Gain calculation of a free-electron laser operating with a non-uniform ion-channel guide

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Amplification of an electromagnetic wave by a free electron laser (FEL) with a helical wiggler and an ion channel with a periodically varying ion density is examined. The relativistic equation of motion for a single electron in the combined wiggler and the periodic ion-channel fields is solved and the classes of possible trajectories in this configuration are discussed. The gain equation for the FEL in the low-gain-per-pass limit is obtained by adding the effect of the periodic ion channel. Numerical calculation is employed to analyse the gain induced by the effects of the non-uniform ion density. The variation of gain with ion-channel density is demonstrated. It is shown that there is a gain enhancement for group I orbits in the presence of a non-uniform ion-channel but not in a uniform one. It is also shown that periodic ion-channel guiding is used to reach the maximum peak gain in a low ion-channel frequency (low ion density).

Keywords: free electron laser, nonuniform ion-channel, helical wiggler

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1. Introduction

Free electron lasers (FELs) are devices that amplify intense electromagnetic waves by passing electron beams through a wiggler and converting the kinetic energy of the relativistic electron beam to that of the electromagnetic wave.[1] The ion channel in a FEL is used to confine the electrons against the self-repulsive forces generated by the beam. To create a stable ion channel, an electron beam must be injected into a neutral plasma in a blow-out regime when the electron beam density is higher than the plasma density.[2,3] In this regime, within less than a recombination time, the plasma electrons inside the path of the beam are expelled electrostatically by the beam space charge and form the ion channel. The plasma channel is wide and focusing is approximately linear.[4] Unfortunately, the density of a relativistic electron beam cannot be very high because of technological reasons. This leads to a serious limitation on the gain in the radiated power, which is quadratic in plasma density. A way to overcome the blow-out regime relies on the use of a high power laser to ionize a gas and form a plasma column along the beam line. The ponderomotive force of the laser, together with the beam space charge, expels the plasma electrons and creates the ion channel.[5−7] A relativistic electron running along the ion channel undergoes betatron oscillations about the channel axis due to the restoring force acting on the electron by the uncompensated ion charge. Relativistic electrons executing betatron oscillations in the ion channel emit short-wavelength electromagnetic radiation.[8] The successful utilization of an ion channel in a plasma to wiggle an electron beam to obtain broadband X-ray radiation is reported in Ref. [9]. The propagation of a relativistic electron beam in the ion-channel field has been studied extensively for many years. It is known that if the channel is also present, the combined wiggler and channel fields can have profound effects on growth rate and gain.[10−13] The use of ion channels with a periodic variation in ion density has been proposed as a possible method of transporting and inducing oscillations in a propagating electron beam.[14] Several methods have been suggested to create the periodic ion-channel. Differential pumping of a background gas or the use of gas puffs can create a periodic neutral gas density. Laser ionization will then result in a periodic ion-channel.[14,15] Alternatively, a uniform gas density could be periodically ionized using periodic laser beamlets. The other method is generated by a large number of laser beamlets incident

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perpendicular to the electron beam or with a surface flashover plasma source coupled to the beam pulsed power source.[14−17] The periodic ion-channel is similar to a uniform ion-channel guide, in that no magnetic field is required but it does ensure more effective transverse oscillation. Therefore, a periodic ion channel may be used to guide electron beams and excite its transverse oscillations over distances on the order of several betatron lengths of the beam.[14,17] In the present paper, we attempt to investigate the effects of a non-uniform ion-channel on the electron trajectories and the gain of an FEL with a helically polarized magnetic wiggler field. The ion-channel density is assumed to vary sinusoidally in the axial direction. The paper is organized as follows: a description of the configuration is given in Section 2, along with a discussion of the single-particle trajectories of an electron in the presence of a helical wiggler and non-uniform ion-channel. In Section 3, the interaction of an electron with an electromagnetic radiation field is studied and the small signal gain is then derived in the FEL interaction region. Finally, the results of a numerical study of the electron orbits and gain are discussed.

