A method to control the polarization of random terahertz lasing in two-dimensional disordered ruby medium*

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Terahertz (THz) random lasing is studied numerically for two-dimensional disordered media made of ruby grains with a three-level atomic system. A method via the adjustment of the pumping area to control the polarization of the THz wave is proposed. Computed results reveal that transverse electric THz lasing modes could occur if pumping is supplied on the whole medium, while transverse magnetic THz lasing modes could occur if pumping is appropriately supplied on a partial area of the medium.

Keywords: disordered media, lightwave localization, random lasers

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1. Introduction

In the late 1960s, Letokhov[1] predicted that a high concentration of scatterers in a gain medium can provide stimulated emission feedback, thus eliminating the need for an external laser cavity. Those proposed unusual sources of stimulated emission, later called random lasers, have been widely studied both experimentally and theoretically.[2−12] It is worth stating here that in order to study the lasing thresholds of both the transverse magnetic (TM) field and the transverse electric (TE) field, an experiment in a two-dimensional (2D) amplifying and scattering medium consisting of a random array of dye-doped plastic fibre was performed by Ito and Tomita.[5] They found that the lasing threshold of the TE field is higher than that of the TM field. Owing to the different spatial tropisms of the dipole moment in the dye molecular structure, TE and TM fields have different dye molecules. In this case, the characteristics of TE and TM fields can be investigated individually under no competition. However, when the gain medium is solid, the TE and TM fields will share the same population inversion, which could cause competition between the two states. Considering the competition, our previous study pointed out that the TM state is suppressed strongly by the TE state in the optical band.[9]

So far, most studies have focused on the random laser in the optical band. Can random lasing occur in the terahertz (THz) domain? Recently, a one-dimensional (1D) theoretical model was built for active disordered media made of ruby grains with a three-level atomic system and numerical results revealed that THz random lasing could occur under suitable conditions.[12] The proposed scheme is based on the pumping of the $2\bar{A}$ level of ruby via a ruby laser operating on its R_2 line (693.9 nm) at low temperatures and 0.87 THz random lasing is expected on the $2\bar{A}$ to $\bar{E}$ transition of the split ^{2}E level.

In this work, by combining Maxwell equations with three-level rate equations of electronic population, we build a 2D model for active disordered media made of ruby grains to research random lasing in the THz domain. On this basis, we perform many numerical calculations and the results reveal that THz random lasing can occur in 2D disordered medium. Such a 2D model allows us to analyse the property of THz random lasing in detail. We pay attention to the polarization property of the THz wave for the case where the two polarization states will share the same

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population inversion. A method via adjusting pumping area to control the polarization of THz random lasing is presented. Computed results reveal that TE THz lasing modes could occur if pumping is supplied on the whole medium, while TM THz lasing modes could occur if pumping is appropriately supplied on a partial area of the medium. We believe that the results can provide some valuable predictions for further possible experimental observation.

2. Theoretical model

Our model starts with the construction of a 2D square disordered medium, the distribution of which can be seen from Fig. 1(a). The outline dimension of the medium is 10 cm×10 cm, the surface-filling fraction of the particles is $\Phi = 40\%$ and the radius r of circular particles is $r = 60$ μm. In our system, these disordered particles are randomly distributed in a background medium with a refractive index n_1 and are chosen as active ruby grains with a refractive index n_2.

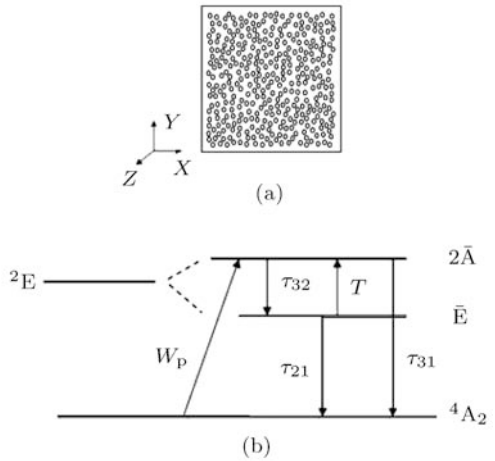


Fig. 1. (a) Schematic diagram of the 2D random medium made of ruby grains, and (b) the scheme of the energy levels for Cr^{3+} in ruby.

