A lattice traffic model with consideration of preceding mixture traffic information*

Li Zhi-Peng(李志鹏)[a)], Liu Fu-Qiang(刘富强)[a)], Sun Jian(孙 剑)[b)†]

[a)] *School of Electronics and Information Engineering, Tongji University, Shanghai* 200092, *China*
[b)] *School of Transportation Engineering, Tongji University, Shanghai* 200092, *China*

(Received 24 September 2010; revised manuscript received 15 March 2011)

In this paper, the lattice model is presented, incorporating not only site information about preceding cars but also relative currents in front. We derive the stability condition of the extended model by considering a small perturbation around the homogeneous flow solution and find that the improvement in the stability of traffic flow is obtained by taking into account preceding mixture traffic information. Direct simulations also confirm that the traffic jam can be suppressed efficiently by considering the relative currents ahead, just like incorporating site information in front. Moreover, from the nonlinear analysis of the extended models, the preceding mixture traffic information dependence of the propagating kink solutions for traffic jams is obtained by deriving the modified KdV equation near the critical point using the reductive perturbation method.

Keywords: traffic flow, lattice model, mixture traffic information, numerical simulation

PACS: 89.40.–a, 64.60.Cn, 02.60.Cb **DOI:** 10.1088/1674-1056/20/8/088901

1. Introduction

Due to the great importance of efficient traffic in modern countries, traffic problems have attracted considerable attention over recent decades. While earlier studies were mostly conducted by traffic engineers, recently, traffic flow problems have been extensively studied from a physical point of view with very gratifying results.[1−11] There are essentially five different types of approach to describe the characteristics of traffic flow, namely, car-following models, cellular automaton models, lattice models, gas-kinetic models, and fluid-dynamic models. The first three types use a microscopic approach while the last type uses a macroscopic one. The approach used in gas-kinetic models is intermediate and may be called mesoscopic.

It is well known that traffic jams occur and propagate as density waves when car density is high on a freeway. In the past, the traffic jam has been investigated using the perturbation method. Kerner and Konhaüser[12] have found the single-pulse density wave in the numerical simulation with the hydrodynamic traffic models. Kurtze and Hong[13] have proved that the single-pulse density wave is a soliton. Komatsu and Sasa[14] derived the modified Korteweg-de Vries (mKdV) equation from the optimal velocity (OV) model to describe traffic jams in terms of a kink-density wave.

In 1998, Nagatani[15] advised the continuum models describing the jamming transition in traffic flow on a freeway, and proposed two simplified versions of the hydrodynamic models. One of them, called Model I, is described as

$$\partial_t \rho + \rho_0 \partial_x(\rho v) = 0, \tag{1}$$

$$\partial_t \rho v = a\rho_0 V(\rho(x+1)) - a\rho v, \tag{2}$$

where ρ_0 is the average density, and a is the sensitivity of a driver; $\rho(x+1)$ is the local density at position $x+1$ at time t. The idea is that the variation in traffic current ρv at position x is determined by the difference between optimal current $\rho_0 V(\rho(x+1))$ at position $x+1$ and actual current ρv at position x. The other is the lattice version of Model I with dimensionless space x,

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0, \tag{3}$$

$$\partial_t \rho_j v_j = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j, \tag{4}$$

where j represents site j on the one-dimensional lattice and $\rho_j(t)$ and $v_j(t)$ indicate the density and velocity, respectively, on site j at time t.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 60904068, 10902076, 11072117, and 61004113).

†Corresponding author. E-mail: sunjian@tongji.edu.cn

© 2011 Chinese Physical Society and IOP Publishing Ltd

From the viewpoint of economy, the most important problem is to maximize the throughput of cars on highways and prevent traffic congestion.[16] Some work has been conducted to suppress traffic jam by use of intelligent transport system facilities, which enable running cars to receive motion information about other cars on the road. Nagatani[17] examined an extended optimal velocity model by introducing the next-nearest-neighbour intersection in modeling. Similarly, Sawada discussed the differential–difference equation, considering not only the headway of each car but also that of the nearest preceding one. Xue[18] also extended the optimal velocity model to incorporate the relative velocity of each car and verify the new consideration that can improve the stability of a traffic jam.[18] In 2005, Ge[19] advised an extended lattice traffic flow model that introduced an arbitrary number of sites ahead on a single-lane freeway. The result of linear stability analysis showed that considering more than one site ahead can lead to stabilization of the traffic system.

