Controlled quantum state sharing of arbitrary two-qubit states with five-qubit cluster states*

Wang Dong(王 东)[a)b)†], Zha Xin-Wei(查新未)[b)], Lan Qian(兰 倩)[b)], Li Ning(李 宁)[a)b)], and Wei Jing(卫 静)[b)]

[a)] *School of Science, Xi'an University of Posts and Telecommunications, Xi'an* 710061, *China*

[b)] *School of Communication and Information Engineering, Xi'an University of Posts and Telecommunications, Xi'an* 710061, *China*

(Received 21 March 2011; revised manuscript received 18 May 2011)

In this paper, we propose a controlled quantum state sharing scheme to share an arbitrary two-qubit state using a five-qubit cluster state and the Bell state measurement. In this scheme, the five-qubit cluster state is shared by a sender (Alice), a controller (Charlie), and a receiver (Bob), and the sender only needs to perform the Bell-state measurements on her particles during the quantum state sharing process, the controller performs a single-qubit projective measurement on his particles, then the receiver can reconstruct the arbitrary two-qubit state by performing some appropriate unitary transformations on his particles after he has known the measured results of the sender and the controller.

Keywords: controlled quantum state sharing, five-qubit cluster state, Bell-state measurements

PACS: 03.67.Dd, 03.67.Hk, 03.65.Ud **DOI:** 10.1088/1674-1056/20/9/090305

1. Introduction

In the quantum information theory, entanglement plays a key role in most quantum information processes.[1,2] Quantum entanglement is a valuable resource for the implementation of quantum computation and quantum communication protocols, such as quantum teleportation,[3−23] quantum key distribution,[24] quantum secure direct communication,[25] dense coding,[26−28] quantum computation,[29] and remote state preparation.[30−33] Quantum teleportation is regarded as one of the most profound results of the quantum information theory. In the original quantum teleportation protocol of Bennett,[3] it was shown that an unknown state of a qubit can be perfectly transported from a sender Alice to a remote receiver Bob with the aid of the long-range Einstein–Podolsky–Rosen (EPR) correlation and the transmission of two bits of classical information without the transmission of the carrier of the quantum state.

Recently, the branch for sharing an unknown quantum state was generated, which is called the quantum state sharing (QSTS). It is the extension of the quantum secret sharing of an unknown state among several agents in use of the multipartite entanglement. In 2005, Deng *et al.*[34] presented a scheme for multi-parties arbitrary two-particle state sharing with the EPR pairs. In the previous work about quantum state sharing, maximally entangled states were used, and the state could be transferred between the agents with fidelity 1. [35,36]

In this paper, we propose a controlled quantum state sharing scheme to share an arbitrary two-qubit state using a five-qubit cluster state and the Bell state measurement. In this scheme, the five-qubit cluster state is shared by a sender (Alice), a controller (Charlie), and a receiver (Bob). The sender only needs to perform the Bell-state measurements on her particles in the quantum state sharing process, and the controller performs a single-qubit projective measurement on his particles, then the receiver can reconstruct the arbitrary two-qubit state by performing some appropriate unitary transformations on his particles after he has known the measured results of the sender and the controller.

*Project supported by the National Natural Science Foundation of China (Grant No. 10902083) and the Natural Science Foundation of Shannxi Province, China (Grant No. 2009JM1007).

†Corresponding author. E-mail: iamwangdong@163.com

© 2011 Chinese Physical Society and IOP Publishing Ltd

2. Controlled quantum state sharing of an arbitrary two-qubit state

Suppose that the sender Alice plans to transmit the following arbitrary two-qubit entangled state to Charlie,

$$|\chi\rangle_{\rm ab} = \alpha|00\rangle_{\rm ab} + \beta|10\rangle_{\rm ab} + \gamma|01\rangle_{\rm ab} + \delta\,|11\rangle_{\rm ab}\,, \quad (1)$$

where coefficients α, β, γ, and δ are complex numbers, and $|\alpha|^2+|\beta|^2+|\gamma|^2+|\delta|^2=1$. We also assume that the quantum channel shared by Alice, Bob, and Charlie is composed of the following five-qubit cluster state,[37]

$$\begin{aligned}|C\rangle_5 \,=\, & \frac{1}{2}(|00000\rangle + |00111\rangle + |11101\rangle \\ & +\, |11010\rangle)_{\rm A_1B_1B_2CA_2}, \end{aligned} \quad (2)$$

where $\rm A_1$, $\rm A_2$; B_1, B_2; and C_1 are states known to Alice, Bob, and Charlie, respectively.

