A novel adaptive finite-time controller for synchronizing chaotic gyros with nonlinear inputs

Mohammad Pourmahmood Aghababa†

Young Researchers Club, Ahar Branch, Islamic Azad University, Ahar 54516, *Iran*

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In this paper, the problem of the finite-time synchronization of two uncertain chaotic gyros is discussed. The parameters of both the master and the slave gyros are assumed to be unknown in advance. The effects of model uncertainties and input nonlinearities are also taken into account. An appropriate adaptation law is proposed to tackle the gyros' unknown parameters. Based on the adaptation law and the finite-time control technique, proper control laws are introduced to ensure that the trajectories of the slave gyro converge to the trajectories of the master gyro in a given finite time. Simulation results show the applicability and the efficiency of the proposed finite-time controller.

Keywords: chaotic gyro, finite-time synchronization, model uncertainty, nonlinear input

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1. Introduction

Recently, the synchronization of chaotic systems and complex networks has attracted the attention of many researchers due to its powerful applications in many areas of physics and engineering science, such as secure communication, chemical reactions, optics, lasers, biological systems and mechanical systems.[1] Until now, many effective control methodologies have been proposed to synchronize chaotic systems and complex dynamical networks.[2−17]

The gyro is one of the most interesting and everlasting dynamical systems. It has found various useful applications in optics, navigation, aeronautics and space engineering. Recent research has recognized different kinds of gyros with linear or nonlinear damping features. It has been shown that these systems display a diverse range of dynamic behaviour including both sub-harmonic and chaotic motion.[18−21]

In recent years, due to its useful applications in areas of secure communication,[22] altitude control of long-duration spacecrafts[23] and signal processing, the synchronization of gyros has become an important and essential research issue. In this regard, Lei *et al.*[24] developed an active control technique for the asymptotic synchronization of two identical gyros. Yau[25,26] has developed fuzzy controllers for synchronizing two uncertain chaotic gyros. In Ref. [27], a modified sliding mode controller was proposed to synchronize two chaotic gyros with stochastic base excitations and uncertain parameters. Yan *et al.*[28] introduced an adaptive sliding mode controller to synchronize chaotic gyros with unknown parameters. Hung *et al.*[29] addressed the problem of the generalized projective synchronization of chaotic gyros with dead-zone nonlinear inputs.

All of the aforementioned methods have been proposed to guarantee the asymptotic stability of the resulting synchronization error system. In other words, in those works, the trajectories of the slave gyro can approach those of the master gyro with an infinite settling time. Nevertheless, in practice, it is more valuable to synchronize the chaotic gyros in a given finite time rather than merely asymptotically. The finite-time control method is an effective technique to achieve faster convergence in the control systems. It has been shown that finite-time control techniques have better robustness and disturbance rejection properties.[30] In real-world applications, the gyros' parameters are inevitably disturbed by external inartificial factors, such as environmental temperature, voltage oscillation and mutual interference among components, and cannot be precisely known in advance. Moreover, due to un-modeled dynamics, structural variation in the systems and modeling errors, there are always some unknowns in the gyros'

†Corresponding author. E-mail: m.pour13@gmail.com; m-Pourmahmood@iau-Ahar.ac.ir

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dynamics. Furthermore, when the controller is implemented in real-world physical systems, the physical limitations of the actuators cause nonlinearities in the control inputs. Since chaotic systems are extremely sensitive to any system parameter variations, the nonlinearities in the control inputs may lead the perturbed chaotic system to unpredictable results.[31] Hence, the effect of the input nonlinearities should not be neglected in the design of a controller for chaos synchronization between two chaotic gyros. So, the finite-time synchronization of the chaotic gyros with fully unknown parameters, model uncertainties and nonlinear inputs is important in practical applications. To the author's best knowledge, there is no work in the literature dealing with the finite-time synchronization of two chaotic gyros in the presence of model uncertainties, unknown parameters and nonlinear inputs. So the problem remains open and challenging.

The main contribution of this paper is to propose an adaptive robust finite-time control approach to realize the chaos synchronization between two uncertain chaotic gyros with fully unknown parameters and input nonlinearities as quickly as possible. In this regard, first, an adaptation law is introduced to estimate the unknown parameters. Afterwards, based on the finite-time control idea, a robust adaptive finite-time controller is designed. The finite-time stability and the convergence of the proposed scheme are analytically proved. An estimation of the convergence time is given. Finally, a numerical simulation is given to demonstrate the efficiency and the usefulness of the proposed synchronization strategy.