For a TM field, the Maxwell's equations read

$$\mu_0 \frac{\partial H_x}{\partial t} = -\frac{\partial E_z}{\partial y}, \tag{1a}$$

$$\mu_0 \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}, \tag{1b}$$

$$\varepsilon_i \varepsilon_0 \frac{\partial E_z}{\partial t} + \frac{\partial P_z}{\partial t} = \frac{\partial H_m}{\partial x} - \frac{\partial H_x}{\partial y}. \tag{1c}$$

For a TE field, the counterparts of the above equations are given in the following form:

$$\frac{\partial H_z}{\partial y} = \varepsilon_0 \varepsilon_i \frac{\partial E_x}{\partial t} + \frac{\partial P_x}{\mathrm{d}t}, \tag{2a}$$

$$-\frac{\partial H_z}{\partial x} = \varepsilon_0 \varepsilon_i \frac{\partial E_y}{\partial t} + \frac{\partial P_y}{\mathrm{d}t}, \tag{2b}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu_0 \frac{\partial H_z}{\partial t}. \tag{2c}$$

Here, μ_0 and ε_0 are the magnetic permeability and electric permittivity of vacuum, respectively, and $\varepsilon_i = n_i^2$, $i = 1, 2$.

If the active part includes ruby, it can be described as a three-level atomic system. For such a system, the three-level rate equations read[12]

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = \frac{N_2}{\tau_{21}} + \frac{N_3}{\tau_{31}} - N_1 W_\mathrm{p}, \tag{3a}$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} - \frac{N_2}{T} + \frac{E}{\hbar\omega_\mathrm{l}} \frac{\mathrm{d}P}{\mathrm{d}t}, \tag{3b}$$

$$\frac{\mathrm{d}N_3}{\mathrm{d}t} = N_1 W_\mathrm{p} \frac{N_3}{\tau_{31}} - \frac{N_3}{\tau_{32}} - \frac{N_2}{T} - \frac{E}{\hbar\omega_\mathrm{l}} \frac{\mathrm{d}P}{\mathrm{d}t}. \tag{3c}$$

In the above rate equations, the system considered here is a simplified yet realistic three-level atomic system (Fig. 1(b)) with energy levels E_1 ($^4A_2$), E_2 ($\bar{E}$), and E_3 ($2\bar{A}$), and populations N_1, N_2 and N_3, respectively. W_p is the external optical pumping rate at 693.9 nm that transfers atoms from the ground state E_1 to level E_3; $1/\tau_{ij}$ is the spontaneous decay rate of an atom from energy level E_i to level E_j. In Eqs. (3b) and (3c), owing to the interaction of the electronic states $2\bar{A}(^2E)$ and $\bar{E}(^2E)$ of the excited Cr^{3+} with the resonant 29 cm^{-1} phonons, T denotes the time of flight between emission and re-absorption of a 29 cm^{-1} phonon and is chosen as 1×10^{-9} s.[12] In fact, T is temperature-dependent and tends to converge to a shorter time at high (i.e. room) temperature.[13] Here, due to a ruby laser operating at its R_2 line (693.9 nm) at low temperature (4 K), T is chosen as 1×10^{-9} s in our simulations. This is because the cooling of the solid single crystalline ruby is considered to reduce the THz losses and phonon densities. The frequency $\omega_\mathrm{l} = (E_3 - E_2)/\hbar = 2\pi \times 0.87$ THz is the transition frequency from level E_3 to level E_2. The term $(E_z/\hbar\omega_\mathrm{l}) \cdot (\mathrm{d}P_z/\mathrm{d}t)$ is the classical expression for instantaneous energy transfer divided by the energy per photon and is equivalent to the stimulated transition probabilities more commonly used in the rate equation above. Note that in our finite-difference time-domain (FDTD) simulation, Eq. (3c) actually uses the conservation law $N_\mathrm{tot} = N_1 + N_2 + N_3$ for the total electron density.

