Rotating consensus of multi-agent systems without relative velocity measurement*

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We study the rotating consensus of multi-agent systems without the relative velocity measurement in this paper. A new protocol is proposed. Then we use the theory of the complex system combined with the function continuity to derive a condition, under which all agents finally reach the rotating consensus. Finally, a numerical example is provided to illustrate our theoretical results.

Keywords: rotating consensus, multi-agent systems, complex systems

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1. Introduction

The consensus problem is a critical problem in the study of multi-agent systems and numerous results have been obtained for this problem.[1−19] For example, Hong *et al.* considered the tracking control for multi-agent consensus with an active leader and gave a local controller together with a neighbour-based state-estimation rule.[1] Lin and Jia studied H_∞ consensus problems for first-order multi-agent systems with external disturbances and uncertainty.[3] Recently, more and more attention has been paid to rotating consensus and formation control problems.[11,14−16] Moreover, in Ref. [13], Ren studied second-order multi-agent systems without relative velocity measurements and gave sufficient conditions for the state consensus of the systems in the case of a fixed topology. In Ref. [15], Sepulchre *et al.* proposed a rotating formation problem and used an approach to make all nodes circle around a common point at a unit speed. In Ref. [11], Pavone *et al.* addressed these circular motions and gave a cyclic pursuit policy to enable one node to track another node. Moreover, Ren extended the results of Ref. [11] by introducing a rotation matrix to an existing control law.[14] Founded on the works of Refs. [11], [14] and [15], Lin *et al.* investigated a collective rotating formation control problem and proposed control laws to make all agents surround a common point.[16]

The motivation of this paper is to extend the work of Ref. [16] to propose a simple model and to derive stability conditions for the above-mentioned circular motions without relative velocity measurement. This work differs from that in Ref. [16] in that the relative velocity information need not be accessed. The approach adopted is to use the theory of complex systems combined with function continuity. It should be noted that all the obtained results can be extended to the time-delay case.

The following notations will be used throughout this paper. The $\mathbb{R}^m$ and $\mathbb{C}^m$ denote the sets of all m dimensional real and complex column vectors, respectively; $\boldsymbol{I}_m$ denotes the m dimensional unit matrix; $\otimes$ denotes the Kronecker product; $\mathbf{1}$ represents $[1,\ldots,1]^{\mathrm{T}}$ with compatible dimensions (we use $\mathbf{1}_n$ to denote $\mathbf{1}$ with a dimension n); $\mathbf{0}$ denotes the zero vector or the zero matrix with appropriate dimensions; $\mathscr{I}=\{1,2,\ldots,n\}$; $\|\cdot\|$ denotes the 2-norm.

2. Graph theory

Let $\mathscr{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ be an undirected graph of order n, where $\mathcal{V}=\{s_1,\ldots,s_n\}$ is the set of nodes, $\mathscr{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges and $\mathscr{A}=[a_{ij}]$ is the weighted adjacency matrix. The node indices belong to a finite index set $\mathscr{I}$. An edge of $\mathscr{G}$ is denoted by $e_{ij}=(s_i,s_j)$. The weighted adjacency matrix is defined as $a_{ii}=0$

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and $a_{ij} \geq 0$, where $a_{ij} = a_{ji} > 0$ if and only if $e_{ij} \in \mathscr{E}$. Since the graph considered is undirected, it means that once $e_{ij} \in \mathscr{E}$, then $e_{ji} \in \mathscr{E}$. Thus, $\mathscr{A}$ is a symmetric nonnegative matrix. The set of neighbours of node s_i is denoted by $N_i = \{s_j \in \mathscr{V} : (s_i, s_j) \in \mathscr{E}\}$. The in and the out degrees of node s_i are defined as $d_{\rm in}(s_i) = \sum_{j=1}^{n} a_{ji}$ and $d_{\rm o}(s_i) = \sum_{j=1}^{n} a_{ij}$, respectively. Then, the Laplacian corresponding to the undirected graph $\mathscr{G}$ is defined as $\boldsymbol{L} = [l_{ij}]$, where $l_{ii} = d_{\rm o}(s_i)$ and $l_{ij} = -a_{ij}$, $i \neq j$. Obviously, the Laplacian of any undirected graph is symmetric. A path is a sequence of ordered edges of the form $(s_{i_1}, s_{i_2}), (s_{i_2}, s_{i_3}), \ldots$, where $i_j \in \mathscr{I}$ and $s_{i_j} \in \mathscr{V}$. If there is a path from every node to every other node, the graph is said to be connected.

