Elimination of spiral waves and spatiotemporal chaos by the synchronization transmission technology of network signals*

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A method to eliminate spiral waves and spatiotemporal chaos by using the synchronization transmission technology of network signals is proposed in this paper. The character of the spiral waves and the spatiotemporal chaos in the Fitzhugh–Nagumo model is presented. The network error evolution equation with spatiotemporal variables and the corresponding eigenvalue equation are determined based on the stability theory, and the global synchronization condition is obtained. Simulations are made in a complex network with Fitzhugh–Nagumo models as the nodes to verify the effectiveness of the synchronization transmission principle of the network signal.

Keywords: synchronization, complex network, spatiotemporal chaos, spiral waves

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1. Introduction

It has been confirmed in recent research that cardiac arrhythmia in humans and other mammals is related to the appearance of spiral waves in cardiac muscle. The breakup of the spirals will lead the system into spatiotemporal chaos, corresponding to ventricular fibrillation, which can cause sudden cardiac death in humans and other mammals. Electrical defibrillation and drugs are usually used to suppress the spiral waves. High voltage electrical signals at high intensity are used in electrical defibrillation to stimulate and inspire the heart system to eliminate spiral waves and ventricular fibrillation, and to bring the heart beat back to normal. However, it is very dangerous to stimulate and inspire the heart with the high voltage signal directly. Since ventricular fibrillation is considered as a major cause of sudden cardiac death, the theoretical research of suppressing the spiral waves and the spatiotemporal chaos in biological physics and life sciences has attracted the attention of many scholars. Some corresponding theoretical models have been built, and methods for suppressing spiral waves and spatiotemporal chaos have been proposed theoretically. A typical model is the Fitzhugh–Nagumo model, and its control methods are mainly continuous and local,[1−10] having great application value and laying an important foundation for the further study of the characteristics and the suppression of spiral waves and spatiotemporal chaos.

A method to eliminate spiral waves and spatiotemporal chaos by using the synchronization transmission technology of a network signal is proposed in this paper. Any type of given external signal is input to any node of the network and when the network achieves synchronization, all nodes in the network are synchronized to the external signal, that is, the given external signal has obtained synchronization transmission, which can effectively suppress the spiral waves and the spatiotemporal chaos in multiple systems.

2. Theoretical model of spiral waves and spatiotemporal chaos

A typical theoretical model of spiral waves and spatiotemporal chaos is the Fitzhugh–Nagumo model. The dynamical equations are[11]

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$$\begin{cases} \dfrac{\partial u(\boldsymbol{r},t)}{\partial t} = \varepsilon^{-1}u(\boldsymbol{r},t)(1-u(\boldsymbol{r},t))\left(u(\boldsymbol{r},t) - \dfrac{v(\boldsymbol{r},t)+b}{a}\right) + D\boldsymbol{\nabla}^2 u(\boldsymbol{r},t), \\ \dfrac{\partial v(\boldsymbol{r},t)}{\partial t} = f(u(\boldsymbol{r},t)) - v(\boldsymbol{r},t) \end{cases} \tag{1}$$

$$f(u(\boldsymbol{r},t)) = \begin{cases} 0 & 0 \le u(\boldsymbol{r},t) < 1/3, \\ 1 - cu(\boldsymbol{r},t)(u(\boldsymbol{r},t)-1)^2, & 1/3 \le u(\boldsymbol{r},t) \le 1, \\ 1, & u(\boldsymbol{r},t) > 1, \end{cases} \tag{2}$$

where $u(\boldsymbol{r},t)$ and $v(\boldsymbol{r},t)$ are system state variables. Two-dimensional coordinate variables are used, i.e., $\boldsymbol{r} = (x,y)$. a, b, c and ε are system parameters, D is a diffusion coefficient and $\boldsymbol{\nabla}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

