Dynamic models of pest propagation and pest control*

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This paper proposes a pest propagation model to investigate the evolution behaviours of pest aggregates. A pest aggregate grows by self-monomer birth, and it may fragment into two smaller ones. The kinetic evolution behaviours of pest aggregates are investigated by the rate equation approach based on the mean-field theory. For a system with a self-birth rate kernel $I(k) = Ik$ and a fragmentation rate kernel $L(i, j) = L$, we find that the total number $M_0^A(t)$ and the total mass of the pest aggregates $M_1^A(t)$ both increase exponentially with time if $L \neq 0$. Furthermore, we introduce two catalysis-driven monomer death mechanisms for the former pest propagation model to study the evolution behaviours of pest aggregates under pesticide and natural enemy controlled pest propagation. In the pesticide controlled model with a catalyzed monomer death rate kernel $J_1(k) = J_1 k$, it is found that only when $I < J_1 B_0$ (B_0 is the concentration of catalyst aggregates) can the pests be killed off. Otherwise, the pest aggregates can survive. In the model of pest control with a natural enemy, a pest aggregate loses one of its individuals and the number of natural enemies increases by one. For this system, we find that no matter how many natural enemies there are at the beginning, pests will be eliminated by them eventually.

Keywords: kinetic evolution behaviour, pest propagation, pest control, scaling law

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1. Introduction

The aggregate growth phenomenon occurs very widely in various fields, such as physics,[1−4] chemistry,[5−8] biology,[9−11] demography,[12] and network growth.[13−16] In the last few decades, many theoretical studies have been devoted to kinetic evolution behaviours of various aggregation processes with different microscopic aggregation mechanisms on the background of the aggregation processes in natural systems.[17−29] Recently, much attention has been devoted to generalized aggregation phenomena in economic and social systems, and various specific microscopic mechanisms have been proposed for the analysis of economic and social activities.[30,31] Fu *et al.* introduced a model of proportional growth to explain the distribution of business-firm growth rates.[32] Plerou *et al.* quantified university research activities, and found the similarities between the growth dynamics of university research and those of competitive economic activities.[33] Pammolli *et al.* and Buldyrev *et al.* introduced a model of proportional growth to explain both the central part and the tails of growth distributions at different levels of aggregation in economic systems.[34,35] Ispolatov, Krapivsky, and Redner proposed random exchange, greedy exchange, and very greedy exchange models to describe the evolution behaviour of wealth distribution in an economically interacting population.[36] Leyvraz and Redner investigated a migration-driven or exchange-driven irreversible aggregate growth model to describe the evolution behaviour of city populations.[37] In these models, irreversible growth of aggregates takes place through biased migration or unbiased exchange mechanisms. In a biased exchange model, one monomer will migrate from a smaller aggregate to a larger one, while in an unbiased exchange model, an aggregate is equally likely to gain or to lose a monomer. After these investigations, Lin *et al.* made a more systematic study of the unbiased exchanged processes,[38,39] and they further proposed an unbiased exchange-driven aggregation model with mutually catalyzed-birth and catalyzed-death mechanisms between aggregates of different species to study the competition roles of the catalyzed-birth and catalyzed-death in the self-exchange driven aggregation growth process on the kinetic evolution behaviours of the aggregates of a two-species system.[40,41] Ke *et al.* also studied the kinetics

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of migration-driven aggregate growth on completely connected scale-free networks.[42] Recently, Ke *et al.* proposed a controlled particle deposition model for cluster growth on the substrate surface and then presented exact results for the cluster size distribution.[28] Furthermore, Ben-Naim *et al.* studied models for the development of disparate social classes in a competitive population, where individuals advance their fitness by competing against those in lower classes, and in parallel, individuals decline due to inactivity.[43,44]

Moreover, researchers have carried out many studies in terms of infection. For example, Yang *et al.* proposed a two-species infection model to study the dynamic behaviour of germ infection.[45] Chen *et al.* proposed two irreversible aggregation growth models to study the interactions between virus aggregates and medicine efficacy aggregates in the virus-medicine cooperative evolution system.[46] Motivated by their studies, we investigate the dynamics of pest propagation and control. In the natural world, some pests will emigrate to another new area where there is abundant food and space for them. Meanwhile they propagate rapidly, generating tremendous damage to agricultural production. We call this phenomenon pest propagation. Generally, to keep pests from propagating themselves rapidly, people often take many pest control measures to prevent pest propagation. For example, they often use pesticide or natural enemies to kill them. In this work, first, we will study the dynamics of pest propagation, and then we will look at the dynamics of pest control with pesticides and natural enemies, respectively.