If undirected graph $\mathscr{G}$ is connected, then the Laplacian $\boldsymbol{L}$ of graph $\mathscr{G}$ has one eigenvalue at zero with the associated eigenvector $\mathbf{1}$. All of its other eigenvalues are positive and real.[20]

3. Model

Consider a multi-agent system consisting of n agents in an undirected graph $\mathscr{G}$. Each edge $(s_i, s_k) \in \mathscr{E}$ corresponds to an available information channel between agents s_i and s_k. Suppose that the i-th agent s_i ($i \in \mathscr{I}$) has the following dynamics,

$$\begin{aligned} \dot{r}_i &= v_i, \\ \dot{v}_i &= u_i, \end{aligned} \tag{1}$$

where $r_i, v_i \in \mathbb{C}$ denote the position and the velocity of agent s_i, and $u_i(t) \in \mathbb{C}$ is the control input (or the protocol).

Multi-agent system (1) reaches rotating consensus if

$$\begin{aligned} &\lim_{t\to+\infty}[v_i(t) - v_k(t)] = 0, \\ &\lim_{t\to+\infty}[(r_i(t) + \omega^{-1}{\rm j}v_i(t)) - (r_k(t) + \omega^{-1}{\rm j}v_k(t))] = 0, \\ &\lim_{t\to+\infty}[\dot{v}_i(t) - {\rm j}\omega v_i(t)] = 0, \end{aligned}$$

for all $i, k \in \mathscr{I}$, where ω denotes the angular velocity. Rotating consensus means that all of the agents finally reach agreement on velocity and position while surrounding a common point. It should be noted that $r_i(t) + \omega^{-1}{\rm j}v_i(t)$ denotes the circle centre which each agent surrounds. For convenience of discussion, we assume $\omega = 1$.

The rotating consensus has important applications. For example, it can be applied to spacecraft docking. In reality, due to the heaviness of a spacecraft, the spacecraft often needs to be split into several parts when it is taken from the Earth into space. All parts of the spacecraft then need to form one body surrounding the Earth when the spacecraft enters space, i.e., all parts of the spacecraft need to reach rotating consensus.

4. Main results

In this section, we study the rotating consensus of multi-agent systems (1). The control input of each agent is given as

$$\begin{aligned} \dot{p}_i &= -\gamma p_i + \sum_{s_k \in N_i} a_{ij}[r_i - r_k], \\ u_i &= {\rm j}v_i - \sum_{s_k \in N_i} a_{ij}[r_i - r_k] + p_i, \end{aligned} \tag{2}$$

for any $i \in \mathscr{I}$, where $p_i \in \mathbb{C}$ is an auxiliary variable used to estimate the effect of the relative velocities and $p_i(0) = 0$.

Denote

$$\boldsymbol{\xi} = [r_1, v_1, p_1, \ldots, r_n, v_n, p_n]^{\rm T},$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & {\rm j} & 1 \\ 0 & 0 & -\gamma \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}.$$

Then using protocol (2), the closed-loop network dynamics can be written as

$$\dot{\boldsymbol{\xi}} = (\boldsymbol{I}_n \otimes \boldsymbol{A} - \boldsymbol{L} \otimes \boldsymbol{B})\boldsymbol{\xi}, \tag{3}$$

where $\boldsymbol{L}$ is the Laplacian of graph $\mathscr{G}$.