2. Dynamics of chaotic gyros

The motion of a symmetric gyro with the linear-plus-cubic damping mounted on a vibrating base in terms of rotation angle θ, i.e. the angle between the spin axis of the gyro and the vertical axis, is governed by[18]

$$\ddot{\theta} + \alpha^2 \frac{(1-\cos\theta)^2}{\sin^3\theta} - \beta\sin\theta + c_1\dot{\theta} + c_2\dot{\theta}^3 = f\sin(\omega t)\sin\theta, \tag{1}$$

where the term $f\sin(\omega t)$ is a parametric excitation that models the base excitation, $c_1\dot{\theta}$ and $c_2\dot{\theta}^3$ are the linear and the nonlinear damping terms, respectively, and the term $\alpha^2(1-\cos\theta)^2/\sin^3\theta - \beta\sin\theta$ is the nonlinear resilience.

Defining $x_1 = \theta$ and $x_2 = \dot{\theta}$, we can rewrite system (1) in the following normalized form,

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\alpha^2\frac{(1-\cos x_1)^2}{\sin^3 x_1} + \beta\sin x_1 \\ &\quad - c_1x_2 - c_2x_2^3 + f\sin(\omega t)\sin x_1. \end{aligned} \tag{2}$$

The dynamics of this gyro have been extensively studied in Refs. [18] and [19] for the values of f in the range of $32 < f < 36$. Particularly, for parameter values of $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $\omega = 2$ and $f = 35.5$, this system exhibits chaotic behaviour.[18] The chaotic motion and the strange attractor of system (2) with initial conditions of $x_1(0) = -1$ and $x_2(0) = 2$ are illustrated in Figs. 1 and 2, respectively.

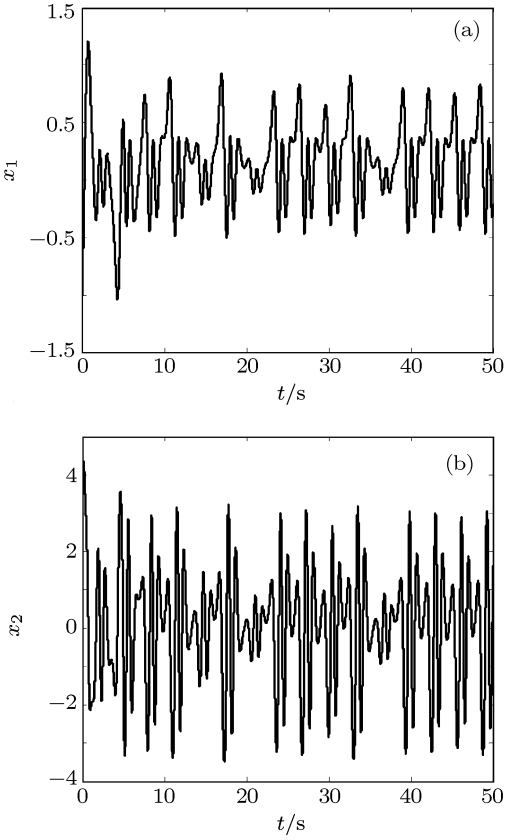


Fig. 1. Trajectories of chaotic gyro: (a) x_1 and (b) x_2.

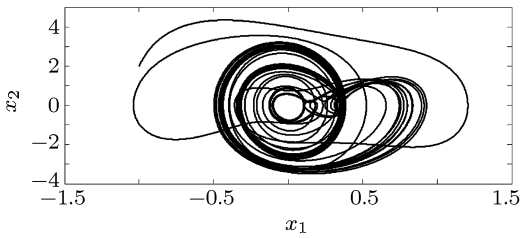


Fig. 2. Strange attractor of a chaotic gyro.

3. Statement of the synchronization problem

Consider the following two coupled chaotic gyros with model uncertainties and nonlinear inputs.