The rest of this paper is organized as follows. In Section 2, we introduce the pest propagation model, and then discuss the kinetic evolution behaviour of pest aggregates. In Section 3, we investigate two different kinds of pest control measure (pest control with pesticides and with natural enemies), and analyse the kinetic evolution of the pest system. In Section 4, we provide a brief summary.

2. Pest propagation

2.1. The pest propagation model

In this model, we consider a kinetic evolution model of pest aggregates with self-birth and fragmentation schemes. The self-birth process can be described by $A_k \xrightarrow{I(k)} A_{k+1}$ with the rate kernel $I(k)$, and the fragmentation process can be described by $A_{i+j} \xrightarrow{L(i,j)} A_i + A_j$ with the rate kernel $L(i,j)$, where A_k denotes a pest aggregate consisting of k-mers.

Generally, the rate kernels are complicated. Here, we consider a simple practical case, in which the monomer birth rate of a pest aggregate is proportional to its size (namely, the number of its individuals) and an aggregate completely randomly fragments into two smaller ones independent of the sizes of the resultant aggregates. So, the rate kernels can be expressed as $I(k) = Ik$ and $L(i,j) = L$. Here, I and L are two proportional constants ($I > 0$ and $L \geq 0$). $L = 0$ denotes the system without the fragmentation of aggregates. From $L(i,j) = L$, we can see that the bigger the pest aggregates, the easier the fragmentation, that is, the fragmentation rate of the pest aggregate A_k is $L(k-1)/2$.

The investigation into the kinetic behaviours of the system is focused mainly on the evolution of the concentrations of pest aggregates. Here, it is the pest aggregate size distribution, $a_k(t)$, that denotes the concentration or the number of pest aggregates A_k at time t. We assume that the system has spatial homogeneity, i.e., the aggregates of the pest are considered to be homogeneously distributed in space throughout the whole evolution process, so that the fluctuations in the densities of the reactant aggregates of the pest are ignored. Thus, the theoretical approach to investigating the kinetic behaviours of the system can be based on the rate equations in the mean-field frame, which assumes that the reaction proceeds at a rate proportional to the concentrations of reactant aggregate. We can write the rate equations for the concentration $a_k(t)$ of our system as follows,

$$\frac{\mathrm{d}a_k}{\mathrm{d}t} = I(k-1)a_{k-1} - Ika_k + \sum_{i=1}^{\infty} La_{k+i} - \frac{1}{2}\sum_{i+j=k} La_k. \quad (1)$$

In Eq. (1), the first and second terms on the right-hand side account for the gain and loss in $a_k(t)$ due to the self-birth schemes $A_{k-1} \xrightarrow{I(k-1)} A_k$ and $A_k \xrightarrow{I(k)} A_{k+1}$ respectively. The third and fourth terms account for the gain and loss in $a_k(t)$ due to the fragmentation schemes $A_{k+i} \xrightarrow{L(k,i)} A_k + A_i$ and $A_{i+j=k} \xrightarrow{L(i,j)} A_i + A_j$, respectively.

We assume that there exist only monomer aggregates of pests at $t = 0$ (more precisely, at $t = 0^+$).

So, the monodisperse initial conditions can be written as

$$a_k(0) = A_0\delta_{k1}, \tag{2}$$

where A_0 is the concentration (or the total number) of initial monomer aggregates of pests.

Several methods have been developed to solve the rate equations in different aggregation processes. A generating function method is used to solve the simple coagulation process $A_i + A_j \xrightarrow{K(i,j)} A_{i+j}$ for $K(i,j) = 1, i+j$, and ij cases.[6–8] In aggregation–annihilation processes, a special *Ansatz*

$$a_k(t) = a_1(t)[a(t)]^{k-1} \tag{3}$$

and the generating function method are used to solve the rate equations.[24,25] In exchange-driven aggregation processes, the rate equations were solved by making some scaling *Ansatzs* directly[36,37] or with the help of the special *Ansatz* (3).[38–40] Here we find that the rate equation (1) can be solved by the *Ansatz* (3).