In Eqs. (2) and (3), $\boldsymbol{P}$ is described as obeying the quantum population equation of motion

$$\frac{\mathrm{d}^2 P}{\mathrm{d}t^2} + \Delta\omega_\mathrm{l} \frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} + \omega_\mathrm{l}^2 \boldsymbol{P} = \kappa \Delta N \boldsymbol{E}, \tag{1}$$

where $\boldsymbol{P} = P_z\boldsymbol{e}_z$ and $\boldsymbol{E} = E_z\boldsymbol{e}_z$ for TM state, while $\boldsymbol{P} = P_x\boldsymbol{e}_x + P_y\boldsymbol{e}_y$ and $\boldsymbol{E} = E_x\boldsymbol{e}_x + E_y\boldsymbol{e}_y$ for TE state; $\Delta\omega_{\rm l} = 1/\tau_{32} + 2/T_2$ is the actual linewidth (gain spectrum width) of the transition centred at the angular frequency $\omega_{\rm l}$ with T_2 being the collision time; κ is given by $\kappa = 6\pi\varepsilon_0 c^3/(\omega_{32}^2\tau_{32})$; and $\Delta N = N_2 - N_3$ is the population difference density between the populations on levels 2 and 3.

It is worth noting that, as mentioned previously, both the TE and the TM states share the same population inversion, which could cause competition on the inversion population between them. Here, from Eqs. 3(b) and 3(c), one can see that the populations N_2 and N_3 are related to both the TE state ($\boldsymbol{P} = P_x\boldsymbol{e}_x + P_y\boldsymbol{e}_y$ and $\boldsymbol{E} = E_x\boldsymbol{e}_x + E_y\boldsymbol{e}_y$) and the TM state ($\boldsymbol{P} = P_z\boldsymbol{e}_z$ and $\boldsymbol{E} = E_z\boldsymbol{e}_z$). That is, the inversed population $\Delta N = N_2 - N_3$ shared by the two states will result in competition for the population.

The values of the parameters used in the simulating calculations are taken as $T_2 = 2 \times 10^{-14}$ s, $\tau_{32} = 1.1 \times 10^{-9}$ s, $\tau_{21} = 3 \times 10^{-3}$ s, $\tau_{31} = 3 \times 10^{-6}$ s, $N_{\rm T} = \sum_{i=1}^{3} N_i = 1.6 \times 10^{25}$ m^{-3}, $n_1 = 1$ and $n_2 = 10$. Note that the refractive index n_2 for THz radiation in ruby is twice larger than the one for optics.[13]

To excite this system, we must introduce sources into the system. Therefore, a broadband Gaussian pulse with a random amplitude as an excitation source is used in our system. Then by FDTD methods and under a perfectly matched layer (PML) absorbing boundary condition, the electromagnetic fields can be calculated through solving the above equations. In the FDTD calculation, we discretize both the space and the time. The discrete time step and space step are chosen to be $\Delta x = 2 \times 10^{-6}$ m and $\Delta t \leq \Delta x/(2c)$, where Δt is taken to be 2×10^{-15} s. The time-domain response of the pulse is observed in a time window of duration $T_{\rm w} = 5.2 \times 10^{-10}$ s at all nodes in the system and Fourier transformed in order to obtain the intensity spectrum.

