Modified coupled map car-following model and its delayed feedback control scheme*

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A modified coupled map car-following model is proposed, in which two successive vehicle headways in front of the considering vehicle is incorporated into the optimal velocity function. The steady state under certain conditions is obtained. An error system around the steady state is studied further. Moreover, the condition for the state having no traffic jam is derived. A new control scheme is presented to suppress the traffic jam in the modified coupled map car-following model under the open boundary. A control signal including the velocity differences between the following and the considering vehicles, and between the preceding and the considering vehicles is used. The condition under which the traffic jam can be well suppressed is analysed. The results are compared with that presented by Konishi *et al.* (the KKH model). The simulation results show that the temporal behaviour obtained in our model is better than that in the KKH model. The simulation results are in good agreement with the theoretical analysis.

Keywords: traffic flow, coupled map car-following model, optimal velocity function, delayed feedback control

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1. Introduction

In 1961, Newell[1] presented a car-following model described in a first-order differential equation and defined the optimal velocity (OV) function. In 1995, Bando *et al.*[2] proposed a very charming microscopic traffic model called the optimal velocity car-following model (OVM). It was based on the idea that each vehicle has an optimal velocity, which depends on the following distance of the preceding vehicle. After that, much attention was focused on the optimal velocity model.[3−6] It is worth pointing out that Konishi *et al.*[7] proposed a coupled map car-following model (KKH model), which describes the dynamical behaviour of a group of road vehicles travelling on a single lane without overtaking. The dynamic version of the decentralized delayed feedback control scheme was introduced to suppress the traffic jam and the procedure for designing the controller was given. The coupled map (CM) traffic model is convenient for computer simulation because of its simple time development algorithm. In 2006, Zhao[8] presented another simple strategy to suppress the congested state in the traffic system. A control signal that incorporated the effect of the velocity difference between the preceding and the considering vehicles was presented. In the model analyzed by Han,[9] the control signal was proportional to the velocity difference between the $(i-1)$-th vehicle in front and the i-th vehicle. The simulation results were in good agreement with the theoretical analysis. Therefore, we will consider the influence on the traffic flow brought by the following vehicle. Some research related to the control signal has recently been carried out[9−11].

Until now, the coupled map car-following model has not been developed further. In particular, the OV function in the model is still a simple piecewise linear function. In order to improve the stability of the model, we pay attention to the modification of the OV function. In the meantime, we propose a new control scheme that takes into account the two velocity differences $\Delta v_i(n)$ and $\Delta v_{i-1}(n)$ that are

$$\Delta v_i(n) = v_{i-1}(n) - v_i(n),$$
$$\Delta v_{i-1}(n) = v_{i-2}(n) - v_{i-1}(n).$$

Detail definitions are in the next section.

The present paper is organized as follows. In Section 2, the modified CM car-following model is given

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and its stability analysis is derived. In Section 3, a new control method for the congested traffic is introduced and its theoretical analysis is given. Section 4 shows the simulation results. Meanwhile, a comparison between our control scheme and the control scheme of the KKH model is made. Section 5 gives a conclusion.

2. CM car-following model and its stability analysis

2.1. CM car-following model

Our investigations are based on the KKH model. The leading vehicle is described as

$$x_0(n+1) = v_0 T + x_0(n), \tag{1}$$

where $x_0(n) > 0$ is the position of the leading vehicle at time $t = nT$, $v_0 > 0$ is its speed, $T > 0$ is the sampling time and $x_0(0) > 0$ is the leading vehicle's initial position.

The following vehicle i is described as

$$x_i(n+1) = v_i(n)T + x_i(n), \quad (i = 1, 2, \ldots, N), \tag{2}$$

where $x_i(n) > 0$ is the position of the i-th vehicle at time $t = nT$, $v_i(n) > 0$ is its speed at time $t = nT$ and N is the total number of the vehicles. The velocity of the following vehicle is governed by

$$\begin{aligned} v_i(n+1) &= \alpha_i[V_i^{\mathrm{op}}(y_i(n), y_{i-1}(n)) - v_i(n)]T \\ &\quad + v_i(n), \end{aligned} \tag{3}$$

where $\alpha_i > 0$ is the sensitivity of the i-th vehicle driver and $V_i^{\mathrm{op}}(y_i(n), y_{i-1}(n))$ is the OV function, which depends on the distance between the $(i-1)$-th and the i-th vehicles,

$$\begin{aligned} y_i(n) &= x_{i-1}(n) - x_i(n), \\ y_{i-1}(n) &= x_{i-2}(n) - x_{i-1}(n). \end{aligned} \tag{4}$$

