Efficient scheme for entangled states and quantum information transfer with trapped atoms in a resonator*

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A protocol is proposed to generate atomic entangled states and implement quantum information transfer in a cavity quantum electrodynamics system. It utilizes Raman transitions or stimulated Raman adiabatic passages between two systems to entangle the ground states of two three-state Λ-type atoms trapped in a single mode cavity. It does not need the measurements on cavity field nor atomic detection and can be implemented in a deterministic fashion. Since the present protocol is insensitive to both cavity decay and atomic spontaneous emission, it may have some interesting applications in quantum information processing.

Keywords: cavity quantum electrodynamics, entangled states, quantum information transfer

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1. Introduction

Entangled states can not only be utilized to test fundamental quantum mechanical principles such as Bell's inequalities,[1] but also play a central role in practical applications of quantum information processing,[2] such as quantum computation,[3,4] quantum teleportation,[5] and quantum cryptography.[6] It is generally believed that atoms are good candidates for storing quantum information and are natural quantum information processors. Therefore, producing atomic entangled states is particularly significant. In the context of cavity quantum electrodynamics(QED),[7,8] numerous proposals have been presented for generating atomic entangled states.[9−23] Though these proposals seem very promising, some rely on measurements on photons, which impair their performance, some are not immune to atomic spontaneous emission.

In this study, we propose a protocol for the realization of atomic entangled states and quantum information transfer with a cavity QED system. It consists of two three-state Λ-type atoms and a single mode cavity. We show that through choosing suitable detunings and intensities of fields, Raman transitions or stimulated Raman adiabatic passages (STIRAP)[24] between the two atoms can be achieved, which can be utilized to produce the atomic entangled states. We also discuss how to transfer arbitrary quantum information encoded in atomic ground states from one atom to the other based on this scheme. This proposal could be implemented in a deterministic fashion, thus requiring no measurements on cavity field and atomic states. Because the atomic excited states and cavity mode excitations are not involved in this process, this protocol is very robust against atomic spontaneous emission and cavity photon decay. Since the STIRAP techniques[24] are utilized, the protocol is very robust against moderate fluctuations of experimental parameters. With the presently available experimental setups in cavity QED, this proposal could be implemented.

2. Theoretical model

Consider the case of two three-state Λ-type atoms trapped in a single mode cavity. As sketched in Fig. 1, each atom has the level structure of a Λ system with two stable ground states $|0\rangle$ and $|1\rangle$, and an excited state $|e\rangle$. The classical field of frequency $\omega_{\rm L}$ drives the transition dispersively $|0\rangle \leftrightarrow |e\rangle$ with the Rabi frequency Ω_i $(i = 1, 2)$ and detuning $\Delta = \omega_{\rm e1} - \omega_{\rm L}$. The cavity mode of frequency ν couples the tran-

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sition $|1\rangle \leftrightarrow |e\rangle$ with the coupling constant g_i and the same detuning $\Delta = \omega_{e0} - \nu$. For simplicity, the coupling constants of both atoms to the cavity mode are taken to be the same, $g_1 = g_2 = g$,

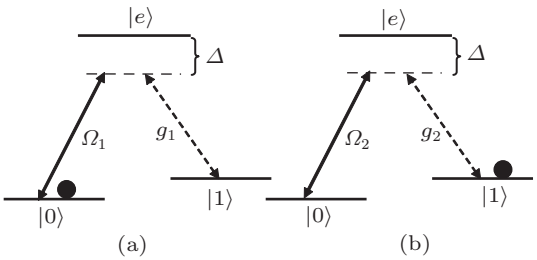


Fig. 1. The atomic levels of two atoms trapped in a single mode cavity: (a) atom 1, (b) atom 2.

but this is not the necessary condition for the analysis. In addition, we neglect the position dependence of the cavity-atom coupling strengths by assuming the Lamb–Dicke limit. In the interaction picture, the associated Hamiltonian under the dipole and rotating wave approximation is given by (let $\hbar = 1$)

$$\hat{H}_{\rm I} = \sum_{i=1,2} (\Omega_i \hat{\sigma}^i_{e0}\, {\rm e}^{{\rm i}\Delta t} + g_i \hat{a}\hat{\sigma}^i_{e1}\, {\rm e}^{{\rm i}\Delta t}) + {\rm H.c.}, \quad (1)$$