The system parameters are fixed at $a = 0.84$, $b = 0.07$, $c = 6.75$, and the diffusion coefficient $D = 1$. For different values of parameter ε, the Fitzhugh–Nagumo model can present very complex and rich spatiotemporal dynamical behaviour. The results show that when parameter ε is in the region of $0 < \varepsilon < 0.06$, a stable rotating spiral wave can be observed in the system with proper initial values. When $\varepsilon > 0.06$, the spiral wave becomes unstable and when $\varepsilon > 0.07$, the system comes into spatiotemporal chaos. The spatiotemporal evolution of the state variable $u(\boldsymbol{r},t)$ and its projection maps for $\varepsilon = 0.05$ and $\varepsilon = 0.09$ are given in Figs. 1–4. Where the system size is 100×100, the space step is 1, the time step is 0.1 s and the total time is 80 s.

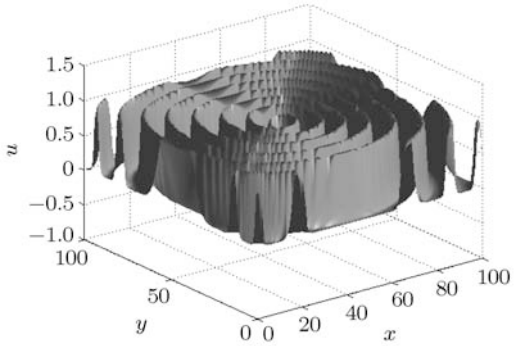


Fig. 1. Spatiotemporal evolution of the state variable at $\varepsilon = 0.05$.

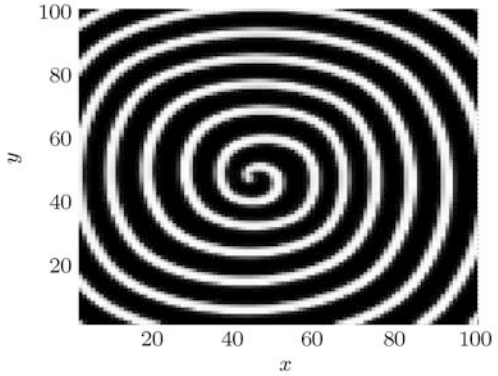


Fig. 2. Projection map of the spatiotemporal evolution of the state variable at $\varepsilon = 0.05$.

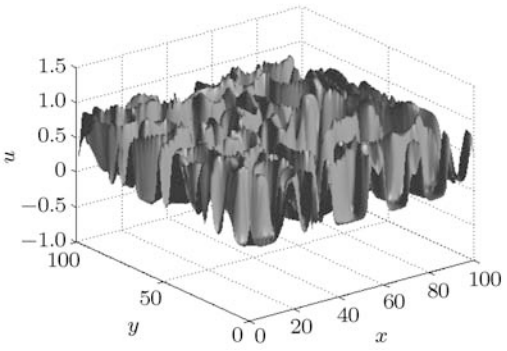


Fig. 3. Spatiotemporal evolution of the state variable at $\varepsilon = 0.09$.

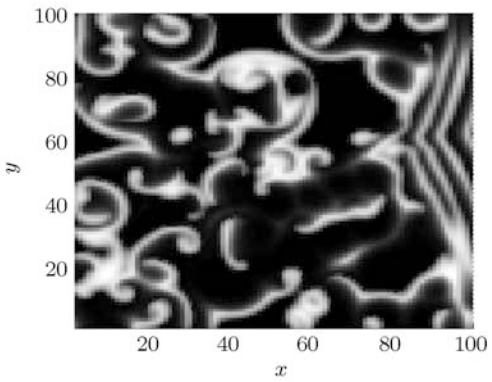


Fig. 4. Projection map of the spatiotemporal evolution of the state variable at $\varepsilon = 0.09$.