The OV function is given as

$$\begin{aligned} &V_i^{\mathrm{op}}(y_i(n), y_{i-1}(n)) \\ &= \frac{v_i^{\max}}{2}\left[1 + \bar{H}_{\mathrm{sat}}\left(2\frac{py_i(n) + (1-p)y_{i-1}(n) - \eta_i}{\zeta_i}\right)\right], \end{aligned} \tag{5}$$

where p is a parameter, which can be determined later by theoretical analysis. Here the traditional piecewise linear function

$$V_i^{\mathrm{op}}(y_i(n)) = \frac{v_i^{\max}}{2}\left[1 + \bar{H}_{\mathrm{sat}}\left(2\frac{y_i(n) - \eta_i}{\zeta_i}\right)\right] \tag{6}$$

is replaced by the above nonlinear function, which incorporates the two successive vehicle headways $y_i(n)$ and $y_{i-1}(n)$. The saturation function $\bar{H}_{\mathrm{sat}}$ is described as

$$\bar{H}_{\mathrm{sat}}(\rho) = \begin{cases} +1, & \rho > +1, \\ \rho, & -1 \le \rho \le +1, \\ -1, & \rho < -1, \end{cases} \tag{7}$$

$v_i^{\max} > 0$ is the maximum speed, $\eta_i > 0$ is the neutral headway distance and $\zeta_i > 0$ is a parameter. To guarantee the existence of a vehicle group, we assume that speed v_0 of the leading vehicle is less than the maximum speed of all the following vehicles, that is

$$v_0 < v_i^{\max}, \quad (i = 1, 2, \ldots, N). \tag{8}$$

To avoid collision, all vehicles adopt the full-braking action, i.e., if

$$py_i(n) + (1-p)y_{i-1}(n) < y_i^{\min},$$

then

$$x_i(n+1) = x_i(n), \; v_i(n+1) = 0.$$

This action shows that the i-th vehicle stops suddenly when the whole headway distance is less than the safety distance $y_i^{\min}$, which implies that $y_i^{\min} < \eta_i - \zeta_i/2$.

2.2. Stability analysis

The dynamics of the following vehicle (e.g., i-th vehicle) can be given as

$$\begin{aligned} v_i(n+1) &= \alpha_i[V_i^{\mathrm{op}}(y_i(n), y_{i-1}(n)) - v_i(n)]T \\ &\quad + v_i(n), \end{aligned} \tag{9}$$

$$y_i(n+1) = v_{i-1}(n)T - v_i(n)T + y_i(n). \tag{10}$$

The steady state of the above system is

$$\begin{aligned} [v_i^*, y_i^*]^{\mathrm{T}} &= \left[v_0, \left(1 - \frac{1}{p}\right)^{i-1}\left(y_1 - \frac{v_0}{r_i} + \frac{\zeta_i}{2} - \eta_i\right)\right. \\ &\quad \left. + \frac{v_0}{r_i} - \frac{\zeta_i}{2} + \eta_i\right]^{\mathrm{T}}, \end{aligned} \tag{11}$$

where $r_i = v_i^{\max}/\zeta_i$. Obviously, the necessary and sufficient condition for the existence of the steady state is the condition $v_0 < v_i^{\max}$.

Let us consider an error system around steady state (11), that is $\delta v_i(n) = v_i(n) - v_i^*$ and $\delta y_i(n) = y_i(n) - y_i^*$. Let $V_i^{\mathrm{op}}(y_i^*(n),\, y_{i-1}^*(n)) = v_i^*$, we have

$$\begin{pmatrix} \delta v_i(n+1) \\ \delta y_i(n+1) \end{pmatrix} = \begin{pmatrix} 1 - \alpha_i T & p\alpha_i r_i T \\ -T & 1 \end{pmatrix} \begin{pmatrix} \delta v_i(n) \\ \delta y_i(n) \end{pmatrix}$$

$$+\begin{pmatrix} 0 & (1-p)\alpha_i r_i T \\ T & 0 \end{pmatrix} \times \begin{pmatrix} \delta v_{i-1}(n) \\ \delta y_{i-1}(n) \end{pmatrix}, \quad (12)$$

$$\delta v_i(n) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \delta v_i(n) \\ \delta y_i(n) \end{pmatrix}. \quad (13)$$

