epsilon) = \tilde{\boldsymbol{G}}_{m-n}(\epsilon - n\omega)$ and ω being the frequency of the AC magnetic field. The Fourier transform of $\boldsymbol{g}^{\rm r}(t,t')$ gives rise to

$$\boldsymbol{g}^{\rm r}_{ij,mn}(\epsilon) = \frac{\delta_{ij}\delta_{mn}}{\epsilon - \epsilon_i - n\omega + i0^+}, \quad (4)$$

where i, j are the matrix element indices, and m, n are the Fourier component indicators.

Under the wide-band approximation, the Fourier transforms of the retarded self-energies are

$$\sum_{\alpha={\rm L,R}}\boldsymbol{\Sigma}^{\rm r}_{\alpha,ij,mn}(\epsilon) = -\frac{\rm i}{2}\delta_{mn}\big(\Gamma_{\rm L}\delta_{i1}\delta_{j1} + \Gamma_{\rm R}\delta_{i\frac{N+1}{2}}\delta_{j\frac{N+1}{2}}\big), \quad (5)$$

and

$$\Sigma^{\rm r}_{{\rm c},ij,mn}(\epsilon) = t_{\rm c}[J_{n-m}(x)\delta_{i,j+1} + J_{m-n}(x)\delta_{i+1,j} + J_{m-n}(x)\delta_{i1}\delta_{jN} + J_{n-m}(x)\delta_{iN}\delta_{j1}], \quad (6)$$

where $x = \Phi/N\Phi_0$ and $J_n(x)$ is the nth Bessel function of the first kind, $\Gamma_\alpha = 2\pi\rho_\alpha t_\alpha t^*_\alpha$ with ρ_α being the density of states of the corresponding lead. The lesser self-energies are $\boldsymbol{\Sigma}^{<}_{{\rm c},mn}(\epsilon) = 0$, and

$$\sum_{\alpha={\rm L,R}}\boldsymbol{\Sigma}^{<}_{\alpha,ij,mn}(\epsilon) = {\rm i}\delta_{mn}\Big[f_{\rm L}(\epsilon - n\omega)\Gamma_{\rm L}\delta_{i1}\delta_{j1} + f_{\rm R}(\epsilon - n\omega)\Gamma_{\rm R}\delta_{i\frac{N+1}{2}}\delta_{j\frac{N+1}{2}}\Big], \quad (7)$$

where $f_\alpha(\epsilon) = 1/({\rm e}^{\frac{\epsilon-\mu_\alpha}{k_B{\rm T}}} + 1)$ denotes the Fermi distribution of electrons in the α lead. After taking the Fourier transformations, the Dyson equation and the Keldysh equation become

$$\boldsymbol{G}^{\rm r}_{mn}(\epsilon) = \boldsymbol{g}^{r}_{mn}(\epsilon) + \sum_{lk}\boldsymbol{G}^{\rm r}_{ml}(\epsilon)\boldsymbol{\Sigma}^{\rm r}_{lk}(\epsilon)\boldsymbol{g}^{\rm r}_{kn}(\epsilon), \quad (8)$$

and

$$\boldsymbol{G}^{<}_{mn}(\epsilon) = \sum_k\boldsymbol{G}^{\rm r}_{mk}(\epsilon)\boldsymbol{\Sigma}^{<}_{kk}(\epsilon)\boldsymbol{G}^{\rm a}_{kn}(\epsilon). \quad (9)$$

With these Green's functions obtained, the time-dependent current can be expressed as

$$I_{\rm L}(t) = \frac{2e}{\hbar}{\rm Re}\sum_m{\rm e}^{{\rm i}m\omega t}\times\Big\{\int\frac{{\rm d}\epsilon}{2\pi}\big[\boldsymbol{G}^{\rm r}_{11,m0}(\epsilon)\boldsymbol{\Sigma}^{<}_{{\rm L},11,00}(\epsilon) + \boldsymbol{G}^{<}_{11,m0}(\epsilon)\boldsymbol{\Sigma}^{\rm a}_{{\rm L},11,00}(\epsilon)\big]\Big\}, \quad (10)$$

and the average current is

$$I_{\rm L} = \langle I_{\rm L}(t)\rangle = \frac{2e}{\hbar}{\rm Re}\Big\{\int\frac{d\epsilon}{2\pi}\big[\boldsymbol{G}^{\rm r}_{11}(\epsilon)\boldsymbol{\Sigma}^{<}_{{\rm L},11}(\epsilon) + \boldsymbol{G}^{<}_{11}(\epsilon)\boldsymbol{\Sigma}^{\rm a}_{{\rm L},11}(\epsilon)\big]_{00}\Big\}. \quad (11)$$

In what follows, we shall give the results of the numerical calculations at zero temperature.

3. Results and discussion

We now numerically study the average current and the dynamical evolution of our system with different parameters when the oscillating magnetic field is applied. In the following discussion, we set $\hbar = e = \Phi_0 = 1, \Gamma_{\rm L} = \Gamma_{\rm R} = \Gamma$ and the driving frequency ω serves as the energy units. The other parameters are chosen as follows: the hopping matrix element $t_{\rm c}/\omega = 0.25$, the applied voltage $\mu_{\rm L} - \mu_{\rm R} = 5\omega$, and the QD levels $\epsilon_i/\omega = 1$, $i = 1, \ldots, N$.