3. Synchronization transmission technology of a network signal

The synchronization transmission of a network signal in a complex network with N Fitzhugh–Nagumo models as nodes is studied. First we rewrite the Fitzhugh–Nagumo model (1) in a matrix form,

$$\frac{\partial}{\partial t}\begin{pmatrix} u(\boldsymbol{r},t) \\ v(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} -b/a\varepsilon & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} u(\boldsymbol{r},t) \\ v(\boldsymbol{r},t) \end{pmatrix} + \begin{pmatrix} U(u,v) \\ V(u,v) \end{pmatrix}. \tag{3}$$

Then $\begin{pmatrix} U(u,v) \\ V(u,v) \end{pmatrix}$ in the above equation is expressed as

$$\begin{pmatrix} U(u,v) \\ V(u,v) \end{pmatrix} = \begin{pmatrix} \dfrac{b}{a\varepsilon}u^2(\boldsymbol{r},t) + \varepsilon^{-1}u(\boldsymbol{r},t)[1-u(\boldsymbol{r},t)]\left[u(\boldsymbol{r},t) - \dfrac{v(\boldsymbol{r},t)}{a}\right] + D\boldsymbol{\nabla}^2 u(\boldsymbol{r},t) \\ f(u(\boldsymbol{r},t)) \end{pmatrix}, \tag{4}$$

where

$$f(u(\boldsymbol{r},t)) = \begin{cases} 0, & 0 \le u(\boldsymbol{r},t) < 1/3, \\ 1 - cu(\boldsymbol{r},t)(u(\boldsymbol{r},t)-1)^2, & 1/3 \le u(\boldsymbol{r},t) \le 1, \\ 1, & u(\boldsymbol{r},t) > 1. \end{cases} \tag{5}$$

The N Fitzhugh–Nagumo models are taken as the nodes of the complex network with an arbitrary structure, in which the state equation satisfied by node i is

$$\frac{\partial}{\partial t}\begin{pmatrix} u_i(\boldsymbol{r},t) \\ v_i(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} -b/a\varepsilon & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} u_i(\boldsymbol{r},t) \\ v_i(\boldsymbol{r},t) \end{pmatrix} + \begin{pmatrix} U_i(u_i,v_i) \\ V_i(u_i,v_i) \end{pmatrix} + \begin{pmatrix} Q_{i1} \\ Q_{i2} \end{pmatrix}, \quad (i = 1,2,\ldots,N), \tag{6}$$

where $\begin{pmatrix} Q_{i1} \\ Q_{i2} \end{pmatrix}$ is the coupling function to connect each node, that is, the edge of the network. If any given external signal S_{sign} is input to the first node of the network, the form of $\begin{pmatrix} Q_{11} \\ Q_{12} \end{pmatrix}$ can be designed as

$$\begin{pmatrix} Q_{11} \\ Q_{12} \end{pmatrix} = \begin{pmatrix} S_{\text{sign1}} \\ S_{\text{sign2}} \end{pmatrix} - \begin{pmatrix} U_1(u_1,v_1) \\ V_1(u_1,v_1) \end{pmatrix}. \tag{7}$$

The errors between the state variables at nodes i and $i+1$ in the Fitzhugh–Nagumo models are defined as

$$\begin{pmatrix} e_{i1}(\boldsymbol{r},t) \\ e_{i2}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} u_i(\boldsymbol{r},t) - u_{i+1}(\boldsymbol{r},t) \\ v_i(\boldsymbol{r},t) - v_{i+1}(\boldsymbol{r},t) \end{pmatrix}, \quad (i = 1,2,\ldots,N-1). \tag{8}$$

The spatiotemporal evolution equation of the error variables can be obtained from the following formula

$$\frac{\partial}{\partial t}\begin{pmatrix} e_{i1}(\boldsymbol{r},t) \\ e_{i2}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} -b/a\varepsilon & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} e_{i1}(\boldsymbol{r},t) \\ e_{i2}(\boldsymbol{r},t) \end{pmatrix} + \begin{pmatrix} \Delta U_i \\ \Delta V_i \end{pmatrix} + \begin{pmatrix} \Delta Q_{i1} \\ \Delta Q_{i2} \end{pmatrix}, \tag{9}$$

where

$$\begin{pmatrix} \Delta U_i \\ \Delta V_i \end{pmatrix} = \begin{pmatrix} U_i(u_i,v_i) - U_{i+1}(u_{i+1},v_{i+1}) \\ V_i(u_i,v_i) - V_{i+1}(u_{i+1},v_{i+1}) \end{pmatrix}, \tag{10}$$

$$\begin{pmatrix} \Delta Q_{i1} \\ \Delta Q_{i2} \end{pmatrix} = \begin{pmatrix} Q_{i1} - Q_{i+11} \\ Q_{i2} - Q_{i+12} \end{pmatrix}. \tag{11}$$