3. Results and discussion

First, for the case of pumping on the whole medium, we begin by calculating the spectral intensity versus the pumping rate for both the TE and the TM states, as shown in Figs. 2 and 3. Note that every peak denotes a localized mode supported by the disordered medium. Here, a TE mode indicated by $\lambda_{\rm TE}$ with $\lambda_{\rm TE} = 333.3$ μm and a TM mode indicated by $\lambda_{\rm TM}$ with $\lambda_{\rm TM} = 332.8$ μm are selected. As can be seen from Figs. 2(a) and 3(a), there are many discrete and weak peaks in the whole spectrum at pumping rate $W_{\rm p} = 10$ s^{-1}, each having the same order of magnitude. As the pumping rate increases, the mode $\lambda_{\rm TE}$ dominates the whole spectrum and is amplified preferentially, and its spectral width becomes broader at $W_{\rm p} = 5 \times 10^2$ s^{-1} than those at lower $W_{\rm p}$. That is, the mode $\lambda_{\rm TE}$ is excited first. When the pumping rate increases further, another TE mode is also excited. However, for TM modes, the spectra are still discrete and their spectral intensities are very weak.

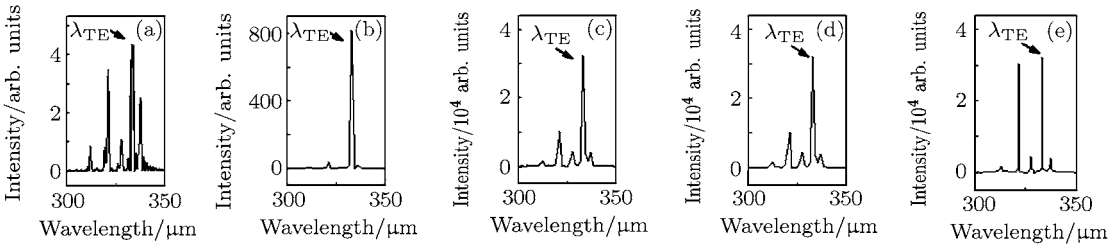


Fig. 2. Power spectra for TE fields: (a) $W_{\rm p} = 10\,{\rm s}^{-1}$, (b) $W_{\rm p} = 5\times10^2\,{\rm s}^{-1}$, (c) $W_{\rm p} = 10^3\,{\rm s}^{-1}$, (d) $W_{\rm p} = 5\times10^3\,{\rm s}^{-1}$ and (e) $W_{\rm p} = 10^4\,{\rm s}^{-1}$.

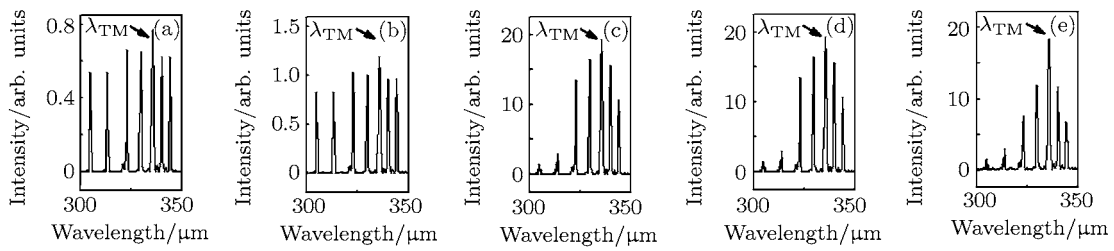


Fig. 3. Power spectra for TM fields: (a) $W_{\rm p} = 10\,{\rm s}^{-1}$, (b) $W_{\rm p} = 5\times10^2\,{\rm s}^{-1}$, (c) $W_{\rm p} = 10^3\,{\rm s}^{-1}$, (d) $W_{\rm p} = 5\times10^3\,{\rm s}^{-1}$ and (e) $W_{\rm p} = 10^4\,{\rm s}^{-1}$.