Under the *Ansatz* (3), the monodisperse initial condition (2) can be transformed correspondingly as follows,

$$a(0) = 0, \quad a_1(0) = A_0. \tag{4}$$

With the *Ansatz* (3), the first two moments of the pest aggregate size distribution $a_k(t)$ can be expressed as

$$\begin{aligned} M_0^A(t) &= \sum_{k=1}^{\infty} a_k(t) = a_1(t)\sum_{k=1}^{\infty}[a(t)]^{k-1} \\ &= \frac{a_1(t)}{1-a(t)}, \end{aligned} \tag{5}$$

$$\begin{aligned} M_1^A(t) &= \sum_{k=1}^{\infty} k a_k(t) = a_1(t)\sum_{k=1}^{\infty} k[a(t)]^{k-1} \\ &= \frac{a_1(t)}{[1-a(t)]^2}. \end{aligned} \tag{6}$$

Obviously, the first two moments $M_0^A(t)$ and $M_1^A(t)$ are the total number and the total mass of the pest aggregates, respectively.

Substituting the *Ansatz* (3) into Eq. (1), we can obtain the following differential equations with the expression of the total number of pest aggregates $M_0^A(t)$,

$$\frac{\mathrm{d}a}{\mathrm{d}t} = I(1-a) - \frac{1}{2}La, \tag{7}$$

$$\frac{\mathrm{d}a_1}{\mathrm{d}t} = -Ia_1 + LaM_0^A. \tag{8}$$

In the next subsection, we study the kinetic evolution behaviour of this system.

2.2. Kinetic evolution behaviour of pest propagation

From Eq. (7), we can derive the solution for $a(t)$ under the initial condition $a(0) = 0$,

$$a(t) = \frac{I}{\beta}(1 - \mathrm{e}^{-\beta t}), \tag{9}$$

where $\beta = I + L/2$.

Substituting Eqs. (5) and (9) into Eq. (8), we can obtain the solution for $a_1(t)$ under the initial condition $a_1(0) = A_0$,

$$a_1(t) = A_0\,\mathrm{e}^{It}\left(\frac{L}{2\beta} + \frac{I}{\beta}\,\mathrm{e}^{-\beta t}\right)^2. \tag{10}$$

Substituting Eqs. (9) and (10) into the *Ansatz* (3), we can derive the pest aggregate size distribution $a_k(t)$ as

$$\begin{aligned} a_k(t) = {} & A_0\,\mathrm{e}^{It}\left(\frac{L}{2\beta} + \frac{I}{\beta}\,\mathrm{e}^{-\beta t}\right)^2 \\ & \times \left[\frac{I}{\beta}(1 - \mathrm{e}^{-\beta t})\right]^{k-1}. \end{aligned} \tag{11}$$

(i) When $L = 0$, it can be rewritten in the scaling region of $k \gg 1$ and $t \gg 1$ as

$$a_k(t) \simeq A_0\,\mathrm{e}^{-It}\exp\left(-\frac{k}{\mathrm{e}^{It}}\right). \tag{12}$$

Obviously, equation (12) satisfies the generalized scaling form,[41,45] as follows:

$$a_k(t) \simeq [f(t)]^{-w}\Phi\left[\frac{k}{S(t)}\right], \quad S(t) \propto [f(t)]^z \tag{13}$$

with the scaling function $\Phi(x) = \mathrm{e}^{-x}$. The $f(t)$ is a time-dependent function (such as e^t, $\ln t$, and so on), and $S(t)$ is the characteristic size of the aggregates. Here the function $f(t)$ is $f(t) = \mathrm{e}^t$ and the scaling exponents are $w = z = I$. The characteristic aggregate size $S(t) = \mathrm{e}^{It}$.

(ii) When $L \neq 0$, it can be rewritten in the scaling region of $k \gg 1$ and $t \gg 1$ as

$$a_k(t) \simeq A_0\left(\frac{L}{2\beta}\right)^2\left(\frac{I}{\beta}\right)^k \mathrm{e}^{It}\exp\left(-\frac{k}{\mathrm{e}^{\beta t}}\right). \tag{14}$$

Apparently, this satisfies the following modified scaling form:[26,27]

$$a_k(t) \simeq b^k[f(t)]^{-w}\Phi\left[\frac{k}{S(t)}\right], \quad S(t) \propto [f(t)]^z. \tag{15}$$

Here, b is a constant ($0 < b < 1$), the scaling function $\Phi(x) = \mathrm{e}^{-x}$, and the function $f(t)$ is an unusual time-dependent function such as e^t, $\ln t$, and so on. In the present case, $b = I/\beta$, the function $f(t) = \mathrm{e}^t$, and the governing exponents are $w = -I$ and $z = \beta$, the characteristic aggregate size $S(t) = \mathrm{e}^{\beta t}$.