If the following relationship exists in the error evolution Eq. (9) with the spatiotemporal variables,

$$\begin{pmatrix} \Delta Q_{i1} \\ \Delta Q_{i2} \end{pmatrix} = -\begin{pmatrix} \Delta U_i \\ \Delta V_i \end{pmatrix}, \tag{12}$$

then

$$\begin{pmatrix} Q_{k1} \\ Q_{k2} \end{pmatrix} = \begin{pmatrix} Q_{k-11} \\ Q_{k-12} \end{pmatrix} + \begin{pmatrix} \Delta U_{k-1} \\ \Delta V_{k-1} \end{pmatrix} = \begin{pmatrix} Q_{11} \\ Q_{12} \end{pmatrix} + \sum_{i=1}^{k-1}\begin{pmatrix} \Delta U_i \\ \Delta V_i \end{pmatrix}, \quad (k = 2,3,\ldots,N), \tag{13}$$

where the coupling function $\begin{pmatrix} Q_{11} \\ Q_{12} \end{pmatrix}$ satisfies Eq. (7).

Taking Eqs. (7) and (13) into account, the error evolution Eq. (9) with the spatiotemporal variables can be simplified as

$$\frac{\partial}{\partial t}\begin{pmatrix} e_{i1}(\boldsymbol{r},t) \\ e_{i2}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} -b/a\varepsilon & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} e_{i1}(\boldsymbol{r},t) \\ e_{i2}(\boldsymbol{r},t) \end{pmatrix}. \tag{14}$$

The coefficient matrix of the error evolution Eq. (14) with the spatiotemporal variables is a Jacobi matrix,

$$\boldsymbol{J} = \begin{pmatrix} -b/a\varepsilon & 0 \\ 0 & -1 \end{pmatrix}. \tag{15}$$

The eigenvalue equation corresponding to the Jacobi matrix is

$$\begin{vmatrix} (-b/a\varepsilon) - \lambda_{i1} & 0 \\ 0 & -1 - \lambda_{i2} \end{vmatrix} = 0. \tag{16}$$

It is obvious that $\lambda_{i1} = -b/a\varepsilon$ and $\lambda_{i2} = -1$. According to the stability theory,[12] because the eigenvalues of the eigenvalue equation corresponding to the Jacobi matrix are all negative, the zero solutions of the error variables of the network for all space lattice coordinates are asymptotically stable, that is, the error variables of the network will be $\begin{pmatrix} e_{i1}(\boldsymbol{r},t) \\ e_{i2}(\boldsymbol{r},t) \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ when $t \to \infty$. The Fitzhugh–Nagumo models at all network nodes are synchronized to the external signal, that is, the given external signal has obtained synchronization transmission and the spiral waves and the spatiotemporal chaos in the multiple systems are suppressed effectively.