The composite quantum system, which consists of the seven particles, is described by

$$\begin{aligned}|\psi\rangle_{\rm s} \,=\, & |\chi\rangle_{\rm ab}\otimes|C\rangle_{\rm A_1B_1B_2CA_2} \\ =\, & \frac{1}{2}(\alpha|0000000\rangle + \beta|0010000\rangle + \gamma|100000\rangle \\ & +\, \delta\,|1010000\rangle + \alpha|0001011\rangle + \beta|0011011\rangle \\ & +\, \gamma|1101110\rangle + \delta\,|1100101\rangle + \alpha|0101110\rangle \\ & +\, \beta|0111110\rangle + \gamma|1101110\rangle + \delta\,|1111110\rangle \\ & +\, \alpha|0100101\rangle + \beta|0110101\rangle + \gamma|1100101\rangle \\ & +\, \delta\,|1110101\rangle)_{\rm aA_1bA_2B_1B_2C}. \end{aligned} \quad (3)$$

Step 1 Alice performs the joint Bell state measurements on her four particles a, $\rm A_1$ and b, $\rm A_2$ respectively. The Bell states are

$$\begin{aligned}|\phi^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \\ |\psi^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle). \end{aligned} \quad (4)$$

Then she announces her results publicly. For example, if Alice's measurement results are $|\phi^+\rangle_{\rm a,A_1}|\phi^+\rangle_{\rm b,A_2}$, the composite system of particles $\rm B_1B_2C_1C_2$ will collapse into

$$|\psi\rangle_{\rm sc} = \frac{1}{4}(\alpha\,|000\rangle + \beta\,|101\rangle + \gamma\,|011\rangle + \delta\,|110\rangle)_{\rm B_1B_2C}. \quad (5)$$

The outcomes of the measurement performed by Alice and the corresponding collapse states of Bob and Charlie's system are shown in Table 1.

Table 1. Outcomes of the measurement performed by Alice and the corresponding collapse states of Bob and Charlie's composite system.