Master gyro:

$$\begin{aligned}\dot{x}_1 &= x_2 + \Delta f_1(\boldsymbol{x}),\\ \dot{x}_2 &= -\alpha^2\frac{(1-\cos x_1)^2}{\sin^3 x_1} + \beta\sin x_1 - c_1x_2 - c_2x_2^3\\ &\quad + f\sin(\omega t)\sin x_1 + \Delta f_2(\boldsymbol{x}); \end{aligned} \tag{3}$$

Slave gyro:

$$\begin{aligned}\dot{y}_1 &= y_2 + \Delta g_1(\boldsymbol{y}) + \varphi_1(u_1),\\ \dot{y}_2 &= -\alpha^2\frac{(1-\cos y_1)^2}{\sin^3 y_1} + \beta\sin y_1 - c_1y_2 - c_2y_2^3\\ &\quad + f\sin(\omega t)\sin y_1 + \Delta g_2(\boldsymbol{y}) + \varphi_2(u_2); \end{aligned} \tag{4}$$

where $\boldsymbol{x} = [x_1, x_2]^{\mathrm{T}}$ is the state vector of the master gyro, $\Delta f_i(\boldsymbol{x})$ with $\Delta f_i(0) = 0$, $i = 1, 2$ represents the unknown model uncertainties of the master gyro, $\boldsymbol{y} = [y_1, y_2]^{\mathrm{T}}$ is the state vector of the slave gyro, $\Delta g_i(\boldsymbol{y})$ with $\Delta g_i(0) = 0$, $i = 1, 2$ represents the unknown model uncertainties of the slave gyro, $\boldsymbol{u}(t) = [u_1(t), u_2(t)]^{\mathrm{T}}$ is the vector of the control inputs, $\varphi_1(u_1)$ and $\varphi_2(u_2)$ with $\varphi_i(0) = 0$, $i = 1, 2$ are continuous nonlinear functions inside the sector $[\rho_i, \nu_i]$, $i = 1, 2$, $\rho_i > 0$, i.e.,

$$\rho_i u_i^2 \le u_i\varphi_i(u_i) \le \nu_i u_i^2, \quad i = 1, 2. \tag{5}$$

Assumption 1 The uncertainties $\Delta f_i(\boldsymbol{x})$ and $\Delta g_i(\boldsymbol{y})$, $i = 1, 2$ are bounded by

$$|\Delta f_i(\boldsymbol{x})| \le a_i^M\|\boldsymbol{x}\|, \quad |\Delta g_i(\boldsymbol{y})| \le a_i^{\mathrm{S}}\|\boldsymbol{y}\|, \quad i = 1, 2, \tag{6}$$

where $\|\cdot\|$ denotes the Euclidean norm in $\mathbf{R}^n$, a_i^{M} and a_i^{S} are given positive constants.

Consequently, using Eq. (6), we can obtain

$$|\Delta f_i(\boldsymbol{x}) - \Delta g_i(\boldsymbol{y})| \le a_i^{\mathrm{M}}\|\boldsymbol{x}\| + a_i^{\mathrm{S}}\|\boldsymbol{y}\|, \quad i = 1, 2. \tag{7}$$

Assumption 2 It is assumed that parameters α^2, c_1, c_2, β, f and ω are unknown in advance. Consider $\boldsymbol{\psi} = [\alpha^2, |c_1|, |c_2|, |\beta + f|]^{\mathrm{T}} = [\psi_1, \psi_2, \psi_3, \psi_4]^{\mathrm{T}}$ as an unknown vector parameter, then we have

$$\|\boldsymbol{\psi}\| \le \Psi, \tag{8}$$

where Ψ is a given positive constant.

If the synchronization error between the master and slave gyros is defined as $\boldsymbol{e}(t) = [e_1, e_2]^{\mathrm{T}} = \boldsymbol{x} - \boldsymbol{y}$, then subtracting Eq. (4) from Eq. (3), we obtain

$$\begin{aligned}\dot{e}_1 &= e_2 + \Delta f_1(\boldsymbol{x}) - \Delta g_1(\boldsymbol{y}) - \varphi_1(u_1),\\ \dot{e}_2 &= \alpha^2 h(x_1, y_1) - c_1e_2 - c_2(x_2^3 - y_2^3)\\ &\quad + [\beta + f\sin(\omega t)](\sin x_1 - \sin y_1)\\ &\quad + \Delta f_2(\boldsymbol{x}) - \Delta g_2(\boldsymbol{y}) - \varphi_2(u_2), \end{aligned} \tag{9}$$

where

$$h(x_1, y_1) = (1-\cos y_1)^2/\sin^3 y_1 - (1-\cos x_1)^2/\sin^3 x_1.$$

It is apparent that the finite-time synchronization problem is transformed to an equivalent problem of the error system's finite-time stabilization. Therefore, the main purpose of this paper is that for any given master chaotic gyro (3) and given slave chaotic gyro (4) with model uncertainties, unknown parameters and input nonlinearities, a suitable control law $\boldsymbol{u}(t)$ is designed such that the stability of the resulting error system (9) is guaranteed in a given finite time.