Substituting Eqs. (9) and (10) into Eqs. (5) and (6), we can obtain the total number of pest aggregates $M_0^A(t)$ and the total mass of pest aggregates $M_1^A(t)$, respectively,

$$M_0^A(t) = A_0\,\mathrm{e}^{It}\left(\frac{L}{2\beta} + \frac{I}{\beta}\,\mathrm{e}^{-\beta t}\right), \tag{16}$$

$$M_1^A(t) = A_0\,\mathrm{e}^{It}. \tag{17}$$

From Eq. (16), we can see that the total number of pest aggregates $M_0^A(t)$ is conserved if $L = 0$ and increases as the exponential function e^{It} in a long time if $L \neq 0$. It is unexpected that $M_0^A(t)$ is not dominated by fragmentation, but it is dominated by self-birth after a long time. From Eq. (17), we can see that the total mass of the pest aggregates $M_1^A(t)$ increases exponentially with time.

3. Pest control measures

In reality, pest propagation can damage agricultural production greatly. In order to prevent pests from multiplying, people always use the corresponding pesticide or natural enemies to kill them. In this section, we will investigate the kinetic evolution behaviours of pest control with pesticides and natural enemies, respectively.

3.1. Pest control with pesticides

3.1.1. Model of pest control with pesticides

Pest control with pesticides can be considered a pesticide-driven pest-monomer death process and can be described by a catalysis-driven monomer death scheme $B + A_k \xrightarrow{J_1(k)} A_{k-1}$ with the rate kernel $J_1(k)$, where B denotes one unit of the corresponding medicine and A_k denotes a pest aggregate consisting of k-mers. Therefore, we add the pesticide-driven pest-monomer death scheme to the pest propagation model to investigate the model of pest control with a pesticide.

Here, we consider a simple but important case in which the concentration of catalyst aggregates B is conserved, i.e., $b(t) = B_0 > 0$. And we select a simple and practical rate kernel $J_1(k) = J_1 k$, where J_1 is a positive proportional constant.

We still assume that the system has spatial homogeneity, thus the rate equations for the concentration $a_k(t)$ can be written as

$$\begin{aligned}\frac{\mathrm{d}a_k}{\mathrm{d}t} &= I(k-1)a_{k-1} - Ika_k \\ &\quad + \sum_{i=1}^{\infty} La_{k+i} - \frac{1}{2}\sum_{i+j=k} La_k \\ &\quad + J_1(k+1)B_0 a_{k+1} - J_1 k B_0 a_k. \end{aligned}\tag{18}$$

In Eq. (18), the first four terms on the right-hand side have the same meaning as those in Eq. (1), and the following two terms account for the gain and loss in $a_k(t)$ due to the pesticide-driven monomer death processes $A_{k+1} + B \xrightarrow{J_1(k+1)} A_k$ and $A_k + B \xrightarrow{J_1(k)} A_{k-1}$, respectively.

We still assume that there exist only monomer aggregates of pests at $t = 0$, i.e., $a_k(0) = A_0\delta_{k1}$.

Utilizing the *Ansatz* (3), we find that the rate equation (18) becomes the differential equations for $a(t)$ and $a_1(t)$,

$$\frac{\mathrm{d}a}{\mathrm{d}t} = J_1 B_0\left(a^2 - \frac{I + L/2 + J_1 B_0}{J_1 B_0}a + \frac{I}{J_1 B_0}\right), \tag{19}$$

$$\frac{\mathrm{d}a_1}{\mathrm{d}t} = -Ia_1 + LaM_0^A + 2J_1 B_0 a_1 a - J_1 B_0 a_1. \tag{20}$$

The evolution equation of $M_1^A(t)$ can be derived from Eqs. (19) and (20),

$$\frac{\mathrm{d}\ln M_1^A}{\mathrm{d}t} = \frac{2}{1-a}\frac{\mathrm{d}a}{\mathrm{d}t} + \frac{1}{a_1}\frac{\mathrm{d}a_1}{\mathrm{d}t} = I - J_1 B_0. \tag{21}$$

From Eq. (21), we can obtain the solution of $M_1^A(t)$ under the initial condition $M_1^A(0) = A_0$,

$$M_1^A(t) = A_0\,\mathrm{e}^{(I - J_1 B_0)t}. \tag{22}$$

Next, we will study the kinetic evolution behaviour of pest control with a pesticide.