Alice's measuring basis	Corresponding collapse states
$\|\phi^+\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$(\alpha\,\|000\rangle+\beta\,\|101\rangle+\gamma\,\|011\rangle+\delta\,\|110\rangle)_{\rm B_1B_2C}$
$\|\phi^+\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$(\alpha\,\|000\rangle+\beta\,\|101\rangle-\gamma\,\|011\rangle-\delta\,\|110\rangle)_{\rm B_1B_2C}$
$\|\phi^+\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$(\alpha\,\|011\rangle+\beta\,\|110\rangle+\gamma\,\|000\rangle+\delta\,\|101\rangle)_{\rm B_1B_2C}$
$\|\phi^+\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$(\alpha\,\|011\rangle+\beta\,\|110\rangle-\gamma\,\|000\rangle-\delta\,\|101\rangle)_{\rm B_1B_2C}$
$\|\phi^-\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$(\alpha\,\|000\rangle-\beta\,\|101\rangle+\gamma\,\|011\rangle-\delta\,\|110\rangle)_{\rm B_1B_2C}$
$\|\phi^-\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$(\alpha\,\|000\rangle-\beta\,\|101\rangle-\gamma\,\|011\rangle+\delta\,\|110\rangle)_{\rm B_1B_2C}$
$\|\phi^-\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$(\alpha\,\|011\rangle-\beta\,\|110\rangle+\gamma\,\|000\rangle-\delta\,\|101\rangle)_{\rm B_1B_2C}$
$\|\phi^-\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$(\alpha\,\|011\rangle-\beta\,\|110\rangle-\gamma\,\|000\rangle+\delta\,\|101\rangle)_{\rm B_1B_2C}$
$\|\psi^+\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$(\alpha\,\|101\rangle+\beta\,\|000\rangle+\gamma\,\|110\rangle+\delta\,\|011\rangle)_{\rm B_1B_2C}$
$\|\psi^+\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$(\alpha\,\|101\rangle+\beta\,\|000\rangle-\gamma\,\|110\rangle-\delta\,\|011\rangle)_{\rm B_1B_2C}$
$\|\psi^+\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$(\alpha\,\|110\rangle+\beta\,\|011\rangle+\gamma\,\|101\rangle+\delta\,\|000\rangle)_{\rm B_1B_2C}$
$\|\psi^+\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$(\alpha\,\|110\rangle+\beta\,\|011\rangle-\gamma\,\|101\rangle-\delta\,\|000\rangle)_{\rm B_1B_2C}$
$\|\psi^-\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$(\alpha\,\|101\rangle-\beta\,\|000\rangle+\gamma\,\|110\rangle-\delta\,\|011\rangle)_{\rm B_1B_2C}$
$\|\psi^-\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$(\alpha\,\|101\rangle-\beta\,\|000\rangle-\gamma\,\|110\rangle+\delta\,\|011\rangle)_{\rm B_1B_2C}$
$\|\psi^-\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$(\alpha\,\|110\rangle-\beta\,\|011\rangle+\gamma\,\|101\rangle-\delta\,\|000\rangle)_{\rm B_1B_2C}$
$\|\psi^-\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$(\alpha\,\|110\rangle-\beta\,\|011\rangle-\gamma\,\|101\rangle+\delta\,\|000\rangle)_{\rm B_1B_2C}$

Step 2 In order to transmit state $|\chi\rangle_{\rm ab}$ to Charlie, Charlie should perform a single-qubit projective measurement on his particle in a set of mutually orthogonal basis vectors $\{|X\rangle_{\pm}\}$, which are

$$|X\rangle_{\pm} = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle). \quad (6)$$

After Charlie's measurement, the composite system of particles B_1B_2 will collapse into

$$\begin{aligned}|\psi\rangle_{\rm scb} &= {}_+\langle X|\,\psi\rangle_{\rm sc}\\ &= \frac{1}{4\sqrt{2}}(\alpha\,|00\rangle+\beta\,|10\rangle+\gamma\,|01\rangle+\delta\,|11\rangle)_{\rm B_1B_2}.\end{aligned}\tag{7}$$

The outcomes of the measurements performed by Alice and Charlie and the corresponding collapse states of Bob are shown in Table 2.

Table 2. Outcomes of the measurements performed by Alice and Charlie and the corresponding collapse states of Bob.