2. Single particle orbits

A relativistic electron beam propagating through a combined helically polarized magnetic wiggler field and an ion-channel with sinusoidal density variation, $n_{\rm i}(z) = n_{\rm i}(1+\cos kz)/2$, where z is the axial coordinate and $k = 2\pi/\lambda$ is the periodic ion-channel wave number, will be considered. The wiggler field in one-dimensional approximation may be described by

$$\boldsymbol{B}_{\rm w} = B_{\rm w}[\hat{e_x}\cos k_{\rm w}z + \hat{e_y}\sin k_{\rm w}z], \tag{1}$$

where $B_{\rm w}$ and $k_{\rm w}$ are the amplitude and the wave number of the wiggler magnetic field, respectively.

The plasma channel generates an electrostatic field. In cylindrical coordinates, it is given by[14]

$$\begin{aligned}\boldsymbol{E}_{\rm i}(r,z) \;=\; & \pi e n_{\rm i} r[1+2K_1(kr)I_1(kr)\cos kz]\hat{e_{\rm r}} \\ & + \frac{2\pi e n_{\rm i}}{k}[1-krK_1(kr)I_0(kr)] \\ & \times \sin kz\hat{e_z},\end{aligned} \tag{2}$$

where I_n and K_n denote the modified Bessel functions of the first and second kinds, respectively. In the limit in which the electron displacements from the axis of symmetry are small in comparison with a channel period (i.e. $kr \ll 1$), the wiggler field is mostly radial.[1] In this limit, the electrostatic field can be given in the particularly simple form

$$\boldsymbol{E}_{\rm i} = \pi e n_{\rm i}(1+\cos kz)(x\hat{e_x} + y\hat{e_y}). \tag{3}$$

In the model presented above, the dynamic of the electron beam is described by the relativistic cold-fluid momentum and energy equations

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma m\boldsymbol{v}) = -e\left[\boldsymbol{E}_{\rm i} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}_{\rm w}\right], \tag{4}$$

and

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = -\frac{e}{mc^2}\boldsymbol{v}\cdot\boldsymbol{E}, \tag{5}$$

where e, m and $\boldsymbol{v}$ denote the charge, mass and the velocity of the electron, respectively. The Lorentz factor corresponding to velocity $\boldsymbol{v}$ is $\gamma = (1-v^2/c^2)^{1/2}$. We shall find it convenient to work in the wiggler frame and we define

$$\hat{e}_1 = \hat{e}_x\cos k_{\rm w}z + \hat{e}_y\sin k_{\rm w}z, \tag{6}$$

$$\hat{e}_2 = -\hat{e}_x\sin k_{\rm w}z + \hat{e}_y\cos k_{\rm w}z, \tag{7}$$

$$\hat{e}_3 = \hat{e}_z. \tag{8}$$

On expressing the normalized velocity $\boldsymbol{\beta} = \boldsymbol{v}/c$ in terms of the orthogonal vectors $\hat{e}_1, \hat{e}_2$ and $\hat{e}_3$ and defining $\boldsymbol{\chi} = k_{\rm w}\boldsymbol{x}$, $\tau = ck_{\rm w}t$, $\overline{k} = k/k_{\rm w}$, $\Omega_{\rm w} \equiv eB_{\rm w}/m\gamma c^2 k_{\rm w}$, and $\omega_{\rm i} \equiv (2\pi e^2 n_{\rm i}/\gamma mc^2 k_{\rm w}^2)^{1/2}$, one can rewrite Eq. (4) as

$$\frac{\mathrm{d}(\gamma\beta_1)}{\mathrm{d}\tau} = -\frac{\gamma\omega_{\rm i}^2}{2}(1+\cos\overline{k}_{\rm i}\chi_3)\chi_1 + \gamma\beta_2\beta_3, \tag{9}$$

$$\frac{\mathrm{d}(\gamma\beta_2)}{\mathrm{d}\tau} = -\frac{\gamma\omega_{\rm i}^2}{2}(1+\cos\overline{k}_{\rm i}\chi_3)\chi_2 - \gamma\beta_1\beta_3 - \gamma\Omega_{\rm w}\beta_3, \tag{10}$$

$$\frac{\mathrm{d}(\gamma\beta_3)}{\mathrm{d}\tau} = \gamma\beta_2\Omega_{\rm w}, \tag{11}$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\tau} = -\gamma\frac{\omega_{\rm i}^2}{2}(1+\cos\overline{k}_{\rm i}\chi_3)(\beta_1\chi_1 + \beta_2\chi_2). \tag{12}$$