4. Simulation of signal synchronization transmission of networks

The number of network nodes is taken as $N = 3$ in order to simulate synchronization transmission. The state equation of each node is described by the Fitzhugh–Nagumo model (1) and the state variables of the Fitzhugh–Nagumo model at the three nodes are $\begin{pmatrix} u_1(\boldsymbol{r},t) \\ v_1(\boldsymbol{r},t) \end{pmatrix}$, $\begin{pmatrix} u_2(\boldsymbol{r},t) \\ v_2(\boldsymbol{r},t) \end{pmatrix}$, and $\begin{pmatrix} u_3(\boldsymbol{r},t) \\ v_3(\boldsymbol{r},t) \end{pmatrix}$. According to Eq. (6) and keeping all the parameter values of the network unchanged, for $\varepsilon = 0.09$, the process of suppressing the spatiotemporal chaos is simulated. Any type of given external signal $S_{\rm sign}$ is input to the first node of the network. The external signal $S_{\rm sign}$ can be a constant signal, linear or nonlinear signals, periodic signals, chaotic signals, etc., and here we take a constant signal as an example. We suppose $S_{\rm sign} = \begin{pmatrix} S_{\rm sign1} \\ S_{\rm sign2} \end{pmatrix} = \begin{pmatrix} 1.2 \\ 1.2 \end{pmatrix}$, let the coupling function $\begin{pmatrix} Q_{11} \\ Q_{12} \end{pmatrix}$ satisfy $\begin{pmatrix} Q_{11} \\ Q_{12} \end{pmatrix} = \begin{pmatrix} S_{\rm sign1} \\ S_{\rm sign2} \end{pmatrix} - \begin{pmatrix} U_1(u_1,v_1) \\ V_1(u_1,v_1) \end{pmatrix}$ and the other coupling functions $\begin{pmatrix} Q_{k1} \\ Q_{k2} \end{pmatrix}$, $(k = 2,3)$, satisfy Eq. (13). The coupling connection of the network is made at the time of 30 seconds. The spatiotemporal evolution of the error variables just at the 30th second is shown in Fig. 5. It can be seen that since the initial values of the state variables at each node of the network are different, the spatiotemporal evolution of error variables $(e_{11}, e_{12})^{\rm T} = (u_1 - u_2, v_1 - v_2)^{\rm T}$, $(e_{21}, e_{22})^{\rm T} = (u_2 - u_3, v_2 - v_3)^{\rm T}$ are quite different. However, when the network is connected, after a period of evolution, the spatiotemporal evolution of the state variables of the Fitzhugh–Nagumo models at the three nodes is exactly the same. This means that global synchronization of the network is achieved and all of the Fitzhugh–Nagumo models at the network nodes are synchronized to the external signal and the

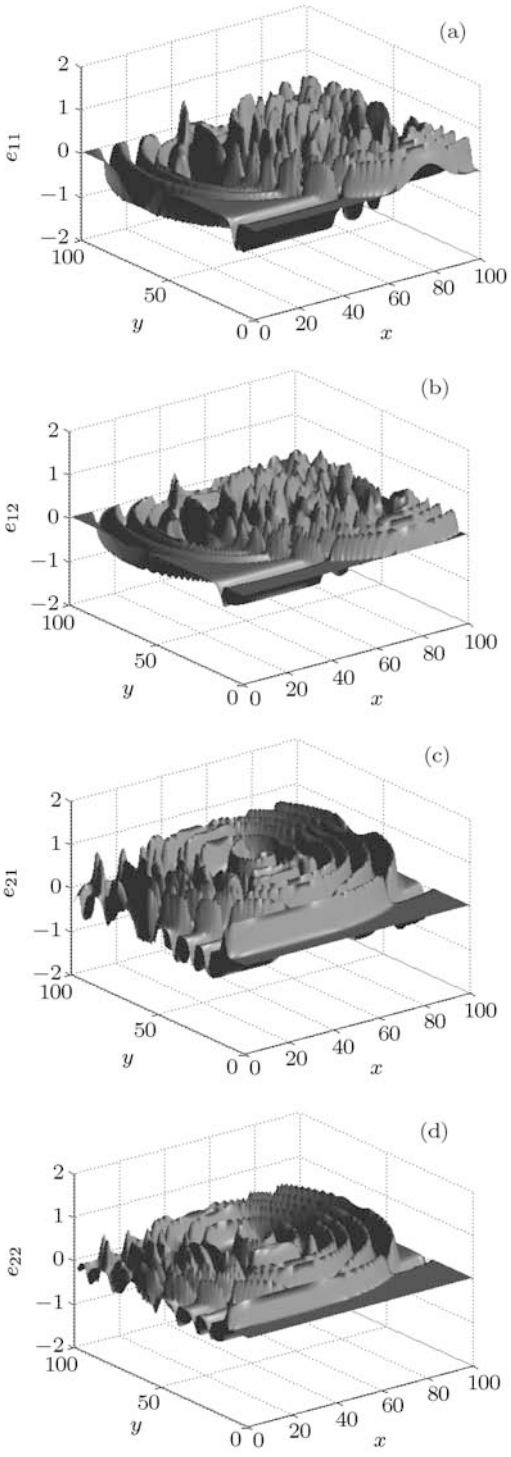


Fig. 5. Spatiotemporal evolution of error variables (t = 30 s): (a) e_{11}, (b) e_{12}, (c) e_{21} and (d) e_{22}.