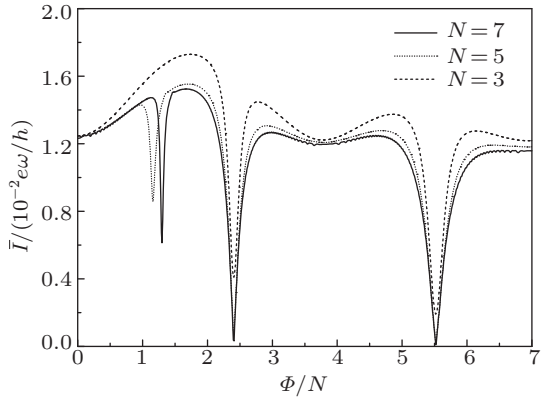


Fig. 1. Average current $\bar{I}$ as a function of the amplitude of the magnetic flux Φ for $N = 3, 5, 7$ and $\Gamma/\omega = 0.05$.

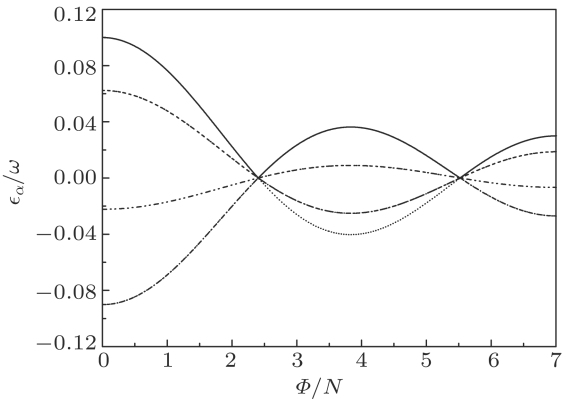


Fig. 2. The quasi-energy spectrum ϵ_α of the system with $N = 7$.

The study of the electron transport properties of the system considered is helpful for understanding the influence of the applied time-dependent magnetic field on the dynamical behaviour of the electron in the QD ring. Figure 1 shows the average current as a function of the magnetic flux caused by the oscillating magnetic field for the dot-lead hopping rate $\Gamma/\omega = 0.05$ and $N = 3, 5, 7$. Obviously one can see two valleys of the average current where the currents are nearly equal to zero. It is interesting that the minima of the currents occur at the same position independent of the QD numbers in the ring. The magnetic flux corresponding to the minima of the currents is the roots of the zeroth order Bessel function, i.e. $\Phi/N = 2.405$ and 5.52. The quasi-energy spectrum ϵ_α can be obtained by numerically diagonalizing the time evolution operator $U(t)$ of the system in one time cycle $T = 2\pi/\omega$. Figure 2 shows the quasi-energy as a function of the magnetic flux for $N = 7$, where three of the seven curves are doubly degenerate. One can find from Figs. 1 and 2 that the quasi-energy also collapses at the same value of the magnetic flux as the average current equals zero, which means that the vanishing of the average current and the collapse of the quasi-energy can result from the dynamical localization.

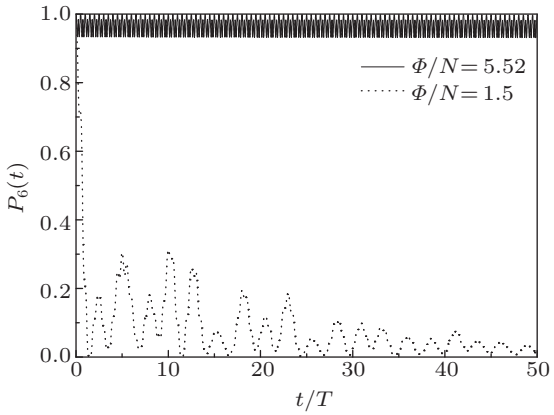


Fig. 3. Time evolution of the occupation probability $P_6(t)$ for $N = 7$, $\Gamma/\omega = 0.05$.

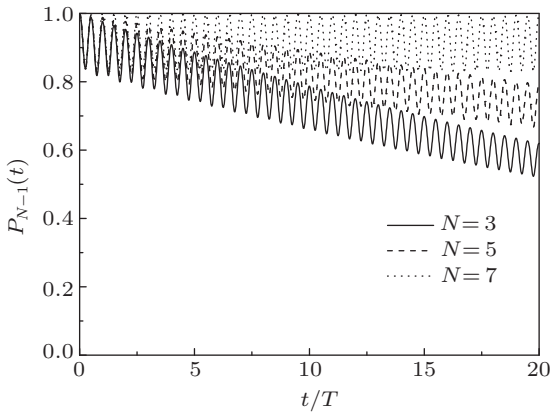


Fig. 4. Time evolution of the occupation probability $P_{N-1}(t)$ with $\Gamma/\omega = 0.05$, $\Phi/N = 2.405$ for $N = 3, 5, 7$.