where $\hat{\sigma}_{jm} = |j\rangle\langle m|$ is the atomic transition operator, and $\hat{a}$ is the annihilation operator for the cavity mode. We consider dispersive detuning $|\Delta| \gg |\Omega_i|, |g|$ for each atom. Using the standard method given in Ref. [25], we can adiabatically eliminate the state $|e\rangle$. Then we obtain the effective Hamiltonian describing the Raman excitations of the atoms

$$\hat{H}_{\rm eff} = \sum_{i=1,2} \Big(\frac{|g|^2}{\Delta} \hat{a}^\dagger \hat{a} \hat{\sigma}^i_{11} + \frac{|g|^2 m}{\Delta} \hat{\sigma}^i_{00} + \frac{|\Omega_i g^*|}{\Delta} \hat{a}^\dagger \hat{\sigma}^i_{10} + \frac{|\Omega_i^* g|}{\Delta} \hat{a} \hat{\sigma}^i_{01} \Big), \quad (2)$$

where we chose Ω_i in phase with g. We have included an energy shift $\Delta_m = (|g|^2 m - |\Omega_i|^2)/\Delta$ $(m = 0, 1, ...)$ to level $|0\rangle$ for each atom, which could be implemented through the action of external classical fields. The number m is introduced for convenience, which can be determined from the expression of the energy shift Δ_m, and can be controlled through tuning the external classical fields. For instance, if we tune the external classical fields such that the energy shift has the value $-|\Omega_i|^2/\Delta$, then this corresponds to the choice $m = 0$. In the same way, we can choose m for other values.

We assume that atoms 1 and 2 are initially prepared in their stable ground states $|0\rangle_1$ and $|1\rangle_2$, and the cavity field is in vacuum state $|0\rangle_{\rm C}$. Then the dynamics is confined to the subspace of the collective energy levels of the two atoms and cavity mode $\{|01;0\rangle, |11;1\rangle, |10;0\rangle\}$, where $|ij;k\rangle = |i\rangle_1 |j\rangle_2 |k\rangle_{\rm C}$ $(i, j, k = 0, 1)$ describes a system with the atoms in state $|i\rangle_1 |j\rangle_2$ and cavity in Fock state $|k\rangle_{\rm C}$. Then we can write the Hamiltonian of Eq. (2) in this subspace as

$$\begin{aligned} \hat{H}_{\rm eff} = {} & \frac{m|g|^2}{\Delta} (|10;0\rangle\langle 0;01| + |01;0\rangle\langle 0;10|) \\ & + \frac{2|g|^2}{\Delta} |11;1\rangle\langle 1;11| + \frac{|g\Omega_1^*|}{\Delta} |01;0\rangle\langle 1;11| \\ & + \frac{|g\Omega_2^*|}{\Delta} |10;0\rangle\langle 1;11| + {\rm H.c.}, \end{aligned} \quad (3)$$

which forms a typical Λ system. Assuming $m = 0$ and performing the unitary transformation

$$U(t) = {\rm e}^{{\rm i}\frac{2|g|^2}{\Delta}t} |11;1\rangle\langle 1;11| + |01;0\rangle\langle 0;10| + |10;0\rangle\langle 0;01|,$$

we obtain

$$\begin{aligned} \hat{H}_{\rm eff} = {} & \frac{|g\Omega_1^*|}{\Delta} {\rm e}^{-{\rm i}\frac{2|g|^2}{\Delta}t} |01;0\rangle\langle 1;11| \\ & + \frac{|g\Omega_2^*|}{\Delta} {\rm e}^{-{\rm i}\frac{2|g|^2}{\Delta}t} |10;0\rangle\langle 1;11| + {\rm H.c.} \end{aligned} \quad (4)$$

If we further assume

$$\frac{2|g|^2}{\Delta} \gg \Big\{ \frac{|g\Omega_1^*|}{\Delta}, \frac{|g\Omega_2^*|}{\Delta} \Big\},$$

then we can obtain the Raman transition between states $|01;0\rangle$ and $|10;0\rangle$. In this case, using the time-average method given in Ref. (25), through adiabatic elimination of the state $|11;1\rangle$, we obtain the new effective Hamiltonian describing the Raman transition

$$\hat{H}_{\rm eff} = \Theta |10;0\rangle\langle 0;10| + {\rm H.c.}, \quad (5)$$

where $\Theta = -\frac{|\Omega_1 \Omega_2|}{2\Delta}$ is the Raman transition rate. The Hamiltonian (5) describes a two photon Raman transition between two distant atoms trapped in a cavity.