Clearly, the spectral intensity of the TE state increases faster than that of the TM state. The peak intensity of the TE mode $\lambda_{\rm TE}$ is higher than that of the TM mode $\lambda_{\rm TM}$ by about three orders of magnitude, which means that TE modes stand on an advantage point in the competition of inversed population with the TM modes. According to the traditional method, two threshold curves for both the TE mode $\lambda_{\rm TE}$ and the TM mode $\lambda_{\rm TM}$ are plotted in Fig. 4, indicating that the TM state is strongly restrained. This result is in good agreement with the previous discussion on optical bands.[9]

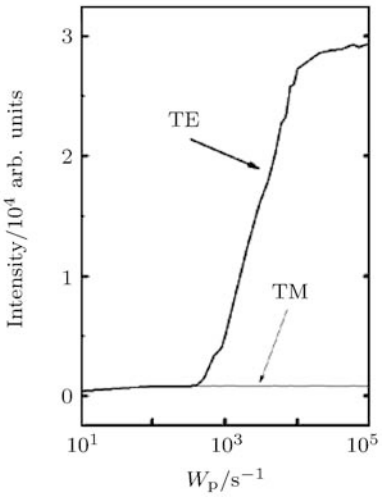


Fig. 4. Power spectra that have the minimum lasing threshold versus pump rate for both TE and TM cases.

As mentioned above, we know that a TM polarized THz wave is strongly suppressed. That is, a TE mode has a much lower lasing threshold than that of TM modes in the competition for THz random lasing systems. By the use of this method, we can easily obtain the TE localized THz wave. Unfortunately, it is unsuitable to obtain TM cases under the whole pumping. To obtain the TM mode individually, we have employed an effective technique of supplying a local pump, which provides with us a feasible way to supply localized gain for a special region in a 2D random medium to efficiently obtain TM polarized THz waves.

In the case of pumping on the whole medium, the spatial distribution maps of the TE and TM modes are obtained, as shown in Figs. 5(a) and 5(b). One can see that the spatial distribution of the TM mode is much more centralized than that of the TE mode. In this special centralized region, marked by the white circle for TM cases, the TM mode has a higher energy density and the radius of the white circle for this selected region is defined as R. Now we take this selected region as the localized pump. Note that Ref. [13] expected amplification in the range of 0.1–0.5 cm^{-1} while the THz losses were estimated to be 0.4–0.5 cm^{-1}. Under this assumption, relatively long samples are needed to maintain a sufficient net gain, at least 0.5 cm at very low temperature (4 K). So we choose the minimal pumping region R as 0.5 cm.

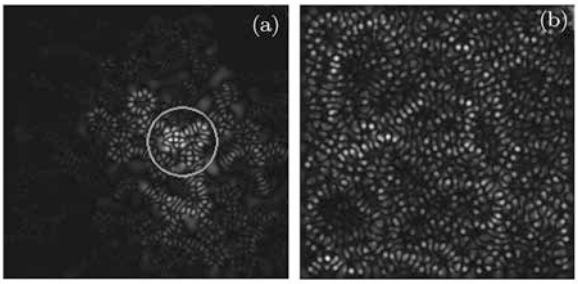


Fig. 5. Spatial distribution of the field amplitude corresponding to indicated modes: (a) mode $\lambda_{\rm TM}$, in which a white circle denotes the local pumping region we select, (b) mode $\lambda_{\rm TE}$.

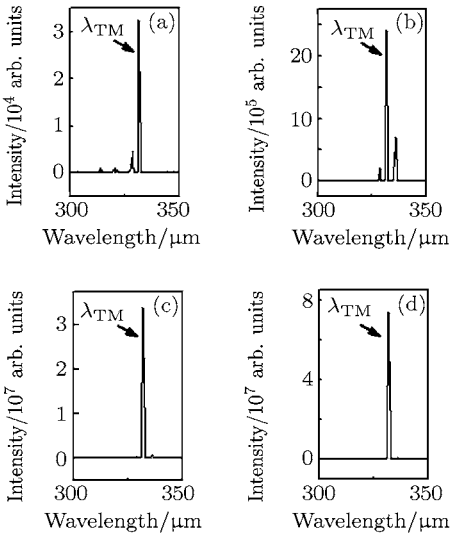


Fig. 6. Power spectra of TM polarized THz waves at different R with $W_{\rm p} = 10^5$ s^{-1}, (a) $R = 0.5$ cm, (b) $R = 1$ cm, (c) $R = 2$ cm, (d) $R = 4$ cm.