4. Main results

Before proceeding to the main results, two useful lemmas are presented.

Lemma 1[30] Consider the system

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}), \quad \text{with } f(0),\ \boldsymbol{x} \in \mathbf{R}^n, \tag{10}$$

where $f(\boldsymbol{x}) : D \to \mathbf{R}^n$ is continuous on an open neighborhood $D \subset \mathbf{R}^n$. Suppose that there exist a continuous differential positive-definite function $V(\boldsymbol{x}) : D \to \mathbf{R}^n$ and real numbers $p > 0$, $0 < \alpha < 1$, such that

$$\dot{V}(\boldsymbol{x}) + pV^{\alpha}(\boldsymbol{x}) \le 0, \quad \forall \boldsymbol{x} \in D, \tag{11}$$

then the origin of system (10) is the locally finite-time stable equilibrium and the settling time depending on the initial state $\boldsymbol{x}(0) = \boldsymbol{x}_0$ satisfies

$$T(\boldsymbol{x}_0) \le \frac{V^{1-\alpha}(\boldsymbol{x}_0)}{p(1-\alpha)}. \tag{12}$$

In addition, if $D = \mathbf{R}^n$ and $V(\boldsymbol{x})$ is also radially unbounded (i.e., $V(\boldsymbol{x}) \to \infty$ as $\|\boldsymbol{x}\| \to \infty$), then the origin is the globally finite-time stable equilibrium of system (10).

Lemma 2 If we assume $u_i(t) = \eta_i\mathrm{sgn}(e_i)$, $\eta_i > 0$, then the following inequality holds,

$$-\mathrm{sgn}(e_i)\varphi_i(u_i) \le -\rho_i\eta_i. \tag{13}$$

Proof Replacing $u_i(t)$ in the left-hand side of Eq. (5) by $\eta_i\mathrm{sgn}(e_i)$, we have

$$\rho_i\eta_i^2\mathrm{sgn}^2(e_i) \le \eta_i\mathrm{sgn}(e_i)\varphi_i(u_i). \tag{14}$$

Since $\mathrm{sgn}^2(e_i)=1$ and $\eta_i>0$, we have

$$-\mathrm{sgn}(e_i)\varphi_i(u_i)\le-\rho_i\eta_i. \tag{15}$$

Hence, the lemma is proved.

In order to guarantee the finite-time stability of synchronization error system (9), the following finite-time control laws are proposed:

$$\begin{aligned}u_1(t)&=(1/\rho_1)(|e_2|+\lambda(\Psi+\|\hat{\boldsymbol{\psi}}\|)(|e_1|/\|\boldsymbol{e}\|^2)\\&\quad+a_1^{\mathrm{M}}\|\boldsymbol{x}\|+a_1^{\mathrm{S}}\|\boldsymbol{y}\|+k_1)\mathrm{sgn}(e_1)\\&=\eta_1\mathrm{sgn}(e_1),\\u_2(t)&=(1/\rho_2)(|\hat{\psi}_1|\cdot|h(x_1,y_1)|+|\hat{\psi}_2|\cdot|e_2|\\&\quad+|\hat{\psi}_3|\cdot|x_2^3-y_2^3|+|\hat{\psi}_4|\cdot|\sin x_1-\sin y_1|\\&\quad+\lambda(\Psi+\|\hat{\boldsymbol{\psi}}\|)(e_2/\|\boldsymbol{e}\|^2)+a_2^{\mathrm{M}}\|\boldsymbol{x}\|\\&\quad+a_2^{\mathrm{S}}\|\boldsymbol{y}\|+k_2)\mathrm{sgn}(e_2)\\&=\eta_2\mathrm{sgn}(e_2)\end{aligned} \tag{16}$$

where $\hat{\psi}_i$, $i=1,2,3,4$ are estimations for ψ_i, $i=1,2,3,4$, respectively, $\hat{\boldsymbol{\psi}}=[\hat{\psi}_1,\hat{\psi}_2,\hat{\psi}_3,\hat{\psi}_4]^{\mathrm{T}}$, k_1 and k_2 are positive gains, and $\lambda=\min\{k_1,k_2\}>0$ is a constant. To estimate the unknown parameters, a suitable adaptation law is defined as