The system is initially prepared in the state $\psi(0) = |01;0\rangle$, then the state evolution of the system is given by

$$\psi(t) = \cos(\Theta t)|01;0\rangle - {\rm i}\sin(\Theta t)|10;0\rangle, \quad (6)$$

which is an entangled state for the two atoms. If we choose $\Theta t = \pi/4$, we can obtain the maximally entangled two-atom state

$$\psi_{\rm a} = \frac{1}{\sqrt{2}} (|01\rangle - {\rm i}|10\rangle), \quad (7)$$

which is the well-known Einstein–Podolsky–Rosen (EPR) state. This entangled state is very robust because it only involves the ground states of the atoms.

Although the entanglement mechanism given above can work well for a pair of atoms, it relies on off-resonance Raman transitions and is not robust enough. We now extend the idea to the case of on-resonance STIRAP process between two trapped

atoms in a cavity. We will see that this STIRAP protocol is more robust than the Raman excitation based scheme. Different to the general STIRAP process,[24] the present transfer is between ground states of two atoms and keeps the cavity field from exciting.

We assume that the two trapped atoms are far apart so that they can be addressed individually by laser beams with time-dependent Rabi frequencies. We also suppose that the amplitudes of $\Omega_1(t), \Omega_2(t)$ satisfy $\Omega_1 = -\xi\Omega_2$ in this paper, where ξ is a control parameter. If we choose $m = 2$, then from Eq. (3), we can obtain the following Hamiltonian:

$$\hat{H}_{\rm eff} = \frac{|g\Omega_1(t)|}{\Delta}|01;0\rangle\langle 1;11| - \frac{|g\Omega_2(t)|}{\Delta}|10;0\rangle\langle 1;11| + \text{H.c.}, \quad (8)$$

where we have discarded the constant energy terms. The effective Hamiltonian (8) describes a typical Λ system which is on resonance. Therefore, a dark state exists in this system,

$$|D(t)\rangle = \cos\theta(t)|01;0\rangle + \sin\theta(t)|10;0\rangle, \quad (9)$$

with $\tan\theta(t) = |\Omega_1(t)|/|\Omega_2(t)|$. As a consequence, adiabatic transfer of population can occur between states $|01;0\rangle$ and $|10;0\rangle$ by slowly varying the laser amplitudes $\Omega_1(t)$ and $\Omega_2(t)$. This procedure resembles the STIRAP process, which transfers population between ground states of one atom, but the present transfer is between ground states of two atoms and keeps cavity field excitation from being involved in this process. A maximally entangled state can be generated in the particular case $|\Omega_1(t)| = |\Omega_2(t)|$, i.e., $1/\sqrt{2}(|01\rangle + |10\rangle)$. The generated entangled two-atom state is more robust than previously proposed entangled atomic states, due to the fact that the atomic excited states and cavity mode are unpopulated during the process.

The dark state (9) of the two-atom system can be used to transfer arbitrary quantum information encoded in atomic ground states from one atom to the other.[26,27] Suppose that the original information is encoded in atom 1, which is an arbitrary unknown state $\alpha|0\rangle_1 + \beta|1\rangle_1$, and atom 2 is prepared in the state $|0\rangle_2$. The initial Rabi frequencies satisfy $|\Omega_1(t)|/|\Omega_2(t)| \gg 1$. Adiabatic passage following the dark state can be implemented by varying the Rabi frequencies. When the Rabi frequencies are slowly changed to $|\Omega_1(t)|/|\Omega_2(t)| \ll 1$, one has

$$(\alpha|0\rangle_1 + \beta|1\rangle_1)|0\rangle_2 \to (\alpha|0\rangle_2 + \beta|1\rangle_2)|0\rangle_1. \quad (10)$$

This process completes the quantum information transfer from atom 1 to atom 2. This procedure is very robust because of the adiabatic passage, where the excited states of the total system are unpopulated during this process. Furthermore, the mechanism is insensitive to fluctuations of experimental parameters. For example, if we expect $\Omega_2(t)$ to be $\Omega_{2\rm f}$ at the end of this process, it is actually $\Omega_{2\rm f} + \Delta\Omega_{2\rm f}$ due to some fluctuations. However, the mechanism for quantum information transfer still works well because the condition $|\Omega_1(t)|/|\Omega_2(t)| \ll 1$ is still satisfied at this moment when the Rabi frequency $\Omega_1(t)$ is switched off.