In order to confirm our speculation, the dynamical evolution of the system is investigated. In the following, we suppose that the electron is initially localized at the ith QD (counting clockwise from the dot coupling to L lead) in the ring and $P_i(t)$ denotes the time evolution of the occupying probability of the electron in the dot. Figure 3 shows the occupation probability $P_6\,(t)$ as a function of dimensionless time

t/T for $N = 7$. The magnetic flux parameter is chosen as $\Phi/N = 5.52$ at which the quasi-energy spectrum collapses and the average current vanishes. We can see that the probability $P_6(t)$ oscillates between 1 and 0.93 with increasing time, which means that the electron remains localized at the sixth QD and the dynamical localization happens perfectly. When we set the magnetic flux parameter $\Phi/N = 1.5$ at which the quasi-energy does not collapse and the average current dose not vanish, the probability $P_6(t)$ decays quickly with increasing time, implying that the electrons will be scattered to all QDs and delocalization occurs. From the above discussion, we can find that when the magnetic flux parameters make the average current vanish, the electrons initially localized will remain localized, and when the parameters cannot ensure the current to vanish, the electrons will be delocalized, which implies that one can predict the dynamical evolution of the system from the transport properties.

The influence of the QD number in the ring on the localization effect is shown in Fig. 4, where $P_{N-1}(t)$ denotes the dynamical evolution of the occupation probability for $N = 3, 5, 7$, with the corresponding magnetic flux parameter $\Phi/N = 2.405$ and $\Gamma/\omega = 0.05$. For the three-dot system the probability $P_2(t)$ oscillates damply and the electron exhibits the behaviour of dynamical localization only in the earlier few periods. For the seven-dot system, the probability $P_6(t)$ is oscillatory with increasing time and the dynamical localization happens perfectly. Thus, we can deduce that the increase of the QD number in the ring strengthens the localization effect.

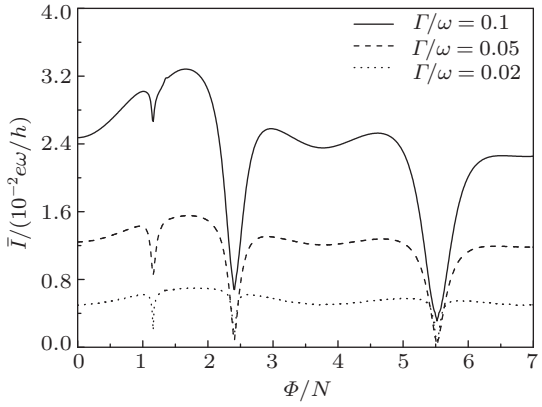


Fig. 5. Average current $\bar{I}$ as a function of the amplitude of the magnetic flux Φ and total quantum dot number N for $\Gamma/\omega = 0.02, 0.05, 0.1$.

The influence of the dot-lead hopping rate Γ on the average current and the dynamical localization are shown in Figs. 5 and 6, respectively. With increasing Γ the average current only increases its amplitude (see Fig. 5). However, the increase of the dot-lead hopping rate Γ influences essentially the localization effect. From Fig. 6 one can find that even if the magnetic flux parameter satisfies the condition of the quasi-energy collapse, for example $\Phi/N = 2.405$, the amplitude of the occupation probability $P_4(t)$ decreases with the time and the delocalization occurs earlier with increasing Γ, which implies that the dot-lead hopping rate Γ is a factor of weakening dynamical localization.

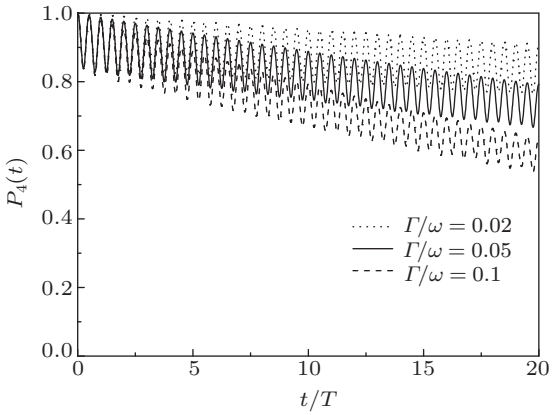


Fig. 6. Time evolution of the occupation probability $P_4(t)$ with $N = 5$, $\Phi/N = 2.405$ for $\Gamma/\omega = 0.02, 0.05, 0.1$.

4. Conclusion

In conclusion, using the non-equilibrium Green's function technique, we have numerically studied the effect of dynamical localization on transport properties of a coupled QD ring connected to leads under the influence of an external sinusoidal magnetic field. We have found that if the ratio of the magnetic flux to the total QDs number is a root of the zeroth order Bessel function, a minimum of the average current appears, which corresponds to the collapse of the quasi-energy spectrum and makes a signal to dynamical localization of one initially localized electron. The dynamical localization effect is strengthened as a result of the increase of the QD number, and it is weakened on account of the increase of the dot-lead hopping rate.

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