The spectral intensities of the marked TM and TE fields under the localized pump at different R with $W_{\rm p} = 10^5$ s^{-1} are calculated, as shown in Figs. 6 and 7. One can see that the intensity of the TE mode is much lower than that of the TM mode, about five orders of magnitude at a fairly small pumping region, $R = 0.5$ cm. For meaningful and direct comparison between Figs. 6 and 7, we plot the peak intensity ratio K of mode $\lambda_{\rm TM}$ to mode $\lambda_{\rm TE}$ at different pumping regions in Fig. 8. With the increase of R, the emission peak ratio K descends gradually. Note that the

spectral intensity of mode $\lambda_{\rm TM}$ climbs up and then declines, as shown in Fig. 6, which denotes that there is a critical value of localized pumping region for mode $\lambda_{\rm TM}$. Therefore, we can obtain a rough idea that at the selected R below this critical value, TM polarized modes would be excited more easily than TE ones. To realize this idea, the threshold gain behaviour for both TM and TE modes with different R is investigated.

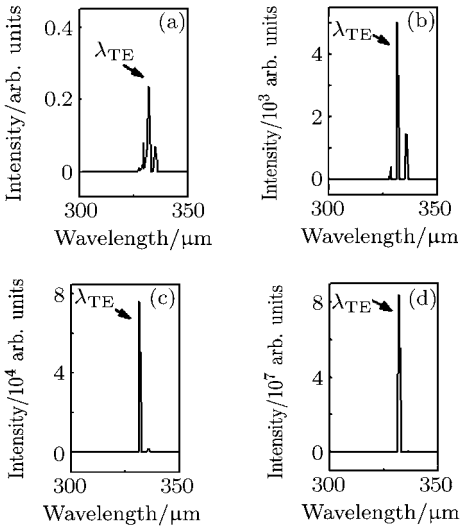


Fig. 7. Power spectra of TE polarized THz waves at different R with $W_{\rm p} = 10^5$ s^{-1}, (a) $R = 0.5$ cm, (b) $R = 1$ cm, (c) $R = 2$ cm, (d) $R = 4$ cm.

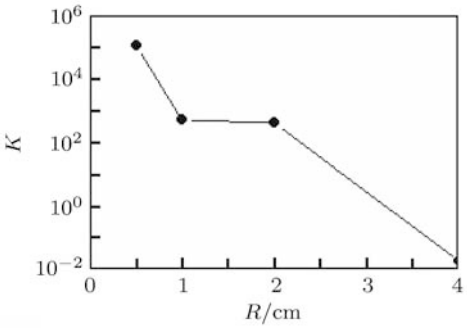


Fig. 8. Plot of the peak intensity ratio K of TM mode $\lambda_{\rm M1}$ to TE model $\lambda_{\rm E1}$ versus R.

We plot the threshold gain curves for the two polarized modes as shown in Figs. 2 and 3 indicated by $\lambda_{\rm TE}$ and $\lambda_{\rm TM}$ with different R, respectively, as shown in Fig. 9. Then their thresholds, marked by $W_{\rm pTM}$ and $W_{\rm pTE}$, are measured in Fig. 9. In order to compare the evolution of the two modes with different R, the thresholds of the two modes varying with different R are also plotted in Fig. 10, from which one can see that with the increase of R, the thresholds of the TE and TM modes decrease simultaneously for $R < 2$ cm. With a further increase of R, the threshold of the TE mode decreases sequentially whereas that of the TM mode begins to increase. Note that the threshold of the TM mode is always lower than that of the TE mode for $R < 2$ cm. This denotes that there exists an optimal value, R, at which the threshold ratio of TE mode to TM mode is the highest. That is, we can obtain effectively TM polarized modes by searching for an optimal value of R. As the pumping region R increases, the threshold of TM mode increases simultaneously and tends to the non-localized pumping case, namely the whole pumping.