3. Numerical simulations

To verify the above approximations and STIRAP process, we numerically simulate the dynamics by solving the master equation. Following the standard quantum theory of damping, we investigate the combined influence of the cavity decay (κ) and atomic spontaneous emission (γ) on the coupled system. After tracing out the degrees of freedom of the reservoir, we obtain the master equation for the density matrix of the atom-cavity system

$$\dot{\rho} = -\,\mathrm{i}[\hat{H}_{\rm I}, \rho] + \kappa(2\hat{a}\rho\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\rho - \rho\hat{a}^\dagger\hat{a}) + \sum_{i=1,2;j=0,1} \frac{\gamma}{2}\left(2\hat{\sigma}^{\rm i}_{je}\rho\hat{\sigma}^i_{ej} - \hat{\sigma}^i_{ee}\rho - \rho\hat{\sigma}^i_{ee}\right). \quad (11)$$

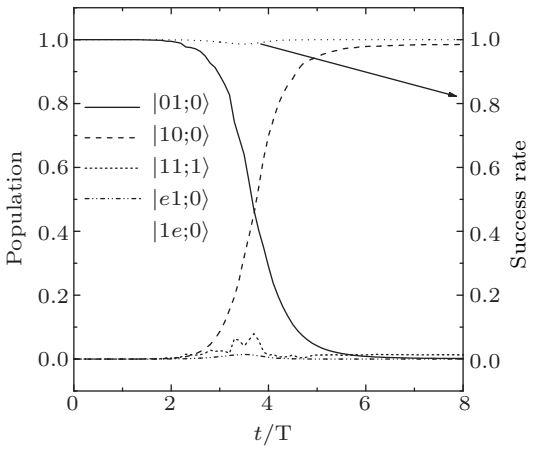


Fig. 2. Populations and success rate versus time for $\gamma = 0.1g$, and $\kappa = 0.1g$, with $T = 50g^{-1}$. The parameters for the simulations are chosen as $|\Omega_1| = 2g$, $|\Omega_2| = g$, $\tau_1 = 0.3 \times 10^3 g^{-1}$, $\tau_2 = 0.15 \times 10^3 g^{-1}$, $\Delta\tau_1 = 0.125 \times 10^3 g^{-1}$, $\Delta\tau_2 = 0.175 \times 10^3 g^{-1}$, and $\Delta = 20g$.

Figure 2 displays the numerical results of the system evolution with decay terms for the atoms ($\gamma = 0.1g$) and cavity modes ($\kappa = 0.1g$). The Rabi frequencies $\Omega_1(t)$ and $\Omega_2(t)$ are assumed to be

Gaussian envelopes for the simulations, i.e., $\Omega_i(t) = \Omega_i\, \mathrm{e}^{-(t-\tau_i)^2/\Delta\tau_i^2} (i = 1, 2)$. The system starts from the state $|01; 0\rangle$. At the end of the STIRAP process, the system ends up in the state $|10; 0\rangle$ with the success rate close to unity. During this process, the occupations of the excited state $|e\rangle$ and the cavity mode stay small. Thus, the numerical simulations clearly verify the analytical results.

In order to further quantify the robustness of this protocol against cavity decay, atomic spontaneous emission, and fluctuations of experimental parameters, we evaluate the success rate P and fidelity F. The success rate P is defined as the probability of producing the entangled state $|\psi\rangle = 1/\sqrt{2}(|01\rangle+|10\rangle)$, and the fidelity $F = \langle\psi|\rho_{\mathrm{a}}|\psi\rangle$, where ρ_{a} is the final reduced density matrix of the atoms. Figure 3 shows the succuss rate and fidelity versus $\kappa\gamma/g^2$ for this scheme. We see that within the relatively strong coupling regime, the success rate and fidelity are always close to unity for this protocol. For instance, when $\kappa\gamma/g^2 \sim 0.01$, we can obtain the target state with the success rate and fidelity higher than 0.9. This is due to the fact that the present proposal does not involve cavity field excitation, is deterministic, and requires no measurement on the cavity field. As the parameter $\kappa\gamma/g^2$ increases, the success rate decreases. However, the fidelity still remains high. When $\kappa\gamma/g^2 \sim 0.1$, the success rate is about 0.8%, but the fidelity is still close to unity. Therefore, the present protocol is insensitive to the cavity decay and atomic spontaneous emission.