Lemma 1 Let

$$\begin{aligned} \alpha(t) &= (1/n)\sum_{i=1}^{n} r_i(0) + \int_0^t \beta(s){\rm d}s, \\ \beta(t) &= (1/n)\sum_{i=1}^{n} v_i(0)\,{\rm e}^{{\rm j}t}, \\ \boldsymbol{\delta}(t) &= \boldsymbol{\xi}(t) - \mathbf{1}_n \otimes [\alpha(t)\ \beta(t)\ 0]^{\rm T}, \end{aligned}$$

then $(\mathbf{1}_n^{\rm T} \otimes \boldsymbol{I}_3)\boldsymbol{\delta}(t) = 0$ and system (3) is equivalent to

$$\dot{\boldsymbol{\delta}}(t) = (\boldsymbol{I}_n \otimes \boldsymbol{A} - \boldsymbol{L} \otimes \boldsymbol{B})\boldsymbol{\delta}(t). \tag{4}$$

Moreover, $(1/n)\sum_{i=1}^{n} r_i(t) + {\rm j}(1/n)\sum_{i=1}^{n} v_i(t)$ is an invariant quantity and equal to $(1/n)\sum_{i=1}^{n} r_i(0) + {\rm j}(1/n)\sum_{i=1}^{n} v_i(0)$.

Proof Since $\mathscr{G}$ is undirected, it follows from Eq. (2) that

$$\sum_{i=1}^{n} \dot{p}_i(t) = -\gamma \sum_{i=1}^{n} p_i(t),$$

which implies $\sum_{i=1}^{n} p_i(t) = \mathrm{e}^{-\gamma t}\sum_{i=1}^{n} p_i(0) = 0$. Similarly, it can easily be obtained that

$$\sum_{i=1}^{n} \dot{v}_i(t) = \mathrm{j}\sum_{i=1}^{n} v_i(t),$$

which implies $\sum_{i=1}^{n} v_i(t) = \mathrm{e}^{\mathrm{j}t}\sum_{i=1}^{n} v_i(0)$. That is $(1/n)\sum_{i=1}^{n} v_i(t) = \beta(t)$. Consequently,

$$(1/n)\sum_{i=1}^{n} r_i(t) = \alpha(t).$$

Evidently, $(\mathbf{1}_n^{\mathrm{T}} \otimes \boldsymbol{I}_3)\boldsymbol{\delta}(t) = 0$. By simple calculations, we have

$$(1/n)\sum_{i=1}^{n} \dot{r}_i(t) + \mathrm{j}(1/n)\sum_{i=1}^{n} \dot{v}_i(t) = 0.$$

Thus,

$$(1/n)\sum_{i=1}^{n} r_i(t) + \mathrm{j}(1/n)\sum_{i=1}^{n} v_i(t)$$

is an invariant quantity and equal to

$$(1/n)\sum_{i=1}^{n} r_i(0) + \mathrm{j}(1/n)\sum_{i=1}^{n} v_i(0).$$

Moreover, by simple calculations, system (3) is equivalent to system (4).

Remark 1 It is shown in lemma 1 that the original system is decomposed into two subsystems, a disagreement dynamic system and a agreement dynamic system. It is also shown that the average values of the positions of all agents change periodically around a fixed point $(1/n)\sum_{i=1}^{n} r_i(0) + \mathrm{j}(1/n)\sum_{i=1}^{n} v_i(0)$ due to the existence of the term $\mathrm{j}v_i$ in protocol (2).

Lemma 2 If $\gamma > 1$, all roots of the equation

$$s^3 + (\gamma - \mathrm{j})s^2 + (\lambda - \gamma\mathrm{j})s - \lambda + \lambda\gamma = 0 \tag{5}$$

have negative real parts, where $\lambda \in \mathbb{R}$ is a positive number.

Proof Consider the equation

$$s^3 + (\gamma - a\mathrm{j})s^2 + (\lambda - a\gamma\mathrm{j})s - \lambda + \lambda\gamma = 0, \tag{6}$$

where $a \in \mathbb{R}$ and $0 \leq a$. Take $s = \mathrm{j}\omega$, where ω is a positive number. Then Eq. (6) is transformed into the following two equations,

$$-\omega^3 + a\omega^2 + \lambda\omega = 0, \tag{7}$$

$$-\gamma\omega^2 + a\gamma\omega - \lambda + \lambda\gamma = 0. \tag{8}$$

By simple calculations, it is easy to see that Eqs. (7) and (8) have no solution with respect to ω. That is, Eq. (6) has no root on the imaginary axis. Moreover, when $a = 0$, Eq. (6) becomes

$$s^3 + \gamma s^2 + \lambda s - \lambda + \lambda\gamma = 0. \tag{9}$$

According to the Routh–Hurwitz criterion, all roots of Eq. (9) have negative real-parts if $\gamma > 1$. Due to the continuity of the functions, the real part of each root of Eq. (6) does not change sign when a changes in $[0, +\infty)$. Therefore, all roots of Eq. (5) have negative real parts.