But, are there other regulations for incorporating the information of many cars ahead? Here we concentrate our attention on such a direction. We are interested in the enhancement and stabilization of traffic flow with more related information about interactions in a single lane. In particular, the relative currents of many cars ahead may also have an important effect on the traffic flow. It may be possible to prevent the traffic jam and enhance the flux of cars on a freeway by use of this traffic information. To our knowledge, until now, the lattice model with the consideration of relative currents preceding has not been investigated, and we do not know whether or not the relative currents of many cars ahead can affect the traffic flow effectively.

In this paper, we propose a difference–difference version of the discrete lattice model with the consideration of mixture traffic information of many cars preceding. We apply linear stability theory and nonlinear analysis to the generalized models and derive the modified KdV equation near the critical point by means of the perturbation method. We analyse the effect of the mixture traffic information in front on the traffic stability and jamming transition, and investigate whether or not more than one relative current can enhance the traffic flux by simulation.

In the following section, the extended lattice model is presented by taking into account not only the site information of preceding cars but also the relative currents of an arbitrary number of cars in front to describe the traffic dynamics. In Section 3, the stability condition of traffic flow is investigated in an analytical way. The mixture traffic information dependence of the kink solution for the traffic jam is obtained from the nonlinear analysis in Section 4. To demonstrate the quantitative validity of our analysis, we will compare it with the result of our simulation in Section 5. Section 6 gives the conclusion.

2. Models

We can further change the form of the lattice models (3) and (4). A difference–difference equation in which both space and time are discrete variables can be obtained as follows,

$$\rho_j(t+2\tau)-\rho_j(t+\tau)+\tau\rho_0[\rho_j(t)v_j(t) - \rho_{j-1}(t)v_{j-1}(t)]=0, \tag{5}$$

$$\rho_j\,(t+\tau)\,v_j\,(t+\tau)=\rho_0 V(\rho_{j+1}(t)), \tag{6}$$

where τ is introduced to denote the delay time that it takes for the traffic current to reach the optimal current and τ is the inverse of the sensitivity a.

From Eqs. (5) and (6), the lattice models described by the difference–difference equation, which are named as Model II, can be classified as microscopic approaches pertaining to single lane traffic with no passing, and the idea is that a driver adjusts the car velocity according to the observed headway $h(x,t)$ [or density ahead $\rho_{j+1}(t)$]. It preferably reflects the traffic characteristic that the drivers of cars react in some specific fashion to a stimulus from the cars ahead. Moreover, the new lattice traffic theory can be easily programmed and simulated on a computer.

In accordance with the idea mentioned in the introduction, on the basis of model II, taking not only the site information of cars ahead but also the relative currents of many preceding cars into account, we obtain a more systematic model, given as

$$\rho_j(t+2\tau)-\rho_j(t+\tau) + \tau\rho_0(\rho_j v_j-\rho_{j-1}v_{j-1})=0, \tag{7}$$

$$\begin{aligned}&\rho_j\,(t+\tau)\,v_j\,(t+\tau)\\ &=\rho_0 V(\rho_{j+1},\rho_{j+2},\rho_{j+3},\ldots,\rho_{j+1})\\ &\quad+\sum_{l=1}^{n}\alpha_l(\rho_{j+l}v_{j+l}-\rho_{j+l-1}v_{j+l-1}),\end{aligned} \tag{8}$$

where n denotes the number of the sites and the relative currents of preceding cars; α_l is the weighted function of relative flux $\Delta Q=(\rho_{j+l}v_{j+l}-\rho_{j+l-1}v_{j+l-1})$. It is well known that the influence of the cars in

front on the car motion decreases gradually as the distance between the considered car and the car ahead increases. So in this paper the weighted function is selected tentatively as $\alpha_l = (1/3)^l$. $V(\cdot)$ is the optimal velocity function, which is a monotonically decreasing function, and it has an upper bound.[15] Calibration of the optimal velocity in city traffic was carried out by Helbing and Tilch[20] to accord to the empirical data. They suggested the form of equation as

$$V(\rho_j(t)) = V_1 + V_2 \tanh\left[C_1\left(\frac{1}{\rho_j(t)} - l_c\right) - C_2\right], \quad (9)$$

where l_c is the length of the vehicles, which can be taken as 5 m in simulations. The resulting optimal parameter values are $k = 0.85\ \mathrm{s}^{-1}$, $V_1 = 6.75$ m/s, $V_2 = 7.91$ m/s, $C_1 = 0.13\ \mathrm{m}^{-1}$, and $C_2 = 1.57$. We extended this function as

$$\begin{aligned} &V(\rho_{j+1}, \rho_{j+2}, \ldots, \rho_{j+n}) \\ &= V_1 + V_2 \tanh\left[C_1\left(\frac{1}{\sum_{l=1}^{n} \beta_l \rho_{j+l}(t)} - l_c\right) - C_2\right]. \end{aligned} \quad (10)$$

Here, β_l is the weighting function to $\rho_{j+l}(t)$, which had been noted by Ge[19] that it has two properties as follows: (i) β_l decreases monotonically with increasing l just like α_l. (ii) $\sum_{l=1}^{n} \beta_l = 1$.