The relations between the normalized transverse position coordinates in the wiggler frame and the normalized velocity components are as follows:

$$\beta_1 = \frac{\mathrm{d}\chi_1}{\mathrm{d}\tau} - \beta_3\chi_2, \tag{13}$$

$$\beta_2 = \frac{\mathrm{d}\chi_2}{\mathrm{d}\tau} + \beta_3\chi_1. \tag{14}$$

The steady-state solutions ($\gamma = \gamma_0 = $ const) of Eqs. (9)–(12) with constant normalized axial velocity $\beta_3 = \beta_\parallel$ (to the first order in the wiggler amplitude) is obtained as

$$\beta_1 = \frac{\beta_\parallel^2\Omega_{\rm w}}{\omega_{\rm i}^2(1+\cos\overline{k}_{\rm i}\chi_3) - \beta_\parallel^2} = \beta_{\rm w}, \tag{15}$$

$$\beta_2 = 0, \tag{16}$$

$$\beta_\parallel = (1 - \gamma_0^{-2} - \beta_{\rm w}^2)^{1/2}. \tag{17}$$

In the limit of an uniform ion-channel (i.e., $k \to 0$), Eqs, (15) and (17) reduce to the well-known result,[10,11] which is characterized by two distinct classes of trajectories (i.e., for $\omega_{\rm i} > \beta_\parallel$ and $\omega_{\rm i} < \beta_\parallel$). The simultaneous solution of Eqs. (15) and (17) yields an expression for the normalized axial velocity, $\beta_\parallel$. To find the quasi steady-state trajectories, we first take the average of Eq. (17) to eliminate rapid oscillations over an FEL interaction length L. The result is

$$2(kL)^{-1}\beta_\parallel^2 \tan(kL/2)[-\Lambda \pm \Omega_{\rm w}\gamma\sqrt{\Lambda}] + \omega_{\rm i}^2\Lambda = 0, \tag{18}$$

$$\Lambda = -1 - \gamma^2(\beta_\parallel^2 - 1). \tag{19}$$

Although it is easy to evaluate $\omega_{\rm i}^2$ from Eqs. (18) and (19), once $\beta_\parallel$ is given, the inverse problem, finding $\beta_\parallel$ when $\omega_{\rm i}^2$ is known, can only be solved numerically.

3. Gain formula

We consider a relativistic electron propagating in an FEL interaction region under the influence of radiation fields, which may be written in the wiggler frame as

$$\begin{pmatrix} \boldsymbol{E}_{\rm r} \\ \boldsymbol{B}_{\rm r} \end{pmatrix} = E_{\rm r} \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \end{pmatrix}, \tag{20}$$

where $\psi = ((\overline{k}_{\rm r} + 1)\chi_3 - \overline{\omega}_{\rm r})\tau + \phi$ is the phase, $\overline{k}_{\rm r} = k_{\rm r}/k_{\rm w}$ and $\overline{\omega}_{\rm r} = \omega_{\rm r}/ck_{\rm w}$ are the normalized wave number and angular frequency, respectively. The effects of radiation modify the transverse equations of motion of the electron as

$$\frac{{\rm d}(\gamma\beta_1)}{{\rm d}\tau} = -\frac{\gamma\omega_{\rm i}^2}{2}(1 + \cos\overline{k}_{\rm i}\chi_3)\chi_1 + \gamma\beta_2\beta_3 - \gamma\widetilde{E_{\rm r}}(1 - \beta_3)\cos\psi, \tag{21}$$

$$\frac{{\rm d}(\gamma\beta_2)}{{\rm d}\tau} = -\frac{\gamma\omega_{\rm i}^2}{2}(1 + \cos\overline{k}_{\rm i}\chi_3)\chi_2 - \gamma\beta_1\beta_3 - \gamma\Omega_{\rm w}\beta_3 + \gamma\widetilde{E_{\rm r}}(1 - \beta_3)\sin\psi, \tag{22}$$

where

$$\widetilde{E_{\rm r}} \equiv \frac{eE_{\rm r}}{\gamma mc^2 k_{\rm w}}. \tag{23}$$