Remark 2 Lemma 2 gives a condition for the complex cubic equations, which is different from the current study, where only real cubic equations are considered.

Theorem 1 Consider a multi-agent system with fixed topology that is connected. If $\gamma > 1$, multi-agent system (1) with protocol (2) reaches rotating consensus. Moreover,

$$\lim_{t\to+\infty}[r_i(t) - \alpha(t)] = 0,$$

$$\lim_{t\to+\infty}[v_i(t) - \beta(t)] = 0,$$

$$\lim_{t\to+\infty}\left[r_i(t) + \mathrm{j}v_i(t) - \frac{1}{n}\sum_{i=1}^{n} r_i(0) - \mathrm{j}\frac{1}{n}\sum_{i=1}^{n} v_i(0)\right] = 0,$$

for any $i \in \mathscr{I}$, where $\alpha(t)$ and $\beta(t)$ are defined in lemma 1.

Proof From the graph theory, there exists an orthogonal matrix $\boldsymbol{W} \in \mathbb{R}^{n\times n}$ such that

$$\boldsymbol{W}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{W} = \mathrm{diag}\{0, \lambda_2, \ldots, \lambda_n\}$$

with $0 < \lambda_2 \leq \lambda_3 \leq \ldots \leq \lambda_n$, and the first column of $\boldsymbol{W}$ is $\mathbf{1}_n/\sqrt{n}$. It follows that

$$\begin{aligned}
&(\boldsymbol{W}\otimes\boldsymbol{I}_2)^{\mathrm{T}}\dot{\boldsymbol{\delta}}(t)\\
&= (\boldsymbol{W}\otimes\boldsymbol{I}_3)^{\mathrm{T}}(\boldsymbol{I}_n\otimes\boldsymbol{A} - \boldsymbol{L}\otimes\boldsymbol{B})(\boldsymbol{W}\otimes\boldsymbol{I}_3)\\
&\quad\times(\boldsymbol{W}\otimes\boldsymbol{I}_3)^{\mathrm{T}}\boldsymbol{\delta}(t)\\
&= \mathrm{diag}\{\boldsymbol{A}, \boldsymbol{A} - \lambda_2\boldsymbol{B}, \boldsymbol{A} - \lambda_3\boldsymbol{B}, \ldots, \boldsymbol{A} - \lambda_n\boldsymbol{B}\}\\
&\quad\times(\boldsymbol{W}\otimes\boldsymbol{I}_3)^{\mathrm{T}}\boldsymbol{\delta}(t).
\end{aligned} \tag{10}$$

According to lemma 1, $(\mathbf{1}_n^{\mathrm{T}}\otimes\boldsymbol{I}_3)\boldsymbol{\delta}(t) = 0$. Thus, we define $(\boldsymbol{W}\otimes\boldsymbol{I}_3)^{\mathrm{T}}\boldsymbol{\delta}(t) = [0, \bar{\boldsymbol{\delta}}^{\mathrm{T}}(t)]^{\mathrm{T}}$, where $\bar{\boldsymbol{\delta}}(t) \in \mathbb{C}^{3n-3}$. Then system (4) can be transformed into

$$\begin{aligned}
&\dot{\bar{\boldsymbol{\delta}}}(t)\\
&= \mathrm{diag}\{\boldsymbol{A} - \lambda_2\boldsymbol{B}, \boldsymbol{A} - \lambda_3\boldsymbol{B}, \ldots, \boldsymbol{A} - \lambda_n\boldsymbol{B}\}\bar{\boldsymbol{\delta}}(t).
\end{aligned} \tag{11}$$