$$\begin{aligned}\dot{\hat{\boldsymbol{\psi}}}(t)&=[\dot{\hat{\psi}}_1(t),\dot{\hat{\psi}}_2(t),\dot{\hat{\psi}}_3(t),\dot{\hat{\psi}}_4(t)]^{\mathrm{T}}\\&=[h(x_1,y_1)e_2,-e_2^2,(y_2^3-x_2^3)e_2,\\&\quad|e_2|\cdot|\sin x_1-\sin y_1|]^{\mathrm{T}}.\end{aligned} \tag{17}$$

Theorem 1 For error system (9) controlled with control laws (15) and adaptation law (16), the trajectories of the error system will converge to zero equilibrium and the closed-loop system is finite-time stable with the convergence time T determined by

$$T\le\sqrt{2}/\lambda[1/2(e_1^2(0)+e_2^2(0)+\|\hat{\boldsymbol{\psi}}(0)-\boldsymbol{\psi}\|^2)]^{1/2}. \tag{18}$$

Proof Choose the candidate Lyapunov function of the error system (9) in the form of

$$V(t)=1/2\big(e_1^2(t)+e_2^2(t)+\|\tilde{\boldsymbol{\psi}}(t)\|^2\big), \tag{19}$$

where $\tilde{\boldsymbol{\psi}}=\hat{\boldsymbol{\psi}}-\boldsymbol{\psi}$ is the parameter's errors. It is clear that $\dot{\tilde{\boldsymbol{\psi}}}=\dot{\hat{\boldsymbol{\psi}}}$. Taking the time derivative of $V(t)$, we have

$$\dot{V}(t)=e_1\dot{e}_1+e_2\dot{e}_2+\tilde{\boldsymbol{\psi}}^{\mathrm{T}}\dot{\hat{\boldsymbol{\psi}}}. \tag{20}$$

Introducing $\dot{e}_1$ and $\dot{e}_2$ from Eq. (9) into the right-hand side of Eq. (20), we have

$$\begin{aligned}\dot{V}(t)&=e_1(e_2+\Delta f_1(\boldsymbol{x})-\Delta g_1(\boldsymbol{y})-\varphi_1(u_1))\\&\quad+e_2(\alpha^2h(x_1,y_1)-c_1e_2-c_2(x_2^3-y_2^3)\\&\quad+[\beta+f\sin(\omega t)](\sin x_1-\sin y_1)+\Delta f_2(\boldsymbol{x})\\&\quad-\Delta g_2(\boldsymbol{y})-\varphi_2(u_2))+(\hat{\boldsymbol{\psi}}-\boldsymbol{\psi})^{\mathrm{T}}\dot{\hat{\boldsymbol{\psi}}}.\end{aligned} \tag{21}$$

It is obvious that

$$\begin{aligned}\dot{V}(t)&\le|e_1e_2|+|e_1|(|\Delta f_1(\boldsymbol{x})-\Delta g_1(\boldsymbol{y})|\\&\quad-\mathrm{sgn}(e_1)\varphi_1(u_1))+e_2[\alpha^2h(x_1,y_1)\\&\quad-c_1e_2-c_2(x_2^3-y_2^3)]\\&\quad+|e_2|(|\beta+f|\cdot|\sin x_1-\sin y_1|\\&\quad+|\Delta f_2(\boldsymbol{x})-\Delta g_2(\boldsymbol{y})|\\&\quad-\mathrm{sgn}(e_2)\varphi_2(u_2))+(\hat{\boldsymbol{\psi}}-\boldsymbol{\psi})^{\mathrm{T}}\dot{\hat{\boldsymbol{\psi}}}.\end{aligned} \tag{22}$$