Considering β_z to be close to 1 (or $\gamma \gg 1$), the last term in Eqs. (21) and (22) generated by the electromagnetic radiation fields can be safely neglected in comparison with others. The last terms in Eqs. (21) and (22) can be safely neglected in comparison with the other terms. As a result, the solutions of the above equations yield the same results as the steady-state equations, Eqs. (9) and (10). An expression for the χ_3 in the presence of the radiation field can be obtained as follows.

The energy exchange of the electron with the radiation field is given by

$$\dot{\gamma} = \frac{{\rm d}\gamma}{{\rm d}\tau} = -\frac{e}{mc^2k_{\rm w}}\beta\cdot\boldsymbol{E}_{\rm r} = -\beta_{\rm w}\gamma\widetilde{E_{\rm r}}\cos\psi. \tag{24}$$

Differentiating Eq. (18) with respect to the normalized time, we have $\dot{\beta}_3 = -\dot{\gamma}\eta$, where

$$\eta = \frac{\pm\beta_3\Omega_{\rm w}}{2\sqrt{\Lambda} - 2(1+\Lambda)\sqrt{\Lambda} \pm \gamma\Omega_{\rm w}(-1 + \Lambda + \gamma^2)}. \tag{25}$$

By employing the method used in Ref. [18], that is

$$\chi_3(\tau) = \beta_\parallel\tau + \int_0^\tau\int_0^{\tau''} \dot{\beta}_3(\tau')\,{\rm d}\tau'\,{\rm d}\tau'', \tag{26}$$

and substituting Eqs. (24) and (25) into Eq. (26), we obtain

$$\chi_3(\tau) = \beta_\parallel\tau + \frac{\eta\beta_{\rm w}\gamma\widetilde{E_{\rm r}}}{\alpha^2} \times [-\cos(\alpha\tau + \phi) + \cos\phi - \alpha\tau\sin\phi], \tag{27}$$

where

$$\alpha = (\overline{k}_{\rm r} + 1)\beta_\parallel - \overline{\omega}_{\rm r}.$$

Using the corrected formula for χ_3, Eq. (27) and discarding the small terms in Eq. (24), we have

$$\langle\dot{\gamma}\rangle_\phi = \frac{1}{2\pi}\int_0^{2\pi}\dot{\gamma}\,{\rm d}\phi = -\frac{\beta_{\rm w}^2\gamma^2}{2\alpha^2}\widetilde{E_{\rm r}}^2\eta(1 + \overline{k}_{\rm r}) \times (-\sin\alpha\tau + \alpha\tau\cos\alpha\tau). \tag{28}$$

Integrating Eq. (28) over the electron transit time through the wiggler, we obtain the average change in γ per electron as

$$\langle\Delta\gamma\rangle_\phi = \int_0^{Lk_{\rm w}/\beta_\parallel}\langle\dot{\gamma}\rangle_\phi\,{\rm d}\tau = -\frac{\beta_{\rm w}^2\gamma^2}{2\alpha^3}\widetilde{E_{\rm r}}^2(1 + \overline{k}_{\rm r})\eta \times (2 - 2\cos\alpha\tau - \alpha\tau\sin\alpha\tau). \tag{29}$$