Alice's result	Charlie's result	Collapse states of Bob	Alice's result	Charlie's result	Collapse states of Bob
$\|\phi^+\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|00\rangle+\beta\|10\rangle+\gamma\|01\rangle+\delta\|11\rangle$	$\|\phi^+\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\alpha\|00\rangle-\beta\|10\rangle-\gamma\|01\rangle+\delta\|11\rangle$
$\|\phi^+\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|00\rangle+\beta\|10\rangle-\gamma\|01\rangle-\delta\|11\rangle$	$\|\phi^+\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\alpha\|00\rangle-\beta\|10\rangle+\gamma\|01\rangle-\delta\|11\rangle$
$\|\phi^+\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|00\rangle-\beta\|10\rangle+\gamma\|01\rangle-\delta\|11\rangle$	$\|\phi^+\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\alpha\|00\rangle+\beta\|10\rangle-\gamma\|01\rangle-\delta\|11\rangle$
$\|\phi^+\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|00\rangle-\beta\|10\rangle-\gamma\|01\rangle+\delta\|11\rangle$	$\|\phi^+\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\alpha\|00\rangle+\beta\|10\rangle+\gamma\|01\rangle+\delta\|11\rangle$
$\|\phi^-\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|01\rangle+\beta\|11\rangle+\gamma\|00\rangle+\delta\|10\rangle$	$\|\phi^-\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$-\alpha\|01\rangle+\beta\|11\rangle+\gamma\|00\rangle-\delta\|10\rangle$
$\|\phi^-\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|01\rangle+\beta\|11\rangle-\gamma\|00\rangle-\delta\|10\rangle$	$\|\phi^-\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$-\alpha\|01\rangle+\beta\|11\rangle-\gamma\|00\rangle+\delta\|10\rangle$
$\|\phi^-\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|01\rangle-\beta\|11\rangle+\gamma\|00\rangle-\delta\|10\rangle$	$\|\phi^-\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$-\alpha\|01\rangle-\beta\|11\rangle+\gamma\|00\rangle+\delta\|10\rangle$
$\|\phi^-\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|01\rangle-\beta\|11\rangle-\gamma\|00\rangle+\delta\|10\rangle$	$\|\phi^-\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$-\alpha\|01\rangle-\beta\|11\rangle-\gamma\|00\rangle-\delta\|10\rangle$
$\|\psi^+\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|10\rangle+\beta\|00\rangle+\gamma\|11\rangle+\delta\|01\rangle$	$\|\psi^+\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$-\alpha\|10\rangle+\beta\|00\rangle+\gamma\|11\rangle-\delta\|01\rangle$
$\|\psi^+\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|10\rangle+\beta\|00\rangle-\gamma\|11\rangle-\delta\|01\rangle$	$\|\psi^+\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$-\alpha\|10\rangle+\beta\|00\rangle-\gamma\|11\rangle+\delta\|01\rangle$
$\|\psi^+\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|10\rangle-\beta\|00\rangle+\gamma\|11\rangle-\delta\|01\rangle$	$\|\psi^+\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$-\alpha\|10\rangle-\beta\|00\rangle+\gamma\|11\rangle+\delta\|01\rangle$
$\|\psi^+\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|10\rangle-\beta\|00\rangle-\gamma\|11\rangle+\delta\|01\rangle$	$\|\psi^+\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$-\alpha\|10\rangle-\beta\|00\rangle-\gamma\|11\rangle-\delta\|01\rangle$
$\|\psi^-\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|11\rangle+\beta\|01\rangle+\gamma\|10\rangle+\delta\|00\rangle$	$\|\psi^-\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\alpha\|11\rangle-\beta\|01\rangle-\gamma\|10\rangle+\delta\|00\rangle$
$\|\psi^-\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|11\rangle+\beta\|01\rangle-\gamma\|10\rangle-\delta\|00\rangle$	$\|\psi^-\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\alpha\|11\rangle-\beta\|01\rangle+\gamma\|10\rangle-\delta\|00\rangle$
$\|\psi^-\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|11\rangle-\beta\|01\rangle+\gamma\|10\rangle-\delta\|00\rangle$	$\|\psi^-\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\alpha\|11\rangle+\beta\|01\rangle-\gamma\|10\rangle-\delta\|00\rangle$
$\|\psi^-\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\alpha\|11\rangle-\beta\|01\rangle-\gamma\|10\rangle+\delta\|00\rangle$	$\|\psi^-\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\alpha\|11\rangle+\beta\|01\rangle+\gamma\|10\rangle+\delta\|00\rangle$

Step 3 After Alice and Charlie announce the measurement results publicly over the classical channel, Bob performs the unitary operations on his two particles. The correspondence between the measurement results and the operations are shown in Table 3.

Table 3. Correspondence between measurement results and operations.