The Jury scheme allows us derive the necessary and sufficient conditions for the above error system to be stable, which are (i) $\widetilde{P}_i(z)$ is stable; (ii) $\max_{|z|=1} |\widetilde{G}_i(z)| \leq 1$, $(i = 1, 2, \ldots, N)$. The $\widetilde{P}_i(z)$ is the characteristic polynomial of the coefficient matrix in system (12), which is

$$\widetilde{P}_i(z) = z^2 + \widetilde{a}_i z + \widetilde{b}_i, \quad (14)$$

where $\widetilde{a}_i = \alpha_i T - 2$, $\widetilde{b}_i = 1 - \alpha_i T + \alpha_i r_i T^2$. Condition (i) means that all the roots of $\widetilde{P}_i(z) = 0$ are within a unit circle. So we have

$$0 < r_i < \frac{1}{pT}, \quad 0 < \alpha_i < \frac{4}{(2 - p r_i T)T}. \quad (15)$$

The $\widetilde{G}_i(z)$ is the transfer function from $(\delta v_{i-1}(n), \delta y_{i-1}(n))^T$ to $\delta v_i(n)$,

$$\begin{aligned} \widetilde{G}_i(z) &= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} z - 1 + \alpha_i T & -p\alpha_i r_i T \\ T & z - 1 \end{pmatrix}^{-1} \\ &\quad \times \begin{pmatrix} 0 & (1-p)\alpha_i r_i T \\ T & 0 \end{pmatrix} \\ &= \frac{\begin{pmatrix} p\alpha_i r_i T^2 & (z-1)(1-p)\alpha_i r_i T \end{pmatrix}}{\widetilde{P}_i(z)}, \end{aligned}$$

Theorem 1 If all the vehicles satisfy the following conditions,

$$0 < r_i \leq \left\{ \frac{1}{pT}, \frac{pT(2 - \alpha_i T) + \sqrt{p^2 T^2 + 4(1-p)^2} |2 - \alpha_i T|}{2(1-p)^2 \alpha_i T} \right\}, \quad (16)$$

$$0 < \alpha_i < \frac{4}{(2 - p\alpha_i T)T}, \quad (17)$$

there is no traffic jam in systems (12) and (13).

Proof See Appendix A.

The conditions mean that the traffic system is robust. The robustness enhances the stability of the system. In a robust system, the amplitude of the transient disturbance decreases with the development of time even if the structure and the parameters of the system are variational. We confirm that a traffic jam never occurs as long as the vehicles satisfy conditions (16) and (17).

In order to determine parameter p, the modified CM car-following model parameters are set as[7]

$$\eta = 25.0 \text{ m}, \quad \zeta = 23.3 \text{ m}, \quad v^{\max} = 33.6 \text{ m/s},$$
$$\alpha = 2.0 \text{ s}^{-1}, \quad v_0 = 20 \text{ m/s}, T = 0.1 \text{ s},$$
$$y_1 = 8 \text{ m}, \quad y^{\min} = 7.02 \text{ m}, \quad r = v^{\max}/\zeta \approx 1.44 \text{ s}^{-1}.$$

It is assumed that all vehicles have the same parameters. The initial conditions are the steady state of the model. The initial positions and the initial speeds are set as

$$x_i(0) = \sum_{j=i+1}^{N} y_j^*, \quad y_j(0) = y_j^*,$$
$$v_i(0) = v_i^*, \quad i = 1, 2, \ldots, N. \quad (18)$$

Consider a situation in which the leading vehicle stops suddenly, i.e.,

$$v_0(n) = 0, \quad \text{for} \quad nT = 100 - 102.$$

Figure 1 shows the velocity–time patterns of the 1th, the 25th and the 50th vehicles with different parameter p. For $p = 1$, the modified CM car-following model reduces to the KKH model, which corresponds to Fig. 1(a). As p decreases from 1 to 0.65, the stability of the model is strengthened. For $p = 0.6$, there is little difference from the case of $p = 0.65$, however the maximum velocities of the 25th and the 50th vehicles are lowered. So we determine parameter p to be 0.65 in the new OV function. It is reasonable that the nearest preceding vehicle has the strongest influence on the considering one.