Also, $\lim_{t\to+\infty}\boldsymbol{\delta}(t) = 0$, if $\lim_{t\to+\infty}\bar{\boldsymbol{\delta}}(t) = 0$. The characteristic polynomial of each $\boldsymbol{A} - \lambda_i\boldsymbol{B}$ is $\det(sI - \boldsymbol{A} + \lambda_i\boldsymbol{B}) = s^3 + (\gamma - \mathrm{j})s^2 + (\lambda_i - \gamma\mathrm{j})s - \lambda_i + \lambda_i\gamma = 0$, $i = 2, 3, \ldots, n$. Then, according to lemma 2, system (11) is Hurwitz stable and $\lim_{t\to+\infty}\bar{\boldsymbol{\delta}}(t) = 0$.

Hence, $\lim_{t\to+\infty}\boldsymbol{\delta}(t)=0$. Then by using lemma 1, we have

$$\lim_{t\to+\infty}[r_i(t)-\alpha(t)]=0,$$
$$\lim_{t\to+\infty}[v_i(t)-\beta(t)]=0,$$
$$\lim_{t\to+\infty}\left[r_i(t)+\mathrm{j}v_i(t)-\frac{1}{n}\sum_{i=1}^{n}r_i(0)-\mathrm{j}\frac{1}{n}\sum_{i=1}^{n}v_i(0)\right]=0,$$

for any $i\in\mathscr{I}$. Calculating $u_i(t)$ in Eq. (2), we have $\lim_{t\to+\infty}[\dot{v}_i(t)-\mathrm{j}v_i(t)]=0$ for any $i\in\mathscr{I}$. Thus, multi-agent system (1) with protocol (2) reaches rotating consensus.

Remark 3 The $\lim_{t\to+\infty}[r_i(t)-\alpha(t)]=0$ means that all agents finally reach the agreement in position; $\lim_{t\to+\infty}[v_i(t)-\beta(t)]=0$ means that all agents finally reach the agreement in velocity; $\lim_{t\to+\infty}[r_i(t)+\mathrm{j}v_i(t)-(1/n)\sum_{i=1}^{n}r_i(0)-\mathrm{j}(1/n)\sum_{i=1}^{n}v_i(0)]=0$ means the circle centres that all agents surround finally tend to be the same.

Remark 4 In this paper, the communication topology is assumed to be undirected. Our further research will be conducted with regard to the directed communication topology.

Remark 5 In this paper, the protocol only uses relative position information, which is different from that of Ref. [16], where the relative velocity information need to be accessed.

5. Numerical simulations

Numerical simulations are given to illustrate the theoretical results obtained in the previous sections. Figure 1 shows one connected undirected graph with four agents. Suppose that the weight of each edge is 1 and $\gamma=2$. Figure 2 shows the state trajectories of all agents using protocol (3). It is clear that the multi-agent system reaches rotating consensus, which is consistent with theorem 1.

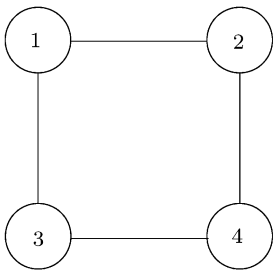


Fig. 1. Communication topology of a multi-agent systems.

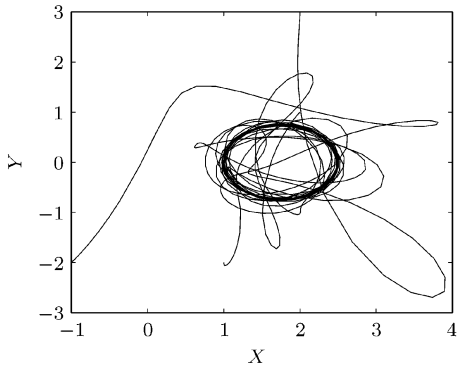


Fig. 2. State trajectories of all agents.

6. Conclusion

In this paper, the rotating consensus of multi-agent systems is studied. A new protocol without using relative velocity measurement is proposed. With the help of the theory of the complex system and function continuity, a condition is derived for the rotating consensus. Finally, a numerical example is included to show our theoretical results.

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