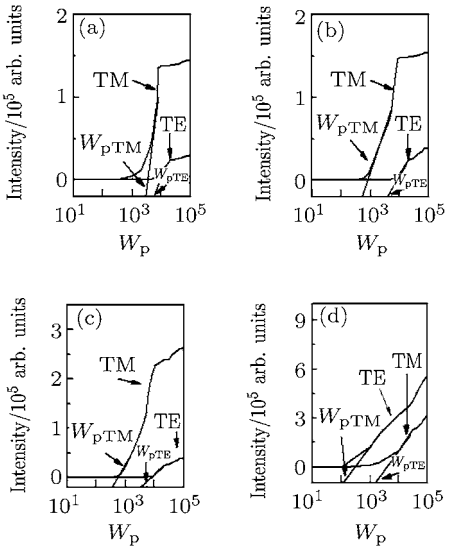


Fig. 9. Power spectra of the mode that has the minimum lasing threshold versus the pump rate for both the TE mode $\lambda_{\rm TE}$ and the TM mode $\lambda_{\rm TM}$: (a) $R = 0.5$ cm, (b) $R = 1$ cm, (c) $R = 2$ cm, (d) $R = 4$ cm.

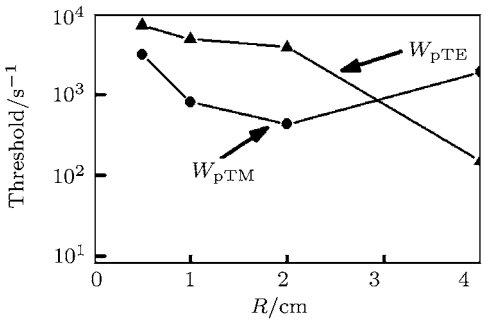


Fig. 10. The threshold value of the two polarized modes versus R.

4. Conclusion

In summary, we have preformed a numerical study on THz random lasing in two-dimensional disordered media made of ruby grains with a three-level atomic system. A method via the adjustment of the pumping area to control the polarization of the THz

wave is proposed. Computed results reveal that transverse electric THz lasing modes could occur as pumping is supplied on the whole medium, while transverse magnetic THz lasing modes could occur if pumping is appropriately supplied on a partial area of the medium. This method is very effective to obtain a polarized THz wave and such a disordered medium has the added benefits of easy fabrication and low cost.

References

[1] Letokhov V S 1968 *Sov. Phys. JETP* **26** 835

[2] Lawandy N M, Balachandra R M, Gomes A S L and Sauvain E 1994 *Nature* **368** 436

[3] Jiang X and Soukoulis C M 2000 *Phys. Rev. Lett.* **85** 70

[4] Sebbah P and Vanneste C 2002 *Phys. Rev.* B **66** 144202

[5] Ito T and Tomita M 2002 *Phys. Rev.* E **66** 027601

[6] Liu J S and Wang H 2004 *Acta Phys. Sin.* **53** 4224 (in Chinese)

[7] Liu J S, Liu H and Wang C 2005 *Acta Phys. Sin.* **54** 3116 (in Chinese)

[8] Wang H 2007 *Acta Phys. Sin.* **56** 2616 (in Chinese)

[9] Liu H, Liu J S, Feng B, Zhang Q, Lü J T and Zhu H 2008 *Opt. Commun.* **281** 2964

[10] Du Z M, Liu J S, Lü J T, Wang K J and Zhang Q Q 2008 *Acta Phys. Sin.* **57** 39289 (in Chinese)

[11] Türeci H E, Ge L, Rotter S and Stone A D 2008 *Science* **320** 643

[12] Liu J S, Liu Y, Lü J T and Wang K J 2010 *Opt. Express* **18** 22880

[13] Lawandy N M 1979 *IEEE J. Quantum Electron.* **6** 401