Equation (7) is the lattice version of a continuity equation, while equation (8) is the evolution equation. By eliminating the velocity in Eqs. (7) and (8), we obtain the density equation for the extended model,

$$\begin{aligned} &\rho_j(t+2\tau) - \rho_j(t+\tau) \\ &+ \tau\rho_0^2\left[V\left(\sum_{l=1}^{n} \beta_l \rho_{j+l}(t)\right) - V\left(\sum_{l=1}^{n} \beta_l \rho_{j+l-1}(t)\right)\right] \\ &- \tau \sum_{l=1}^{n} \alpha_l \left[\Delta\rho_{j+l}(t+\tau) - \Delta\rho_{j+l}(t)\right] = 0, \end{aligned} \quad (11)$$

where $\Delta\rho_{j+1} = \rho_{j+1} - \rho_j$.

3. Linear stability analysis

To explore the impact of the forward traffic information dependence on the traffic flow, we conduct the linear stability analysis for the extended traffic model. It is obvious that the homogeneous traffic flow is defined by such a state that all cars run with a constant density ρ_0 and a constant velocity $V(\rho_0, \rho_0, \ldots, \rho_0)$. The solution of the homogeneous state is given by

$$\rho_j(t) = \rho_0; \quad v_j(t) = V(\rho_0, \rho_0, \ldots, \rho_0). \quad (12)$$

To see whether the homogeneous solution is stable or not, we add a small disturbance $y_j(t)$, so we have

$$\rho_j(t) = \rho_0 + y_j(t). \quad (13)$$

Substituting Eq. (13) into Eq. (11) and linearizing them yield

$$\begin{aligned} &y_j(t+2\tau) - y_j(t+\tau) \\ &+ \rho_0^2 \tau V'(\rho_0)\left[\sum_{l=1}^{n} \beta_l (y_{j+l}(t) - y_{j+l-1}(t))\right] \\ &- \tau \sum_{l=1}^{n} \alpha_l \left(\Delta y_{j+l}(t+\tau) - \Delta y_{j+l}(t)\right) = 0, \end{aligned} \quad (14)$$

where

$$V'(\rho_0) = \left.\frac{\mathrm{d}V(\rho_j)}{\mathrm{d}\rho}\right|_{\rho_j=\rho_0}.$$

By taking $y_j(t) = \exp(\mathrm{i}kj + zt)$, the following equations of z are derived

$$\begin{aligned} &\exp(2z\tau) - \exp(z\tau) + \tau\rho_0^2 V'(\rho_0) \\ &\times \left[\sum_{l=1}^{n} \beta_l (\exp(\mathrm{i}kl) - \exp(\mathrm{i}k(l-1))\right] \\ &- \tau k (\exp(z\tau) - 1) \\ &\times \sum_{l=1}^{n} \alpha_l \left[\exp(\mathrm{i}kl) - \exp(\mathrm{i}k(l-1))\right] = 0. \end{aligned} \quad (15)$$

For simplicity, $V'(\rho_0)$ is written as V' in the above equation and hereafter. By expanding $z = z_1(\mathrm{i}k) + z_2(\mathrm{i}k)^2 + \cdots$ and inserting it into Eq. (15), the first- and second-order terms of the coefficient in the expression of $\mathrm{i}k$ are obtained,

$$z_1 = -\rho_0^2 V', \quad (16)$$

$$\begin{aligned} z_2 = &-\frac{3}{2}\tau\left(\rho_0^2 V'\right)^2 - \tau\rho_0^2 V' \sum_{l=1}^{n} \alpha_l \\ &- \rho_0^2 V' \sum_{l=1}^{n} \beta_l \left(l - \frac{1}{2}\right). \end{aligned} \quad (17)$$