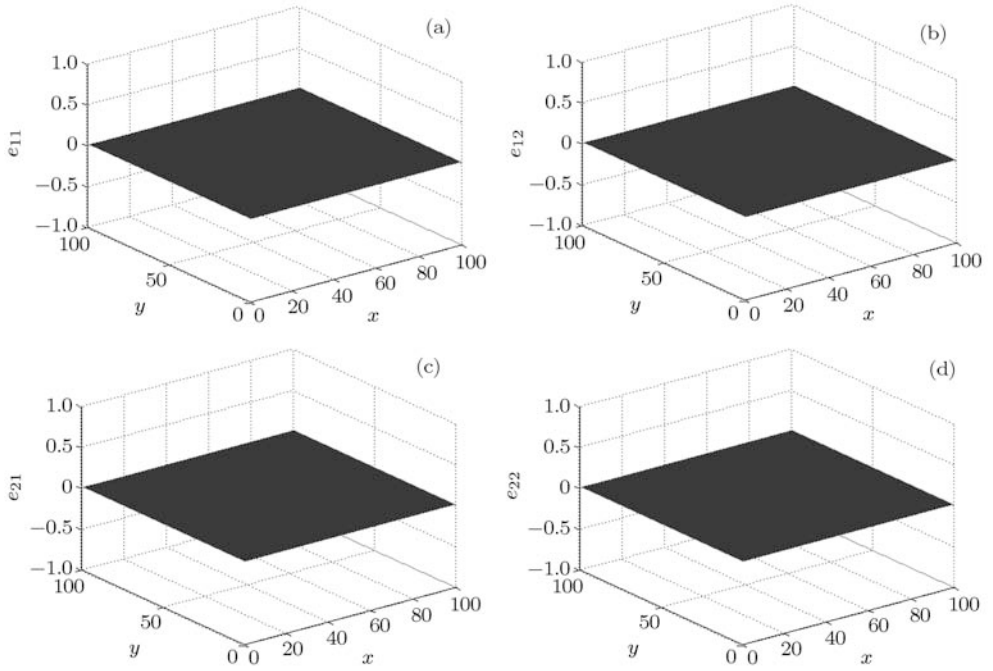


Fig. 6. Spatiotemporal evolution of error variables ($t = 100$ s): (a) e_{11}, (b) e_{12}, (c) e_{21} and (d) e_{22}.