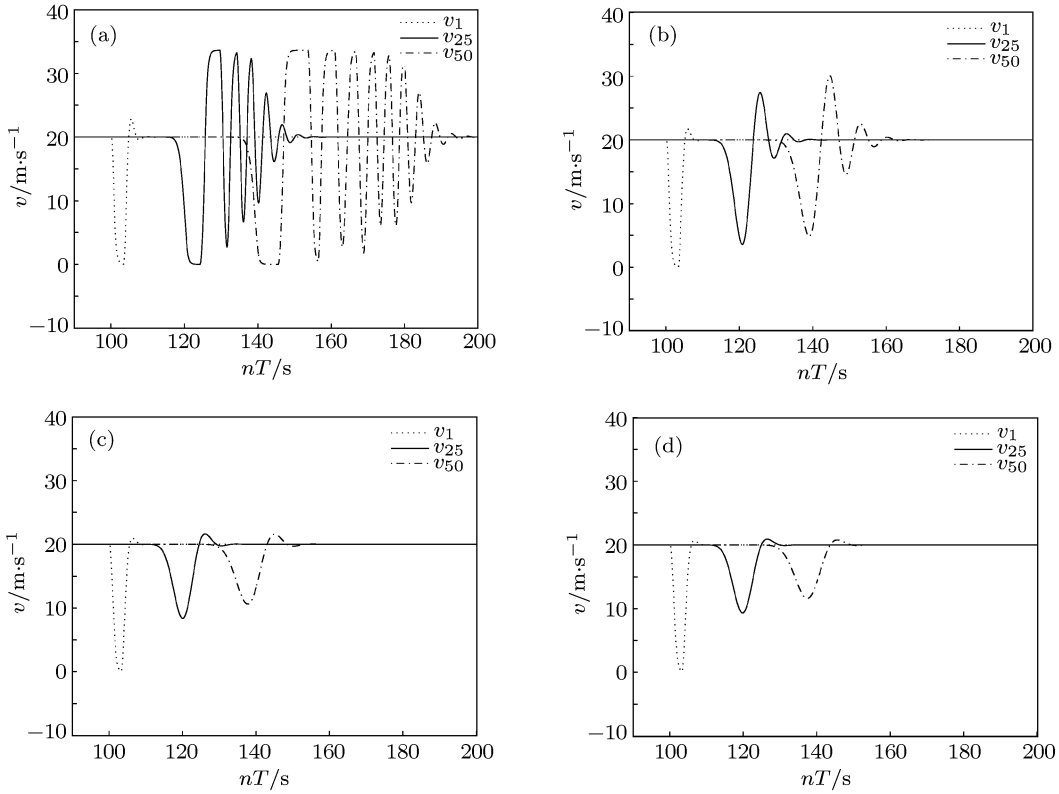


Fig. 1. Numerical simulations for the modified CM car-following model with $(a)p = 1$, (b) $p = 0.8$, (c) $p = 0.65$, and (d) $p = 0.6$.

In the next section, we consider how to suppress the traffic jam by proposing a simple control scheme.

3. The control scheme

We add a feedback control signal $u_i(n)$ to system (3), so we have

$$\begin{aligned} v_i(n+1) &= \alpha_i[V_i^{\mathrm{op}}(y_i(n), y_{i-1}(n)) - v_i(n)]T \\ &\quad + v_i(n) + u_i(n), \qquad (19) \\ u_i(n) &= 0.85\Delta v_i(n) \\ &\quad - kH(-\Delta v_{i+1}(n))\Delta v_{i+1}(n). \qquad (20) \end{aligned}$$

The two velocity differences between $v_{i-1}(n)$ and $v_i(n)$ and between $v_{i+1}(n)$ and $v_i(n)$ at time $t = nT$ are taken into account. The $k > 0$ is the feedback gain, function H is described as

$$H(-\Delta v_{i+1}(n)) = \begin{cases} 0, \ \Delta v_{i+1}(n) \geq 0, \\ 1, \ \Delta v_{i+1}(n) < 0. \end{cases}$$