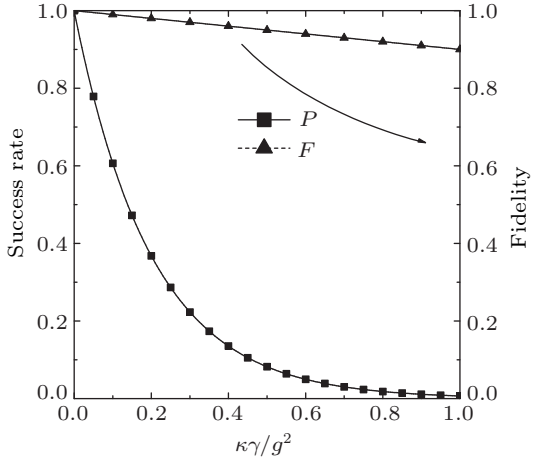


Fig. 3. Succuss rate P and fidelity F versus $\kappa\gamma/g^2$ at the time when $|\Omega_1(t)| = |\Omega_2(t)|$. Other parameters are chosen as those in Fig. 2.

4. Experimental considerations

For experimental implementation of the proposal, one could utilize the recently performed experimental setups.[28] There, trapped Cesium atoms could couple to a high-finesse Fabry-Perot cavity. The Cs atoms are dropped from a magneto-optical trap into the cavity and cooled into a far off-resonant trap by an optical lattice. The states used in the setup are ground $|6\mathrm{S}_{1/2}, F = 3, 4\rangle$ and excited $|6\mathrm{P}_{3/2}, F' = 3\rangle$ manifolds. We can choose $|0\rangle = |6\mathrm{S}_{1/2}, F = 4\rangle$, $|1\rangle = |6\mathrm{S}_{1/2}, F = 3\rangle$, and $|e\rangle = |6\mathrm{S}_{1/2}, F' = 3\rangle$. The experimental parameters are $g_1 \sim g_2 \sim g/(2\pi) = 16$ MHz, $(\gamma, \kappa)/2\pi = (3.8, 2.6)$ MHz, and we choose $\Delta = 10g$, $\Omega_1 \sim 100$ MHz, $\Omega_2 \sim 100$ MHz, $\tau_1 \sim 3$ μs, $\tau_2 \sim 1.5$ μs, and $\Delta\tau_1 \sim 1.3$ μs, $\Delta\tau_2 \sim 1.8$ μs. With these parameters we obtain a total preparation time $t \sim 2$ μs. The lifetime of the entangled state generated by the scheme is estimated to be about 20 μs. The effective lifetime of the photons in the experiment is about $1/(0.001\kappa) \sim 60$ μs. Thus, the present protocol can be implemented with these setups.

5. Discussion and conclusions

It is noted that different from the previously proposed entanglement mechanisms based on dark states,[10] the present proposal does not involve cavity field excitation, so it is more robust against cavity decay. Furthermore, this entanglement mechanism is deterministic and does not need the measurement on cavity field. Comparing this protocol with those presented in Refs. [11] and [12], we see that this protocol utilizes STIRAP techniques and trapped atoms, while the schemes proposed in Refs. [11] and [12] rely on flying atoms. Although the scheme proposed in Ref. [11] utilizes dissipation-assisted adiabatic passage techniques, it requires one to turn off the laser field when the two atoms have equal coupling with the cavity mode, which is a very difficult task in experiments. Moreover, that proposal involves the decoherence channels during the time evolution of the system, which impairs its performance. The scheme in Ref. [12] relies on fractional STIRAP techniques, which have quite different features from STIRAP techniques. The distinct feature of STIRAP is the robustness of its control with respect to the precise tuning of pulse delay, pulse widths, pulse shapes, and detunings, while fractional STIRAP requires a precise ratio of pulse endings, thus impairs its performance if two different pulses are used. Compared with the general STIRAP process,[24] which is utilized to prevent atomic excited states from being involved in the process, we use the present adiabatic Raman transfer to keep the cavity field from exciting. Comparing

the two mechanisms for generating atomic entangled states given in this work, we note that though the STIRAP based mechanism is more robust against cavity decay and atomic spontaneous emission than Raman transitions based mechanism, it needs to individually address the two atoms, which may require more advanced technologies in cavity QED experiments.

To conclude, we have presented a protocol for the generation of atomic entangled states in a cavity QED system. It is based on Raman transitions or adiabatic Raman passages between two systems to entangle the ground states of two three-state Λ-type atoms trapped in a single mode cavity. This scheme needs neither the measurements on cavity field nor atomic detection. Because the atomic excited states and cavity field excitations are never involved in this proposal, our scheme is insensitive to both cavity decay and atomic spontaneous emission. Experimentally, this protocol can be realized with the presently available technology in cavity QED.

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