3.1.2. Kinetic evolution behaviour of pest control with a pesticide

The polynomial, in the brackets on the right-side of Eq. (19), is a quadratic polynomial with variable a. We discuss the evolution behaviours of the

model of pest control with pesticide in the region of $\Delta = (I+L/2+J_1B_0)^2/(J_1B_0)^2 - 4I/(J_1B_0) \geq 0$. The condition $\Delta < 0$ is not possible.

(I) $\Delta = 0$ case

When $\Delta = 0$, we can derive $L = 0$ and $I = J_1B_0$. In this system, there is no fragmentation process.

Substituting $L = 0$ and $I = J_1B_0$ into Eq. (19), we can obtain the solution for $a(t)$ under the initial condition $a(0) = 0$ as

$$a(t) = 1 - \frac{1}{1+It}. \tag{23}$$

Due to $I = J_1B_0$, equation (22) can be rewritten as $M_1^A(t) = A_0$. Substituting Eq. (23) and $M_1^A(t) = A_0$ into Eq. (6), we can obtain the solution for $a_1(t)$ as

$$a_1(t) = A_0(1+It)^{-2}. \tag{24}$$

Substituting Eqs. (23) and (24) into *Ansatz* (3), we can derive the pest aggregate size distribution $a_k(t)$ as

$$a_k(t) = A_0(1+It)^{-2}\left(1 - \frac{1}{1+It}\right)^{k-1}. \tag{25}$$

In the scaling region of $k \gg 1$ and $t \gg 1$, it can be rewritten as

$$a_k(t) \simeq A_0 I^{-2} t^{-2} \exp\left[-\frac{k}{S(t)}\right], \tag{26}$$

where the characteristic aggregate size $S(t) = It$, which grows with time as $S(t) \propto t$. Obviously, equation (26) approaches the generalized scaling form as Eq. (13).

Substituting Eqs. (23) and (24) into Eq. (5), we can obtain the total number of pest aggregates $M_0^A(t)$ as

$$M_0^A(t) = A_0(1+It)^{-1}. \tag{27}$$

In this system, we find that the total number of pest aggregates $M_0^A(t)$ decreases with time as $M_0^A(t) \propto t^{-1}$, and the total mass of pest aggregates $M_1^A(t)$ is conservative. Hence, all pests will concentrate together.

(II) The $\Delta > 0$ case

When $\Delta > 0$, the two roots of the parabola function can be solved as

$$C_1 = \frac{I + L/2 + J_1B_0 - \sqrt{(I+L/2+J_1B_0)^2 - 4IJ_1B_0}}{2J_1B_0},$$
$$C_2 = \frac{I + L/2 + J_1B_0 + \sqrt{(I+L/2+J_1B_0)^2 - 4IJ_1B_0}}{2J_1B_0}. \tag{28}$$

We can see that $0 < C_1 \leq 1$ and $C_2 > C_1$. Substituting the above expressions into Eq. (19), we can derive the solution for $a(t)$ under the initial condition $a(0) = 0$,

$$a(t) = C_1\left[1 - \frac{C_2 - C_1}{C_2\,\mathrm{e}^{J_1B_0(C_2-C_1)t} - C_1}\right]. \tag{29}$$

Using the relation of $a_1(t) = M_1^A(t)[1-a(t)]^2$, Eqs. (22) and (29), and the *Ansatz* (3), we can obtain the accurate solution of the pest aggregate size distribution $a_k(t)$,

$$a_k(t) = A_0\,\mathrm{e}^{(I-J_1B_0)t}\left\{1 - C_1\left[1 - \frac{C_2 - C_1}{C_2\,\mathrm{e}^{J_1B_0(C_2-C_1)t} - C_1}\right]\right\}^2 \times \left\{C_1\left[1 - \frac{C_2 - C_1}{C_2\,\mathrm{e}^{J_1B_0(C_2-C_1)t} - C_1}\right]\right\}^{k-1}. \tag{30}$$