For $\Delta v_{i+1}(n) \geq 0$, our feedback control signal $u_i(n)$ can be reduced to

$$u_i(n) = 0.85\Delta v_i(n).$$

The traffic system exhibits better temporal behaviour when the feedback gain is fixed at 0.85.[8] For $\Delta v_{i+1}(n) < 0$, our feedback control signal is

$$u_i(n) = 0.85\Delta v_i(n) - k\Delta v_{i+1}(n), \qquad (21)$$

which means that the following vehicle will accelerate the considering vehicle as $v_{i+1}(n) > v_i(n)$. The closed-loop system, consisting of Eqs. (2),(4), (19), (20) and (21), has the same steady state as that of Eqs. (9) and (10). Around the steady state, the controlled system can be described by

$$\begin{pmatrix} \delta v_i(n+1) \\ \delta y_i(n+1) \end{pmatrix} = \begin{pmatrix} 1 - \alpha_i T - (k + 0.85) & p\alpha_i r_i T \\ -T & 1 \end{pmatrix} \begin{pmatrix} \delta v_i(n) \\ \delta y_i(n) \end{pmatrix} + \begin{pmatrix} 0.85 & (1-p)\alpha_i r_i T & k \\ T & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta v_{i-1}(n) \\ \delta y_{i-1}(n) \\ \delta v_{i+1}(n) \end{pmatrix}, \quad (22)$$

$$\delta v_i(n) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \delta v_i(n) \\ \delta y_i(n) \end{pmatrix}. \tag{23}$$

The transfer function from $(\delta v_{i-1}(n), \delta y_{i-1}(n), \delta v_{i+1}(n))^{\mathrm{T}}$ to $\delta v_i(n)$ is described by

$$\begin{aligned} \widehat{\boldsymbol{G}}_i(z) &= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} z-1+\alpha_i T+(k+0.85) & -p\alpha_i r_i T \\ T & z-1 \end{pmatrix}^{-1} \begin{pmatrix} 0.85 & (1-p)\alpha_i r_i T & k \\ T & 0 & 0 \end{pmatrix} \\ &= \frac{\begin{pmatrix} (z-1)0.85+p\alpha_i r_i T^2 & (z-1)(1-p)\alpha_i r_i T & k(z-1) \end{pmatrix}}{\widehat{P}_i(z)}, \end{aligned} \tag{24}$$

where $\widehat{P}_i(z) = z^2 + \widehat{a}_i z + \widehat{b}_i$, $\widehat{a}_i = \alpha_i r_i T + k + 0.85 - 2$, and $\widehat{b}_i = 1 - \alpha_i T + \alpha_i r_i T^2 - (k + 0.85)$. Similarly, in order to suppress the traffic jam in the CM delayed feedback car-following model, feedback gain k is designed so that (i) $\widehat{P}_i(z)$ is stable; (ii) $\max_{|z|=1} |\widehat{G}_i(z)| \leq 1,\ (i = 1, 2, \ldots, N)$.

Theorem 2 There is no traffic jam in the controlled system given by Eqs. (19) and (20) if the following conditions are satisfied:

Condition I:

$$\begin{aligned} &0 < p\alpha_i r_i T^2 < \min\{2,\ 2\alpha_i T - 2.3\}, \\ &\max\{0.15 + p\alpha_i r_i T^2 - \alpha_i T,\ R_1\} < k < \frac{p\alpha_i r_i T^2 - 2\alpha_i T + 2.3}{2}, \end{aligned}$$

where

$$R_1 = \frac{-0.6 + 2.3\alpha_i T - 2p\alpha_i r_i T^2 - \alpha_i^2 T^2 + p\alpha_i^2 r_i T^3 + (1-p)^2 \alpha_i^2 r_i^2 T^2}{2\alpha_i T - p\alpha_i r_i T^2 - 2.3}.$$

Condition II:

$$\begin{aligned} &\max\{2\alpha_i T - 2.3, 0\} < p\alpha_i r_i T^2 < 2, \\ &0.15 + p\alpha_i r_i T^2 - \alpha_i T < k < \min\{R_1,\ \frac{p\alpha_i r_i T^2 - 2\alpha_i T + 2.3}{2}\}. \end{aligned}$$

Condition III:

$$\begin{aligned} &0 < p\alpha_i r_i T^2 < \min\{4,\ 2\alpha_i T + 1.7\}, \\ &\max\{-0.85 + p\alpha_i r_i T^2 - \alpha_i T,\ R_2\} < k < \min\left\{\frac{p\alpha_i r_i T^2 - 2\alpha_i T + 2.3}{2}, 0.15 + p\alpha_i r_i T^2 - \alpha_i T\right\}, \end{aligned}$$

where

$$R_2 = \frac{-1.7\alpha_i T + 2p\alpha_i r_i T^2 - \alpha_i^2 T^2 + p\alpha_i^2 r_i T^3 + (1-p)^2 \alpha_i^2 r_i^2 T^2}{2\alpha_i T - p\alpha_i r_i T^2 + 1.7}.$$