Alice's result	Charlie's result	Unitary transformation of Bob	Alice's result	Charlie's result	Unitary transformation of Bob
$\|\phi^+\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$	$\|\phi^+\rangle_{\rm a,A_1}\|\phi^+\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}$
$\|\phi^+\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$	$\|\phi^+\rangle_{\rm a,A_1}\|\phi^-\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}$
$\|\phi^+\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}$	$\|\phi^+\rangle_{\rm a,A_1}\|\psi^+\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&0&-1&0\\0&0&0&1\\1&0&0&0\\0&-1&0&0\end{pmatrix}$
$\|\phi^+\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix}$	$\|\phi^+\rangle_{\rm a,A_1}\|\psi^-\rangle_{\rm b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&0&-1&0\\0&0&0&1\\-1&0&0&0\\0&1&0&0\end{pmatrix}$

Table 3 *Continued*

Alice's result	Charlie's result	Unitary transformation of Bob	Alice's result	Charlie's result	Unitary transformation of Bob
$\|\phi^-\rangle_{a,A_1}\|\phi^+\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}$	$\|\phi^-\rangle_{a,A_1}\|\phi^+\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$
$\|\phi^-\rangle_{a,A_1}\|\phi^-\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}$	$\|\phi^-\rangle_{a,A_1}\|\phi^-\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$
$\|\phi^-\rangle_{a,A_1}\|\psi^+\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{pmatrix}$	$\|\phi^-\rangle_{a,A_1}\|\psi^+\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&0&-1&0\\0&0&0&-1\\1&0&0&0\\0&1&0&0\end{pmatrix}$
$\|\phi^-\rangle_{a,A_1}\|\psi^-\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}$	$\|\phi^-\rangle_{a,A_1}\|\psi^-\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&0&-1&0\\0&0&0&-1\\-1&0&0&0\\0&-1&0&0\end{pmatrix}$
$\|\psi^+\rangle_{a,A_1}\|\phi^+\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$	$\|\psi^+\rangle_{a,A_1}\|\phi^+\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}$
$\|\psi^+\rangle_{a,A_1}\|\phi^-\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&-1&0\end{pmatrix}$	$\|\psi^+\rangle_{a,A_1}\|\phi^-\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}$
$\|\psi^+\rangle_{a,A_1}\|\psi^+\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}$	$\|\psi^+\rangle_{a,A_1}\|\psi^+\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix}$
$\|\psi^+\rangle_{a,A_1}\|\psi^-\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}$	$\|\psi^+\rangle_{a,A_1}\|\psi^-\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&0&0&1\\0&0&-1&0\\0&1&0&0\\-1&0&0&0\end{pmatrix}$
$\|\psi^-\rangle_{a,A_1}\|\phi^+\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}$	$\|\psi^-\rangle_{a,A_1}\|\phi^+\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&-1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$
$\|\psi^-\rangle_{a,A_1}\|\phi^-\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}$	$\|\psi^-\rangle_{a,A_1}\|\phi^-\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&-1&0&0\\-1&0&0&0\\0&0&0&-1\\0&0&-1&0\end{pmatrix}$
$\|\psi^-\rangle_{a,A_1}\|\psi^+\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&0&0&1\\0&0&-1&0\\0&1&0&0\\-1&0&0&0\end{pmatrix}$	$\|\psi^-\rangle_{a,A_1}\|\psi^+\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}$
$\|\psi^-\rangle_{a,A_1}\|\psi^-\rangle_{b,A_2}$	$\|X\rangle_+$	$\begin{pmatrix}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix}$	$\|\psi^-\rangle_{a,A_1}\|\psi^-\rangle_{b,A_2}$	$\|X\rangle_-$	$\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}$

3. Conclusion

In summary, we have proposed a controlled quantum state sharing scheme to share an arbitrary two-qubit state using a five-qubit cluster state and the Bell state measurement. Using the five-qubit cluster state, the receiver recovers the original state with a certain fidelity. In the quantum state sharing scheme for sharing an arbitrary two-qubit state, Alice can in principle prevent the receiver from eavesdropping on her quantum state, as Alice can share a quantum channel with the receiver beforehand.

Compared with the previous quantum state sharing schemes, the present one has the following advantages. First, our protocol has investigated the probabilistic quantum state sharing of an arbitrary two-qubit state from a sender to the receiver, while the previous quantum state sharing schemes considered only the quantum state sharing with two parties from a sender to a receiver. Second, the quantum channels are different in form. In our present scheme, the five-qubit cluster state is used as the quantum channel, whereas the quantum channel in the previous schemes was composed of the EPR pairs or the Greenberger–Horne–Zeilinger (GHZ) states. Nowadays, it has been shown that the five-qubit cluster state is important for the sharing scheme and can be utilized to construct a controlled quantum channel, which may be useful in the future quantum computation.