If z_2 is a negative value, the uniformly steady-state flow becomes unstable for long-wavelength modes, while the uniform flow is stable when z_2 is a positive value. Thus the neutral stability criterion for this steady state is given by

$$\tau = -\frac{\sum_{l=1}^{n} \beta_l (2l-1)}{3\rho_0^2 V' + 2\sum_{l=1}^{n} \alpha_l}. \quad (18)$$

For small disturbances with long wavelengths, the homogeneous traffic flow is unstable in the condition that

$$\tau > -\frac{\sum_{l=1}^{n} \beta_l (2l-1)}{3\rho_0^2 V' + 2\sum_{l=1}^{n} \alpha_l}. \quad (19)$$

Comparing the result with that from Ge's models,[19] we can conclude that the traffic flow in the extended model is stabilized in the region

$$-\left(3\rho_0^2V' + 2\sum_{l=1}^{n}\alpha_l\right)\Big/\sum_{l=1}^{n}\beta_l(2l-1) < a < -3\rho_0^2V'\Big/\sum_{l=1}^{n}\beta_l(2l-1)$$

due to the effect of the relative currents of many cars ahead, which means that the dependence of traffic flow on the preceding mixture traffic information becomes more stable than that without taking into account the relative currents of cars in front in the evolution equation. Furthermore, the neutral stability lines in the parameter space (ρ, a) are shown in Fig. 1 for the model newly considered. The peak of each curve indicates the critical point ($\rho_{\rm c}$, $a_{\rm c}$). The traffic flow is stable above the neutral stability line and no traffic congestion will happen, while the traffic flow below the line is unstable and congestion waves emerge. In Fig. 1, we only consider the effect of the forward relative currents' dependence on traffic flow (i.e. $\beta_l = 0$ $(l \neq 1)$) because the effect of sites information of many cars had been investigated and verified in Ge's model.[21] From Fig. 1, it can be observed that neutral stability curves become lower with the increase of n. This means that the more we consider the relative currents of cars ahead, the more stable the traffic flow will be. The participation of the relative currents of many cars ahead indeed enhances the stability of the traffic flow and the congestion can be suppressed efficiently.

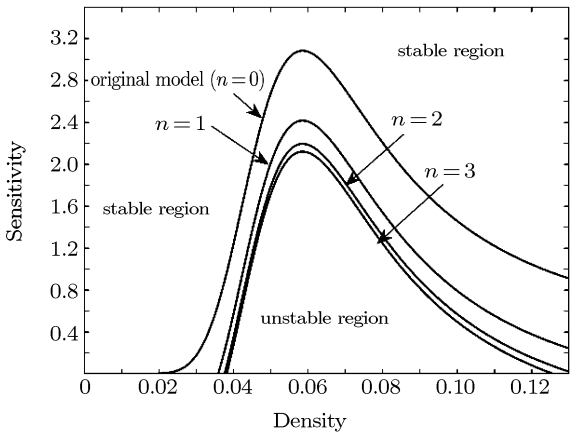


Fig. 1. The phase diagram in the density-sensitivity for $n = 0, 1, 2, 3$ ($\beta_l = 0$ $(l \neq 1)$).

4. Nonlinear analysis

Car density could fluctuate for various reasons and form a density wave in traffic flow, which leads to traffic jams. As indicated in the above section, the kink density wave appears when the car density is higher than the critical one. We now consider the extended lattice model in traffic flow on coarse-grained scales, and apply the reductive perturbation method to Eq. (11), focusing on the system behaviour near the critical point ($\rho_{\rm c}$, $a_{\rm c}$). We will consider the slowly varying behaviour in the unstable region with the help of a small positive scaling parameter ε.

In order to extract slow scales for the space variable j and the time variable t, here we define the slow variables X and T for $0 < \varepsilon < 1$ as follows,

$$X = \varepsilon(j + bt), \quad T = \varepsilon^3 t, \tag{20}$$

where b is a constant to be determined. Let

$$\rho_j = \rho_{\rm c} + \varepsilon R(X, T). \tag{21}$$

Substituting Eqs. (20) and (21) into Eq. (11) and making the Taylor expansions to the fifth order of ε, we can obtain the following nonlinear partial differential equation,