Condition IV:

$$\begin{aligned} &2\alpha_i T + 1.7 < p\alpha_i r_i T^2 < 4, \\ &-0.85 + p\alpha_i r_i T^2 - \alpha_i T < k < \min\left\{\frac{p\alpha_i r_i T^2 - 2\alpha_i T + 2.3}{2}, 0.15 + p\alpha_i r_i T^2 - \alpha_i T, R_2\right\}. \end{aligned}$$

Condition V:

$$\begin{aligned} &0 < p\alpha_i r_i T^2 < 2,\ k = 0.15 + p\alpha_i r_i T^2 - \alpha_i T, \\ &\max\{-0.85 + p\alpha_i r_i T^2 - \alpha_i T,\ R_1\} < k < \frac{p\alpha_i r_i T^2 - 2\alpha_i T + 2.3}{2}, \end{aligned}$$

or

$$-0.85 + p\alpha_i r_i T^2 - \alpha_i T < k < \min\left\{\frac{p\alpha_i r_i T^2 - 2\alpha_i T + 2.3}{2},\ R_1\right\}.$$

The proof is similar to the one given in Appendix A.

The theorem provides with us the ability to design feedback gain k to suppress the traffic jam in our model. Three main differences between our method and the method of the KKH model are obvious. First, our scheme has a simpler form than that of the KKH model. Second, our control signal is more reasonable than that in the KKH model. Third, four feedback gains in the delayed feedback KKH model must be fixed at exact values, while the feedback gain in our model can be set in a range. A comparison between the two models will be carried out in the next section.

4. Simulation results

In this section, we will investigate whether the result of our theoretical analysis in Section 3 holds. The simulation is based on the modified CM and the KKH delayed feedback car-following models, and the parameters are given beforehand.

Through the theoretical analysis in Section 3, we determine that the range of k is from 0 to 0.1061. We set $k = 0.04$ in the following simulation.

First, we consider a situation in which the lead vehicle stops suddenly for a time,[3] i.e., $v_0(n) = 0$ for $nT = 100$–102. Figure 2 shows the temporal velocity behaviour of the 1th, 25th and 50th vehicles for the modified CM (Fig. 2(a)) and the KKH (Fig. 2(b)) models, and the space–time plots of the system for the modified CM (Fig. 2(c)) and the KKH (Fig. 2(d)) models with the control signal. From Figs. 2(a)–2(d), we do not observe oscillating behaviour or traffic jam phenomena. All the vehicles are running smoothly without full braking. We can see that the minimum speed of the following vehicle increases with the increasing vehicle number i. Compared with the result shown in Fig. 1, we can see that the traffic flow is smoother with the control signal. Figure 2 shows that the modified CM model is prior to the KKH model.

Then we consider a situation in which the lead vehicle stops suddenly four times,[3] i.e., $v_0(n) = 0$ for $nT = 100$–103, 120–123, 140–143 and 160–163. The external disturbance is used to examine the effect of the two methods on the traffic flow. We consider 100 vehicles. Figure 3 shows the temporal velocity behaviour and the space–time plots of the 1th and the 100th vehicles for the modified CM model and the KKH model.