Using assumption 1, Eq. (7), lemma 2 and adaptation law (17), we have

$$\begin{aligned}\dot{V}(t)&\le|e_1e_2|+|e_1|(a_1^{\mathrm{M}}\|\boldsymbol{x}\|+a_1^{\mathrm{S}}\|\boldsymbol{y}\|-\rho_1\eta_1)\\&\quad+e_2[\alpha^2h(x_1,y_1)-c_1e_2-c_2(x_2^3-y_2^3)]\\&\quad+|e_2|(|\beta+f|\cdot|\sin x_1-\sin y_1|+a_2^{\mathrm{M}}\|\boldsymbol{x}\|\\&\quad+a_2^{\mathrm{S}}\|\boldsymbol{y}\|-\rho_2\eta_2)+(\hat{\boldsymbol{\psi}}-\boldsymbol{\psi})^{\mathrm{T}}[h(x_1,y_1)e_2,\\&\quad-e_2^2,(y_2^3-x_2^3)e_2,|e_2|\cdot|\sin x_1-\sin y_1|]^{\mathrm{T}}.\end{aligned} \tag{23}$$

Since $\boldsymbol{\psi}^{\mathrm{T}}\dot{\hat{\boldsymbol{\psi}}}=\alpha^2h(x_1,y_1)e_2-c_1e_2^2+c_2(y_2^3-x_2^3)e_2+(|\beta+f|)|e_2||\sin x_1-\sin y_1|$, we have

$$\begin{aligned}\dot{V}(t)&\le|e_1e_2|+|e_1|(a_1^{\mathrm{M}}\|\boldsymbol{x}\|+a_1^{\mathrm{S}}\|\boldsymbol{y}\|-\rho_1\eta_1)\\&\quad+|e_2|(a_2^{\mathrm{M}}\|\boldsymbol{x}\|+a_2^{\mathrm{S}}\|\boldsymbol{y}\|-\rho_2\eta_2)+\hat{\boldsymbol{\psi}}^{\mathrm{T}}\dot{\hat{\boldsymbol{\psi}}}.\end{aligned} \tag{24}$$

Substituting η_1 and η_2 from Eq. (16) into the right-hand side of the above inequality yields

$$\begin{aligned}\dot{V}(t)&\le|e_1e_2|+|e_1|\{a_1^{\mathrm{M}}\|\boldsymbol{x}\|+a_1^{\mathrm{S}}\|\boldsymbol{y}\|\\&\quad-\rho_1(1/\rho_1)[|e_2|+\lambda(\Psi+\|\hat{\boldsymbol{\psi}}\|)(|e_1|/\|\boldsymbol{e}\|^2)\\&\quad+a_1^{\mathrm{M}}\|\boldsymbol{x}\|+a_1^{\mathrm{S}}\|\boldsymbol{y}\|+k_1]\}\\&\quad+|e_2|\{a_2^{\mathrm{M}}\|\boldsymbol{x}\|+a_2^{\mathrm{S}}\|\boldsymbol{y}\|-\rho_2(1/\rho_2)\\&\quad\times[|\hat{\psi}_1|\cdot|h(x_1,y_1)|+|\hat{\psi}_2|\cdot|e_2|\\&\quad+|\hat{\psi}_3|\cdot|x_2^3-y_2^3|+|\hat{\psi}_4|\cdot|\sin x_1-\sin y_1|\\&\quad+\lambda(\Psi+\|\hat{\boldsymbol{\psi}}\|)(e_2/\|\boldsymbol{e}\|^2)\\&\quad+a_2^{\mathrm{M}}\|\boldsymbol{x}\|+a_2^{\mathrm{S}}\|\boldsymbol{y}\|+k_2]\}+\hat{\boldsymbol{\psi}}^{\mathrm{T}}\dot{\hat{\boldsymbol{\psi}}}.\end{aligned} \tag{25}$$

Using the fact that

$$\begin{aligned}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\dot{\hat{\boldsymbol{\psi}}}&\le|\hat{\boldsymbol{\psi}}|^{\mathrm{T}}[|h(x_1,y_1)e_2|,e_2^2,|(y_2^3-x_2^3)e_2|,\\&\quad|e_2|\cdot|\sin x_1-\sin y_1|]^{\mathrm{T}}\\&=|\hat{\psi}_1|\cdot|h(x_1,y_1)e_2|+|\hat{\psi}_3|\cdot|(y_2^3-x_2^3)e_2|\\&\quad+|\hat{\psi}_4|\cdot|e_2|\cdot|\sin x_1-\sin y_1|+|\hat{\psi}_2|e_2^2,\end{aligned}$$

we obtain

$$\begin{aligned}\dot{V}(t)&\le-k_1|e_1|-k_2|e_2|\\&\quad-\lambda(\Psi+\|\hat{\boldsymbol{\psi}}\|)[(e_1^2+e_2^2)/\|\boldsymbol{e}\|^2].\end{aligned} \tag{26}$$