$$\begin{aligned}
&\varepsilon^2\left(b+\rho_{\rm c}^2V'\right)\partial_X R + \varepsilon^3\left(\frac{3}{2}b^2\tau + \frac{\rho_{\rm c}^2V'}{2}\sum_{l=1}^{n}\beta_l(2l-1) - \tau b\sum_{l=1}^{n}\alpha_l\right)\alpha_X^2 R \\
&+ \varepsilon^4\left\{\partial_T R + \left[\frac{7b^3\tau^2}{6} + \frac{\rho_{\rm c}^2V'}{6}\sum_{l=1}^{n}\beta_l(3l^2-3l+1) - \frac{\tau}{6}\sum_{l=1}^{n}3\alpha_l\,(2l-1+b\tau)b\right]\partial_X^3 R + \frac{\rho_{\rm c}^2V'''}{6}\partial_X R^3\right\} \\
&+ \varepsilon^5\left\{3b\tau\partial_T\partial_X R + \left[\frac{5b^4\tau^3}{8} + \frac{\rho_{\rm c}^2V'}{24}\sum_{l=1}^{n}\beta_l(4l^3-6l^2+4l-1)\right.\right. \\
&\left.\left. - \frac{\tau}{24}\sum_{l=1}^{n}\alpha_l(4b^3\tau^2 + 6b^2\tau(2l-1) + 4b(3l^2-3l+1))\right]\partial_X^4 R + \frac{\rho_{\rm c}^2V'''}{12}\partial_X^2 R^3\right\} = 0,
\end{aligned} \tag{22}$$

where

$$\partial_T = \frac{\partial}{\partial_T}, \quad \partial_X \partial_T = \frac{\partial^2}{\partial_X \partial_T}, \quad V' = \left.\frac{\mathrm{d}V(\rho_j)}{\mathrm{d}\rho_j}\right|_{\rho_j=\rho_\mathrm{c}}, \quad \text{and} \quad V''' = \left.\frac{\mathrm{d}^3 V(\rho_j)}{\mathrm{d}\rho_j^3}\right|_{\rho_j=\rho_\mathrm{c}}.$$

Near the critical point, we have $\tau = (1+\varepsilon^2)\tau_\mathrm{c}$; by taking $b = -\rho_\mathrm{c}^2 V'$, and eliminating the second- and third-order terms of ε, we can obtain a simplified equation

$$\begin{aligned}
\varepsilon^4 \Bigg\{ & \partial_T R + \left[\frac{7b^3\tau_\mathrm{c}^2}{6} + \frac{\rho_\mathrm{c}^2 V'}{6} \sum_{l=1}^{n} \beta_l (3l^2 - 3l + 1) - \frac{\tau_\mathrm{c}}{6} \sum_{l=1}^{n} 3\alpha_l \left(2l - 1 + b\tau_\mathrm{c}\right) b \right] \partial_X^3 R + \frac{\rho_\mathrm{c}^2 V'''}{6} \partial_X R^3 \Bigg\} \\
& + \varepsilon^5 \Bigg\{ \frac{3b^2\tau_\mathrm{c}}{2} \partial_X^2 R + \Bigg[-\frac{23b^4\tau_\mathrm{c}^3}{8} + \frac{\rho_\mathrm{c}^2 V'}{24} \sum_{l=1}^{n} \beta_l (4l^3 - 6l^2 + 4l - 1) - \frac{\rho_\mathrm{c}^2 V' b\tau_\mathrm{c}}{2} \sum_{l=1}^{n} \beta_l (3l^2 - 3l + 1) \\
& + \frac{b\tau_\mathrm{c}^2}{2} \sum_{l=1}^{n} 3\alpha_l \left(1 + b\tau_\mathrm{c}\right) b - \frac{\tau_\mathrm{c}}{24} \sum_{l=1}^{n} \alpha_l (4b^3\tau_\mathrm{c}^2 + 6b^2\tau_\mathrm{c}(2l-1) + 4b(3l^2 - 3l + 1)) \Bigg] \partial_X^4 R \\
& + \left(\frac{\rho_\mathrm{c}^2 V'''}{12} - \frac{\rho_\mathrm{c}^2 V''' b\tau_\mathrm{c}}{2} \right) \partial_X^2 R^3 \Bigg\} = 0. \qquad (23)
\end{aligned}$$

Replacing the coefficients of $\partial_X^3 R$, $\partial_X R^3$, $\partial_X^2 R$, $\partial_X^4 R$, $\partial_X^2 R^3$ by $-g_1$, g_2, g_3, g_4, g_5 respectively, equation (23) can be rewritten as

$$\varepsilon^4 \left[\partial_T R - g_1 \partial_X^3 R + g_2 \partial_X R^3 \right] + \varepsilon^5 \left[g_4 \partial_X^4 R + g_3 \partial_X^2 R + g_5 \partial_X^2 R^3 \right] = 0. \qquad (24)$$