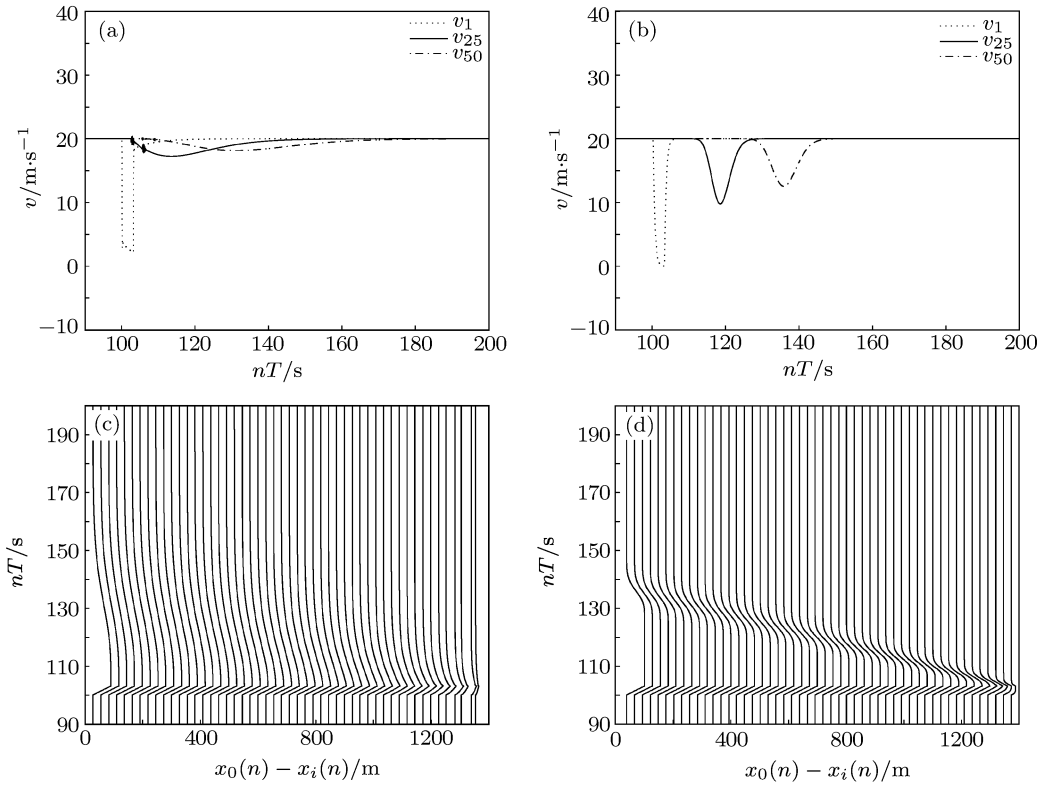


Fig. 2. Numerical simulations for systems with control methods. The figures show the temporal velocity behaviour of the 1th, 25th and 50th vehicles in the modified CM (a) and the KKH (b) models, and the space–time plots of the system in the modified CM (c) and the KKH (d) models. The leading vehicle stops at $nT = 100$–102.