According to $(e_1^2+e_2^2)=\|\boldsymbol{e}\|^2$, we have

$$\dot{V}(t)\leq -k_1|e_1|-k_2|e_2|-\lambda(\Psi+\|\hat{\boldsymbol{\psi}}\|). \quad (27)$$

Using assumption 2 and since $\|\hat{\boldsymbol{\psi}}\|+\Psi\geq\|\hat{\boldsymbol{\psi}}\|+\|\boldsymbol{\psi}\|\geq\|\hat{\boldsymbol{\psi}}-\boldsymbol{\psi}\|$ is always satisfied, we have

$$\begin{aligned}\dot{V}(t) &\leq -k_1|e_1|-k_2|e_2|-\lambda\|\hat{\boldsymbol{\psi}}-\boldsymbol{\psi}\| \\ &\leq -\lambda(|e_1|+|e_2|+\|\hat{\boldsymbol{\psi}}-\boldsymbol{\psi}\|). \quad (28)\end{aligned}$$

Using a basic mathematic fact, $|e_1|+|e_2|+\|\hat{\boldsymbol{\psi}}-\boldsymbol{\psi}\|\geq(e_1^2+e_2^2+\|\hat{\boldsymbol{\psi}}(t)-\boldsymbol{\psi}\|^2)^{1/2}$, we have

$$\begin{aligned}\dot{V}(t) &\leq -\sqrt{2}\lambda[1/2(e_1^2+e_2^2+\|\hat{\boldsymbol{\psi}}(t)-\boldsymbol{\psi}\|^2)]^{1/2} \\ &= -\sqrt{2}\lambda V^{1/2}(t). \quad (29)\end{aligned}$$

Therefore, based on lemma 1, error trajectories e_1 and e_2 converge to zero equilibrium in finite time

$$T\leq\sqrt{2}/\lambda[1/2(e_1^2(0)+e_2^2(0)+\|\hat{\boldsymbol{\psi}}(0)-\boldsymbol{\psi}\|^2)]^{1/2}.$$

Consequently, the slave and master gyros will be synchronized in finite time. This completes the proof.

Remark 1 Since control inputs (17) contain the terms $|e_i|/\|\boldsymbol{e}\|^2$and $\mathrm{sgn}(e_i)=|e_i|/e_i$, $i=1,2$, undesirable chattering may occur. In order to avoid chattering, the terms $|e_i|/\|\boldsymbol{e}\|^2$ and $\mathrm{sgn}(e_i)$, $i=1,2$ are modified to $|e_i|/(\|\boldsymbol{e}\|^2+\varepsilon)$ and $|e_i|/(e_i+\varepsilon)$, $i=1,2$, respectively, where $\varepsilon>0$ is a sufficiently small constant.[32]

5. Simulation results

To verify the effectiveness and the applicability of the proposed finite-time controller in the synchronization of two uncertain chaotic gyros with fully unknown parameters and nonlinear inputs, a numerical simulation is performed. The following uncertainties are considered in the simulation:

$$\begin{aligned}&\Delta f_1(\boldsymbol{x})=-0.35\cos(2t)x_1,\ \Delta f_2(\boldsymbol{x})=0.25\sin(5t)x_2,\\ &\Delta g_1(\boldsymbol{y})=0.3\cos(5t)y_1,\ \Delta g_2(\boldsymbol{y})=-0.2\sin(3t)y_2. \quad (30)\end{aligned}$$

Parameter bound Ψ is chosen to be 160 and ε is selected as 0.001. The input nonlinearities are considered to be $\varphi_i(u_i)=[5+3\cos(t)]u_i$, $i=1,2$. The initial values of the master and slave gyros are randomly chosen as $x_1(0)=-1$, $x_2(0)=1$ and $y_1(0)=1$, $y_2(0)=-1$, respectively. Both gains and k_1 and k_2 are set to 0.1.