In order to derive the regularized equation, we make the following transformations,

$$\begin{aligned}
T' &= -\left[\frac{7b^3\tau_\mathrm{c}^2}{6} + \frac{\rho_\mathrm{c}^2 V'}{6} \sum_{l=1}^{n} \beta_l (3l^2 - 3l + 1) - \frac{\tau_\mathrm{c}}{2} \sum_{l=1}^{n} 3\alpha_l \left(2l - 1 + b\tau_\mathrm{c}\right) b \right] T, \\
R' &= -\left[\frac{7b^3\tau_\mathrm{c}^2}{\rho_\mathrm{c} V'''} + \frac{\rho_\mathrm{c}^2 V'}{\rho_\mathrm{c} V'''} \sum_{l=1}^{n} \beta_l (3l^2 - 3l + 1) - \frac{\tau_\mathrm{c} \sum_{l=1}^{n} 3\alpha_l \left(2l - 1 + b\tau_\mathrm{c}\right) b}{\rho_\mathrm{c} V'''} \right]^{1/2} R.
\end{aligned}$$

Thus, we obtain the regularized mKdV equation with a perturbed term

$$\partial_{T'} R' = \partial_X^3 R' - \partial_X {R'}^3 - \varepsilon M\left[R'\right], \qquad (25)$$

where

$$M\left[R'\right] = \frac{1}{g_1} \left[g_3 \partial_X^2 R' + g_4 \partial_X^4 R' + \frac{g_1 g_5}{g_2} \partial_X^2 {R'}^3 \right].$$

If we ignore $O(\varepsilon)$ terms in Eq. (25), the mKdV equation with a kink solution is obtained as the desired solution,

$${R'}_0 (X, T') = \sqrt{c} \tanh \sqrt{\frac{c}{2}} \left(X - cT'\right). \qquad (26)$$

Next, assuming

$$R'(X, T') = {R'}_0(X, T') + \varepsilon\, {R'}_1(X, T'),$$

we take into account the $O(\varepsilon)$ correction. In order to determine the selected value of the propagation velocity c for the kink solution, it is necessary to satisfy the solvability condition,

$$({R'}_0, M[{R'}_0]) \equiv \int_{-\infty}^{+\infty} \mathrm{d}X {R'}_0 M[{R'}_0] = 0, \qquad (27)$$

where $M[{R'}_0] = M[R']$.

Performing the integration, we obtain the selected velocities according to Ref. [22]

$$c = \frac{5 g_2 g_3}{2 g_2 g_4 - 3 g_1 g_5}. \qquad (28)$$

Hence we obtain the kink–antikink soliton,

$$\begin{aligned}
R(X, T) = & -\Bigg[\Bigg(\frac{7b^3\tau_\mathrm{c}^2}{\rho_\mathrm{c} V'''} + \frac{\rho_\mathrm{c}^2 V'}{\rho_\mathrm{c} V'''} \sum_{l=1}^{n} \beta_l (3l^2 - 3l + 1) \\
& - \frac{\tau_\mathrm{c} \sum_{l=1}^{n} 3\alpha_l \left(2l - 1 + b\tau_\mathrm{c}\right) b}{\rho_\mathrm{c} V'''} \Bigg) c \Bigg]^{1/2} \\
& \times \tanh \sqrt{\frac{c}{2}} \Bigg(X + \Bigg[\frac{7b^3\tau_\mathrm{c}^2}{6} + \frac{\rho_\mathrm{c}^2 V'}{6} \\
& \times \sum_{l=1}^{n} \beta_l (3l^2 - 3l + 1)
\end{aligned}$$

$$-\frac{\tau_{\rm c}\sum_{l=1}^{n}\alpha_l\,(2l-1+b\tau_{\rm c})\,b}{2}\Big]cT'\Big). \quad (29)$$

And the kink solution of the headway is

$$\rho_j=\rho_{\rm c}\pm\sqrt{\frac{\tau}{\tau_{\rm c}}-1}R(X,T). \quad (30)$$

The mKdV equation is derived near the critical point, and the kink wave appears as the density wave in the unstable region, which is described by the solution of the mKdV equation.

5. Simulation

The new extended traffic flow theory for cooperative driving control can be easily programmed and simulated on a computer. While the linear stability analysis in Section 4 tells us that incorporating the relative currents of many preceding cars will further enhance the stability of traffic flow, traffic simulations allow us to study the interesting character of this new theory. We shall conduct the simulation on a ring road, which simplifies the analysis a great deal because we do not have to specify boundary conditions.