The FEL gain is defined as the fractional change in radiation energy. It can be evaluated as

$$G = \frac{8\pi n_{\rm b}e^2mc^2}{E_{\rm r}^2}\beta_\parallel\langle\Delta\gamma\rangle_\phi$$

$$= \frac{2\gamma\eta\omega_{\rm b}^2}{\beta_\parallel^2}(1-\gamma_0^{-2}-\beta_\parallel^2)k_{\rm w}^3L^3(1+\overline{k}_{\rm r})g(\alpha\tau), \quad (30)$$

where $n_{\rm b}$ is the density of the electron beam and the function $g(\alpha\tau)$ describes the frequency dependence of the gain as

$$g(\alpha\tau) = \frac{1}{(\alpha\tau)^3}(2-2\cos\alpha\tau-\alpha\tau\sin\alpha\tau). \quad (31)$$

The maximum gain occurs at $\alpha\tau = 2.6$, corresponding to the normalized frequency

$$\overline{\omega}_{\rm r} = \frac{\overline{\beta}_\parallel}{(1-\overline{\beta}_\parallel)}\left[1 \mp \frac{2.6}{k_{\rm w}L}\right]. \quad (32)$$

Substituting Eqs. (31) and (32) for $\alpha\tau = 2.6$ into Eq. (30) gives the maximum gain equation as

$$G_{\rm m} = \frac{\pm 0.27\omega_{\rm b}^2}{\beta_\parallel^2}(1-\gamma_0^{-2}-\beta_\parallel^2)\gamma k_{\rm w}^3L^3\eta \times\left[1+\frac{\beta_\parallel}{1-\beta_\parallel}\left(1\mp\frac{2.6}{k_{\rm w}L}\right)\right], \quad (33)$$

where $\omega_{\rm b}^2 = 2\pi n_{\rm b}e^2/(\gamma mc^2k_{\rm w}^2)$ is the normalized electron beam frequency. The $\pm$ signs must be chosen to give a positive gain in the gain formula.

4. Numerical results and discussion

In this paper, we have examined the amplification of an electromagnetic wave by an FEL with a helical wiggler and a cosine modulation of the ion density for a cold beam in the low-gain-per pass limit. Numerical studies of the electron trajectories and gain have been carried out in this section. The average of the normalized axial velocity, $\beta_\parallel$, and the maximum gain of the system have been computed by using Eqs. (18) and (33), respectively. The following values of the parameters were chosen. The normalized wiggler magnetic field frequency $\Omega_{\rm w}$ was taken to be 0.008, the Lorentz relativistic factor γ was chosen to be 25 and the FEL interaction length was taken to be 7λ. For calculation of the gain, the normalized axial velocity was taken to be close to 1 ($0.95 < \beta_\parallel < 1$). The numerical results are illustrated in Figs. 1–3. Numerical solutions for $\beta_\parallel$ versus $\omega_{\rm i}^2$ for both uniform ion-channel guiding (dashed lines) and the periodic ion channel (solid lines) are shown in Fig. 1. The solution will be termed the group I and group II orbits, characterized by conditions $\beta_\parallel > \omega_{\rm i}$ and $\beta_\parallel < \omega_{\rm i}$, respectively. Numerical

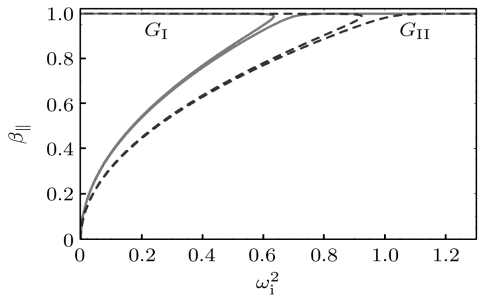


Fig. 1. Graph of the normalized axial velocity versus $\omega_{\rm i}^2$ with a uniform ion-channel (dashed lines)and periodic ion-channel (solid lines).

calculation shows that three real values of axial velocity exist in the region $\omega_{\rm i} < 0.796$. In contrast, there is just one value for $\beta_\parallel$, corresponding to any value of $\omega_{\rm i}$. To demonstrate the effect of the periodic ion-channel on the gain, it is useful to introduce $G_{\rm u}$ and G_0, which describe the peak gain of the system in the presence of a uniform ion-channel and in the absence of it, respectively. If the parameter k or $\omega_{\rm i}$ is set equal to zero in Eq. (18), $\beta_\parallel$ will be obtained for a helical wiggler with uniform ion-channel (without an ion-channel) guiding. Inserting the obtained value of $\beta_\parallel$ into Eq. (33), $G_{\rm u}$ and G_0 are calculated. The ratio of peak gain with nonuniform ion-channel, $G_{\rm m}$, to peak gain without an ion-channel, $\Gamma = G_{\rm m}/G_0$, as a function of $\omega_{\rm i}^2$ is depicted in Fig. 2. It is apparent from this