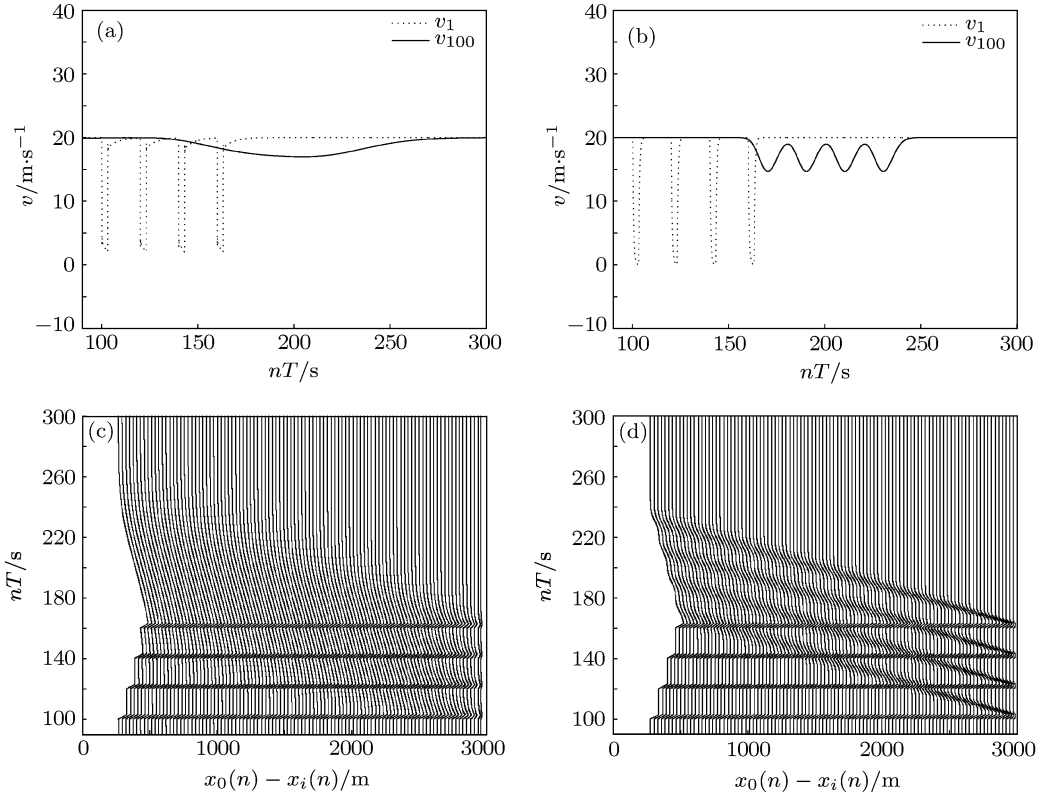


Fig. 3. Numerical simulations for the systems controlled by our scheme and the method proposed by Konishi *et al.* The figures show the temporal velocity behaviour of the 1th and the 100th vehicles in the modified CM (a) and the CM (b) models, and space–time plots of the systems controlled by our scheme (c) and by the method of Konishi *et al.* (d). The lead vehicle stops at $nT = 100$–103, 120–123, 140–143, 160–163.

From Fig. 3, we can see that the 100th vehicle is running smoothly and the amplitude of speed fluctuation decreases with the increase of vehicle number i. It should be noted that there are smaller oscillations in our method compared with the system controlled by the method of Konishi *et al.* These simulations show that both our method and that of Konishi *et al.* can be used to suppress traffic jams, and that the traffic system exhibits better temporal behaviour. From Figs. 2 and 3, we can see that our method is better than that proposed by Konishi *et al.*

5. Conclusion

In this paper, a new CM car-following model is proposed. The stability of the model is investigated. The control signal related to two velocity differences is applied to the modified CM car-following model. Through theoretical and simulated results, we conclude that: (i) our model is more meaningful than the KKH model, and the stability of the traffic system is improved by considering the interactions between the two vehicles in front; (ii) the traffic jam can be well suppressed by introducing signal $u_i(n)$; and (iii) the numerical simulations are in good agreement with the theoretical analysis.

From the above simulations, we find that there is a small difference between the controlled systems with our method and the method in the Zhao–Gao model.[8] It is surprising that our method is not better than the one proposed by Zhao and Gao, although we do add the effect of the following vehicle into the control scheme. The reason may lie in the fact that $\Delta v_{i+1}(n) < 0$ hardly appears under the assumption that all vehicles have the same parameters in our model. We will further investigate the problem.

Our control scheme is simple and feasible for real traffic and is useful for traffic systems of any size.

Appendix A: Proof of Theorem 1

First, according to the stability criterion, $P_i(z)$ is stable if

$$\begin{cases} 1 + a_i + b_i > 0, \\ 1 - b_i > 0, \\ 1 - a_i + b_i > 0. \end{cases}$$

We obtain

$$0 < r_i < \frac{1}{pT}, \quad 0 < \alpha_i < \frac{4}{(2 - pr_iT)T}.$$

Then we consider $\max_{|z|=1} |\widetilde{G}_i(z)| \leq 1$. Let $z = \cos\theta + \mathrm{i}\sin\theta$, we can obtain

$$\begin{aligned} &p^2\alpha_i^2 r_i^2 T^4 + 2(1-\cos\theta)(1-p)^2\alpha_i^2 r_i^2 T^2 \\ &\leq 4b_i\cos^2\theta + 2a_i(1+b_i)\cos\theta + (b_i-1)^2 + a_i^2. \end{aligned}$$

That is

$$\begin{aligned} &2(1-\cos\theta)[2(1-\cos\theta) - 2\alpha_i T(1-\cos\theta) \\ &\quad + \alpha_i^2 T^2 - p\alpha_i^2 r_i T^3 - 2\cos\theta p\alpha_i r_i T^2) \\ &\quad - (1-p)^2\alpha_i^2 r_i^2 T^2] \geq 0, \end{aligned}$$

where $\theta \in [0, 2\pi]$. Since

$$|\cos\theta| \leq 1, \quad \text{for} \quad \forall\theta \in [0, 2\pi],$$

we should consider

$$\begin{aligned} &(-2 + 2\alpha_i T - 2p\alpha_i r_i T^2)\cos\theta + 2 - 2\alpha_i T \\ &\quad + \alpha_i^2 T^2 - p\alpha_i^2 r_i T^3 - (1-p)^2\alpha_i^2 r_i^2 T^2 \geq 0. \end{aligned}$$

Finally, we have Eqs. (16) and (17).

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