Numerical simulations are carried out as follows. Consider a line of cars running on a ring road according to our extended traffic flow model. The initial disturbance is chosen as follows,

$$x_j(0)=\Delta x_j(1)=1/\rho_0=\Delta x,\quad (j\neq 50,51), \quad (31)$$

$$x_j(0)=\Delta x_j(1)=\Delta x-1,\quad (j=50), \quad (32)$$

$$x_j(0)=\Delta x_j(1)=\Delta x+1,\quad (j=51), \quad (33)$$

where the total number of cars is 100.

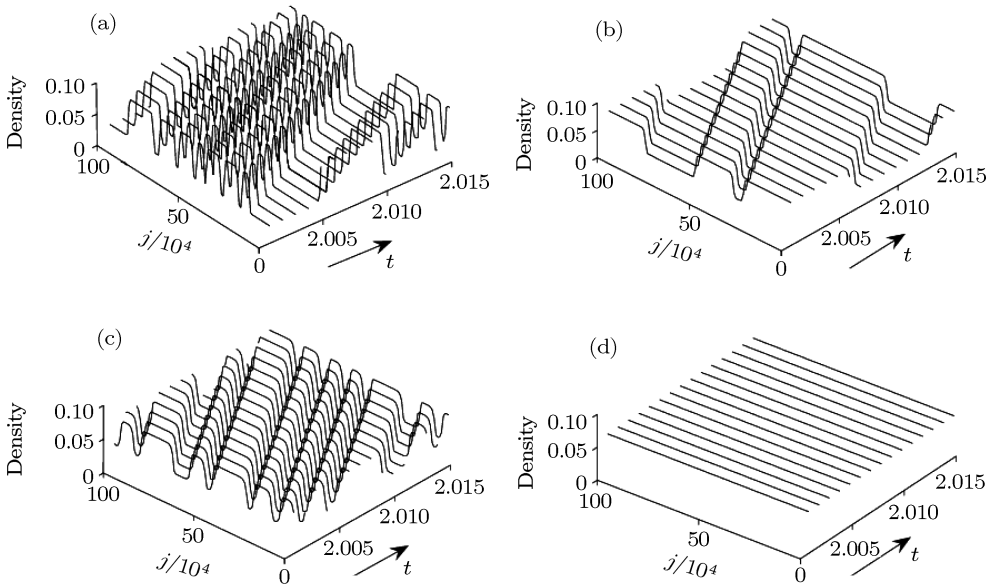


Fig. 2. The spatio-temporal evolution of the headway for (a) $n=0$, (b) $n=1$, (c) $n=2$, and (d) $n=3$ ($a=2.25$, $\beta_l=0$ ($l\neq 1$)).

Figure 2 shows the typical traffic patterns after sufficiently long time steps $t=2\times10^4$. The spatiotemporal evolution of the density for various value of n has different properties. The patterns (a), (b), (c), and (d) show the time evolutions of the density (or headway) profile according to the relative current dependence of traffic flow model for $n=0,1,2,3$ with $a=2.25$ and $\beta_l=0$ ($l\neq 1$). The pattern (a) with $n=0$ corresponds to that of the original lattice model by Nagatani. In patterns (a), (b), and (c), the traffic flow state transition from free flow to congestion traffic is observed, and the congestions propagate backward as the kink–antikink density waves. Figure 3 shows the headway profile obtained at a sufficiently large time $t=2\times10^4$. With increasing number n of the relative currents of the cars ahead considered, the amplitude of the density wave is weakened, initial small disturbances decay, and the initial traffic flow with a non-uniform density profile evolves to a uniform traffic flow, as shown in pattern (d) if we set $n=3$. Therefore, all the results show that the effect of the relative currents of many cars preceding can also stabilize the traffic flow.