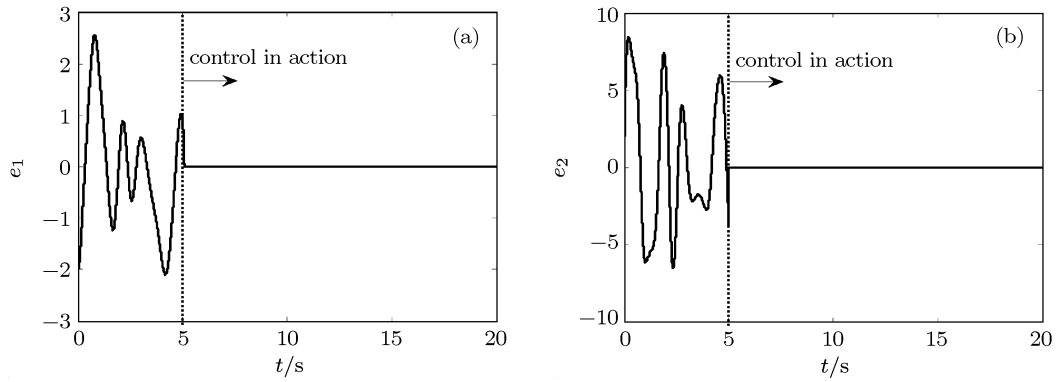


Fig. 3. Trajectories of synchronization error system (9) controlled with the controller (16) and (17): (a) e_1 and (b)e_2.

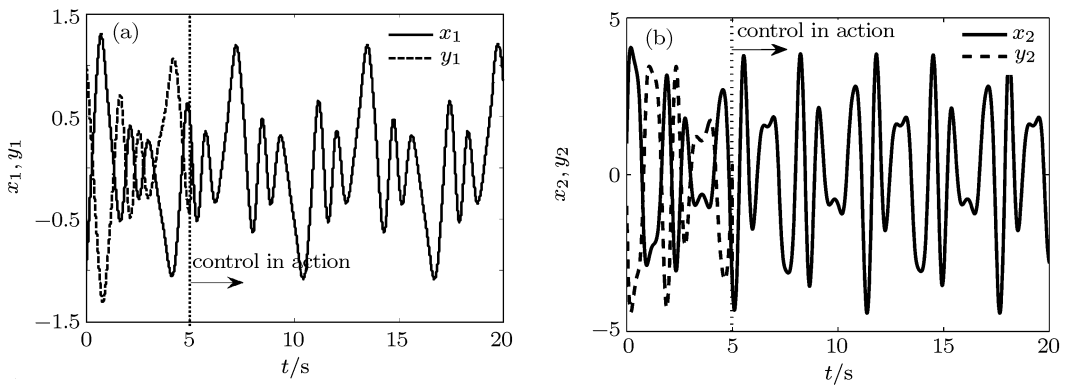


Fig. 4. Trajectories of the master and slave chaotic gyro systems (3) and (4): (a) x_1, y_1 and (b) x_2, y_2.

The trajectories of the synchronization error system (9) controlled with the proposed adaptive finite-time controller (16) and (17) are depicted in Fig. 3, where the control inputs are applied at $t = 5$ s. It can be seen that the synchronization errors quickly converge to zero. As shown in Fig. 4, it is apparent that the trajectories of the slave gyro quickly attain those of the master gyro. Figure 5 displays the time histories of adaptation parameters $\hat{\psi}_i$, $i = 1, 2, 3, 4$. Obviously, all adaptation parameters approach some constants.

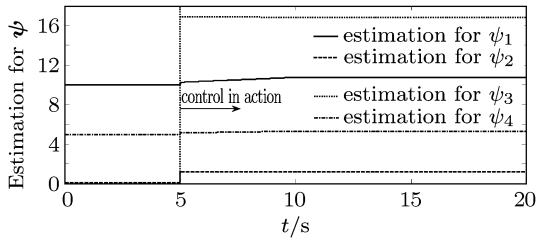


Fig. 5. Time histories of adaptation parameters $\hat{\psi}_i$, $i = 1, 2, 3, 4$.

6. Conclusion

In this paper, an adaptive robust finite-time control technique is introduced to synchronize two chaotic gyros in finite time in spite of unknown uncertainties, fully unknown parameters and input nonlinearities. An adaptation law is proposed to undertake the unknown parameters. Then suitable adaptive robust control laws are derived to guarantee the stability of the synchronization error system in a given finite time. Simulation results indicate that the proposed controller can quickly synchronize two uncertain chaotic gyros with input nonlinearities. It is worth noticing that the proposed approach can be extended to the finite-time synchronization of complex multi-scroll chaotic systems,[33−38] which will be the future work of the author.

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