References

[1] Gisin N, Ribordy G, Tittle W and Zbinden H 2002 *Rev. Mod. Phys.* **74** 145

[2] Hillery M, Buzek V and Berthiaume A 1999 *Phys. Rev.* A **59** 1829

[3] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[4] Zha X W and Song H Y 2010 *Mod. Phys. Lett.* B **24** 19 2069

[5] Zha X W and Song H Y 2010 *Commun. Theor. Phys.* **53** 852

[6] Yang C P and Han S Y 2005 *Phys. Lett.* A **343** 267

[7] An N B 2003 *Phys. Rev.* A **68** 022321

[8] Yan F L and Wang D 2003 *Phys. Lett.* A **316** 297

[9] Yang C P, Chu S I and Han S Y 2004 *Phys. Rev.* A **70** 022329

[10] Gao T, Yan F L and Wang Z X 2005 *Chin. Phys.* **14** 893

[11] Deng F G, Li C Y, Li Y S, Zhou H Y and Wang Y 2005 *Phys. Rev.* A **72** 022338

[12] Li X H, Deng F G and Zhou H Y 2007 *Chin. Phys. Lett.* **24** 1151

[13] Zhou P, Li X H and Deng F G 2007 *J. Phys.* A **40** 13121

[14] Man Z X, Xia Y J and An N B 2007 *J. Phys.* B **40** 1767

[15] Dong J and Teng J F 2008 *Euro. Phys. J.* D **49** 129

[16] Wang Y H and Song H S 2008 *Opt. Commun.* **281** 489

[17] Cao Z L and Li D C 2008 *Chin. Phys.* B **17** 55

[18] Nie D, He G Q and Zeng G H 2008 *J. Phys.* B **41** 175504

[19] Wang J, Hou K, Yuan H and Shi S H 2009 *Phys. Scr.* **80** 015004

[20] Wang T Y and Wen Q Y 2011 *Chin. Phys.* B **20** 040307

[21] Zhou X Q, Wu Y W and Zhao H 2011 *Acta Phys. Sin.* **60** 35 (in Chinese)

[22] Zhang Z J and Man Z X 2005 *Phys. Lett.* A **341** 55

[23] Zhang Z J 2006 *Phys. Lett.* A **352** 55

[24] Wang X B, Hiroshima T, Tomita A and Hayashi M 2007 *Phys. Rep.* **448** 1

[25] Long G L, Deng F G, Wang C, Li X H, Wen K and Wang W Y 2007 *Phys. China* **2** 251

[26] Bennett C H and Wiesner S 1992 *J. Phys. Rev. Lett.* **69** 2881

[27] Wang M Y, Yang L G and Yan F L 2005 *Chin. Phys. Lett.* **22** 1053

[28] Yan F L and Wang M Y 2004 *Chin. Phys. Lett.* **21** 1195

[29] Barenco A C 1996 *Phys.* **37** 375

[30] Pati A K 2000 *Phys. Rev.* A **63** 014302

[31] Chen L B, Jin R B and Lu H 2008 *Chin. Phys.* B **17** 78

[32] Bennett C H, DiVincenzo D P, Shor P W, Smolin J A,Trehal B M and Wootters W K 2001 *Phys. Rev. Lett.* **87** 077902

[33] Zhan Y B 2005 *Commun. Theor. Phys.* **43** 637

[34] Deng F G, Li X H and Li C Y 2005 *Phys. Rev.* A **72** 044301

[35] Lance A M, Symun T and Bowen W P 2004 *Phys. Rev. Lett.* **92** 177903

[36] Li X H, Zhou P and Li C Y 2006 *J. Phys.* B **39** 1975

[37] Briegel H J and Raussendorf R 2001 *Phys. Rev. Lett.* **86** 910