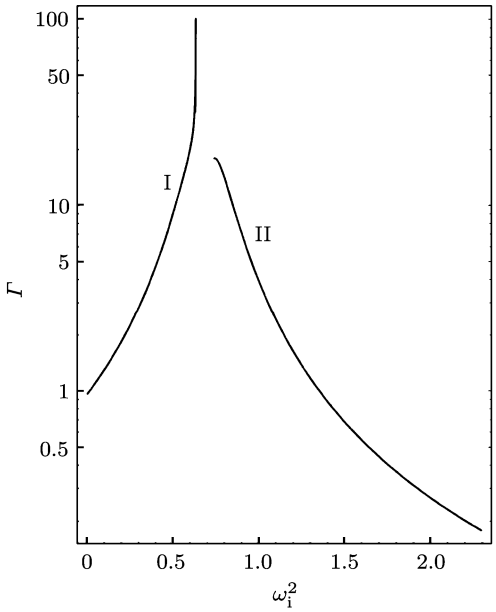


Fig. 2. Graphs of the normalized gain Γ as a function of $\omega_{\rm i}^2$ with $\Omega_{\rm w} = 0.008$, $L = 7\lambda$ and $\gamma = 25$.

curve that for group I orbits, the normalized gain increases monotonically from 1 at $\omega_{\rm i} = 0$ and then goes up to its peak (100.44) near the resonant frequency $\omega_{\rm i} = 0.796$. For group II orbits, if $\omega_{\rm i} < 0.862$,

then $\beta_{\|}$ and the condition of $\beta_{\|}$ being close to 1 (our assumption for the derivation of the gain equation) breaks down. For $\omega_{\rm i} > 0.862$, the normalized gain Γ is equal to 17.85 at $\omega_{\rm i} = 0.862$, goes to 1 at $\omega_{\rm i} = 1.162$ and then starts decreasing to zero with increasing $\omega_{\rm i}$. Consequently, gain enhancement is obtained relative to the absence of an ion-channel for $\omega_{\rm i} < 1.162$. The ratio of the peak gain for a periodic ion-channel to that for a uniform ion-channel, $\Delta = G_{\rm n}/G_{\rm u}$, as a function of $\omega_{\rm i}^2$ is given in Fig. 3.

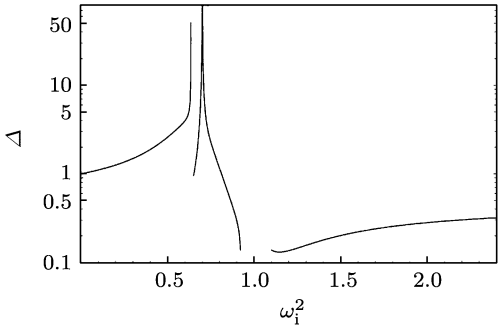


Fig. 3. Graphs of the gain ratio Δ as a function of $\omega_{\rm i}^2$ with $\Omega_{\rm w} = 0.008$, $L = 7\lambda$ and $\gamma = 25$.

It is evident from the figure that throughout the entire range ($\omega_{\rm i} < 0.796$ and $0.803 < \omega_{\rm i} < 0.901$) the value of gain ratio, Δ, is greater than 1, so gain enhancement is obtained due to the periodic ion-channel relative to the uniform ion-channel. By increasing the ion-channel frequency, this gain enhancement increases approximately monotonically up to reach its maximum corresponding to the vicinity of the transverse velocity resonant frequency. This is why periodic ion-channel guiding is used to reach the maximum gain in low ion-channel frequencies (low ion-channel densities).

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