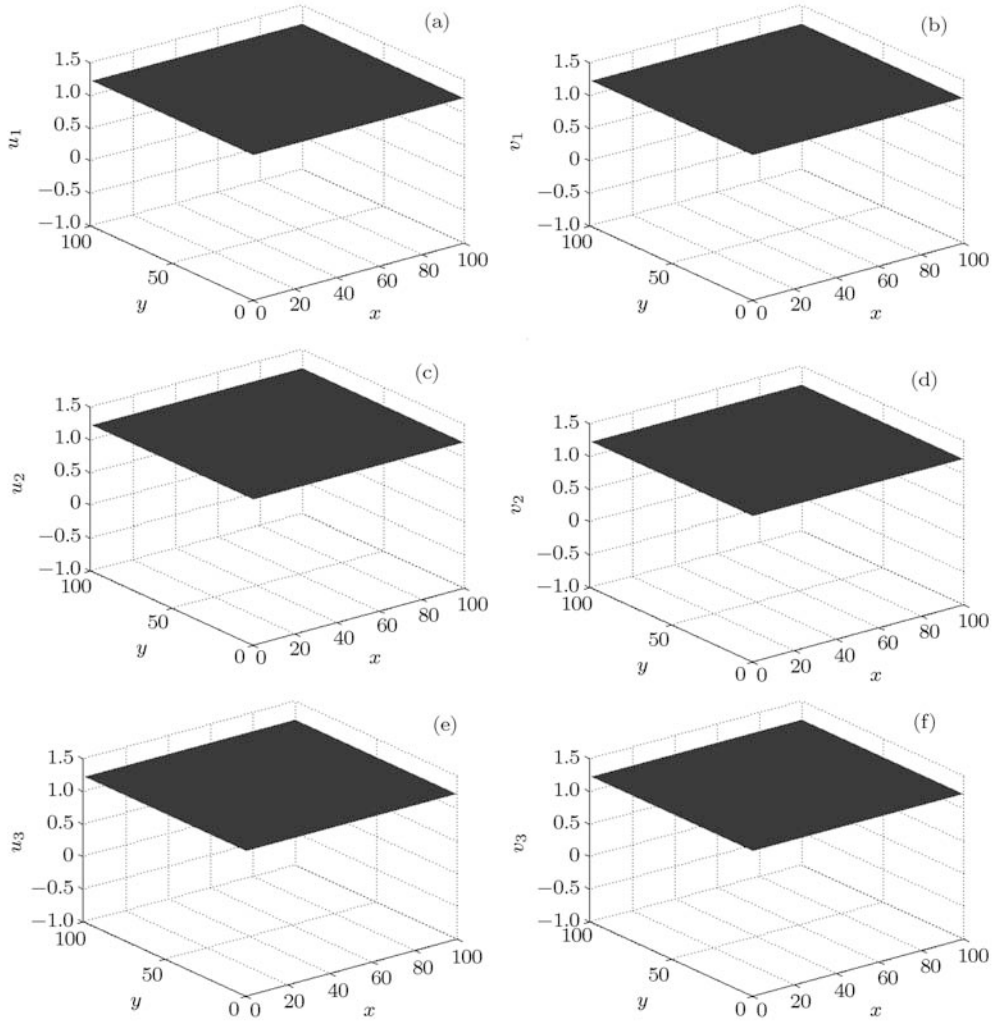


Fig. 7. Spatiotemporal evolution of state variables ($t = 100$ s): (a) u_1, (b) v_1, (c) u_2, (d) v_2, (e) u_3 and (f) v_3.

spatiotemporal chaos at the three nodes are suppressed effectively. The spatiotemporal evolution of the error variables at the 100th second is shown in Fig. 6. It can be seen that the error variables are close to zero after the synchronization. Meanwhile, the evolution of the error variables of the Fitzhugh–Nagumo models at the three nodes has changed from the spatiotemporal chaos shown in Figs. 3 and 4 into a constant signal, which is the same as the given external signal shown in Fig. 7, and the spatiotemporal chaos is suppressed.

When parameter ε is given in the region of $0 < \varepsilon < 0.06$, the simulation process was repeated. It is found that the spiral waves of the Fitzhugh–Nagumo models at all of the nodes of the network are effectively suppressed. The details of the simulation process will not be repeated here.

5. Conclusion

A method to eliminate spiral waves and spatiotemporal chaos by using the synchronization transmission technology of network signals is proposed in this paper. The character of the spiral waves and the spatiotemporal chaos in the Fitzhugh–Nagumo model is presented. The network error evolution equation with spatiotemporal variables and the corresponding eigenvalue equation are determined based on the stability theory, and the global synchronization condition is obtained. Simulations are made in a complex network with Fitzhugh–Nagumo models as the nodes to verify the effectiveness of the synchronization transmission principle of the network signal. The simulation results show that any type of given external signal is input to any node of the network, the external signal can be a constant signal, linear or nonlinear signals, periodic signals, chaotic signals, and so on. When the network is synchronized, all of the network nodes are synchronized to the external signal, that is, the given external signal has obtained synchronization transmission, and the spiral waves and the spatiotemporal chaos in the multiple systems are suppressed effectively.

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