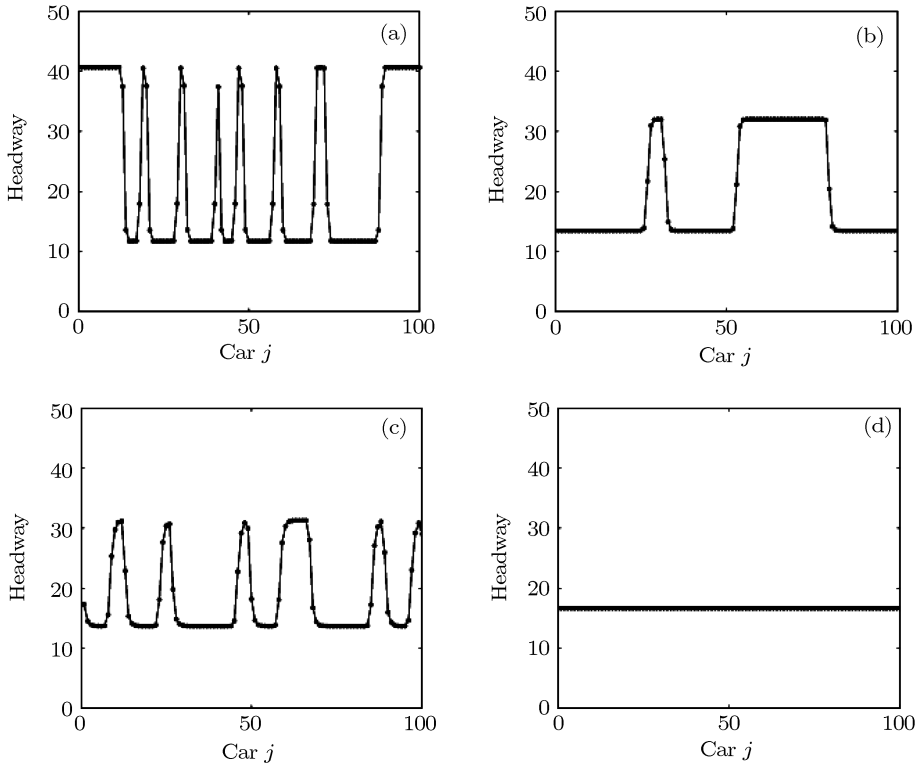


Fig. 3. The snapshots of headway configuration of all cars for (a) $n = 0$, (b) $n = 1$, (c) $n = 2$, and (d) $n = 3$ ($a = 2.25$, $\beta_l = 0$ ($l \neq 1$)), $t = 2 \times 10^4$.

6. Summary

We have proposed an extended lattice model of traffic flow for the purpose of maximizing the throughput of cars on highways and suppressing traffic congestion; and we also give a form of the microscopic model that takes the mixture traffic information of many preceding cars into account. The nature of traffic has been analysed by using linear stability theory, and we find that the new introduction of the forward interactions in relative currents can further improve the stability of the traffic flow, which has been verified by simulations. In addition, from the nonlinear analysis, the forward relative currents' dependence on the propagating kink solution for traffic jam was obtained.

References

[1] Li X L, Kuang H, Song T, Dai S Q and Li Z P 2008 *Chin. Phys.* B **17** 2366
[2] Kerner B S and Rehborn H 1996 *Phys. Rev.* E **53** R4275
[3] Li L and Shi P F 2005 *Chin. Phys.* **14** 576
[4] Peng G H and Sun D H 2009 *Chin. Phys.* B **18** 524
[5] Li Z P, Li X L and Liu F Q 2008 *Int. J. Mod. Phys.* **19** 1163
[6] Ge H X, Zhu H B and Dai S Q 2005 *Acta Phys. Sin.* **54** 4621 (in Chinese)
[7] Jiang R, Wu Q S and Zhu Z J 2001 *Phys. Rev.* E **64** 017101
[8] Tang T Q, Huang H J and Shang H Y 2010 *Chin. Phys.* B **19** 050517
[9] Nagatani T and Nakanishi K 1998 *Phys. Rev.* E **57** 6415
[10] Zhao X and Zhang Z 2005 *Eur. Phys. J.* B **47** 1445
[11] Tian J F, Jia B, Li X G and Gao Z Y 2010 *Chin. Phys.* B **19** 010511
[12] Kerner B S and Konhäuser P 1993 *Phys. Rev.* E **48** 2335
[13] Kurtze D A and Hong D C 1995 *Phys. Rev.* E **52** 218
[14] Komatsu T and Sasa S 1995 *Phys. Rev.* E **52** 5574
[15] Nagatani T 1998 *Physica* A **261** 599
[16] Helbing D and Huberman B A 1998 *Nature* **396** 738
[17] Nagatani T 1999 *Phys. Rev.* E **60** 6395
[18] Xue Y 2002 *Chin. Phys.* **11** 1128
[19] Ge H X, Dai S Q and Dong L Y 2004 *Phys. Rev.* E **70** 066134
[20] Helbing D and Tilch B 1998 *Phys. Rev.* E **58** 133
[21] Ge H X, Dai S Q, Xue X and Dong L Y 2005 *Phys. Rev.* E **71** 066119
[22] Ge H X, Cheng J R and Dai S Q 2005 *Physica* A **357** 466