1) When $C_1 = 1$, i.e., $L = 0$ and $I > J_1B_0$, it can be rewritten in the scaling region of $k \gg 1$ and $t \gg 1$ as

$$a_k(t) \simeq A_0\left(1 - \frac{J_1B_0}{I}\right)^2 \mathrm{e}^{-(I-J_1B_0)t} \times \exp\left[-\frac{k}{S(t)}\right], \tag{31}$$

where the characteristic size of the pest aggregates $S(t) = I\,\mathrm{e}^{(I-J_1B_0)t}/(I-J_1B_0)$, which grows with time as $S(t) \propto \mathrm{e}^{(I-J_1B_0)t}$. This obeys the generalized scal-

ing form as Eq. (13).

2) When $0 < C_1 < 1$, equation (30) can be rewritten in the scaling region of $k \gg 1$ and $t \gg 1$ as

$$a_k(t) \simeq A_0(1-C_1)^2 C_1^k \, \mathrm{e}^{(I-J_1B_0)t} \exp\left[-\frac{k}{S(t)}\right], \quad (32)$$

where the characteristic size of the pest aggregates $S(t) = C_2 \, \mathrm{e}^{J_1B_0(C_2-C_1)t}/(C_2-C_1)$, which grows with time as $S(t) \propto \mathrm{e}^{J_1B_0(C_2-C_1)t}$. Obviously, it approaches the modified scaling form as Eq. (15).

Using the relation of $M_0^A(t) = M_1^A(t)[1-a(t)]$, we can obtain the total number of the pest aggregates $M_0^A(t)$ as

$$M_0^A(t) = A_0 \, \mathrm{e}^{(I-J_1B_0)t} \left\{1 - C_1\left[1 - \frac{C_2 - C_1}{C_2 \, \mathrm{e}^{J_1B_0(C_2-C_1)t} - C_1}\right]\right\}. \quad (33)$$

From Eqs. (22) and (33), we can conclude that a) if $L = 0$ and $I > J_1B_0$, $M_1^A(t)$ will increase rapidly as an exponential function of time, while $M_0^A(t)$ decreases with time and approaches a constant value $A_0(I - J_1B_0)/I$ as $t \to \infty$; b) if $L \neq 0$ and $I = J_1B_0$, $M_1^A(t)$ is equal ever to A_0, and $M_0^A(t)$ will decrease with time and approach to a constant value $A_0(1-C_1)$ as $t \to \infty$; c) if $L \neq 0$ and $I > J_1B_0$, $M_1^A(t)$ and $M_0^A(t)$ both increase rapidly as an exponential function of time; d) if $I < J_1B_0$, $M_1^A(t)$ and $M_0^A(t)$ both decrease exponentially with time. So the pests will be eliminated quickly, if the efficacy is strong and the concentration of the medicine is high.

3.2. Pest control with a natural enemy

3.2.1. Model of pest control with a natural enemy

Pest control by natural enemies can be considered as the natural enemy-driven pest-monomer death process and can be described by another catalysis-driven monomer death scheme $C + A_k \xrightarrow{J_2(k)} A_{k-1} + C + C$ with the rate kernel $J_2(k)$, where C denotes one unit of the corresponding natural enemy and A_k denotes a pest aggregate consisting of k-mers. Therefore, we add the natural enemy-driven pest-monomer death scheme to the pest propagation model to investigate the model of pest control with a natural enemy.

We select a simple and practical rate kernel $J_2(k) = J_2k$, where J_2 is a positive proportional constant.

We still assume that the system has spatial homogeneity, thus the rate equations for the concentrations $a_k(t)$ and $c(t)$ ($c(t)$ denotes the concentration of natural enemy aggregates (C) can be written as

$$\frac{\mathrm{d}a_k}{\mathrm{d}t} = I(k-1)a_{k-1} - Ika_k + \sum_{i=1}^{\infty} La_{k+i} - \frac{1}{2}\sum_{i+j=k} La_k + J_2(k+1)ca_{k+1} - J_2kca_k, \quad (34)$$

$$\frac{\mathrm{d}c}{\mathrm{d}t} = J_2c\sum_{l=1}^{\infty} la_l. \quad (35)$$

In Eq. (34), the first four terms on the right-hand side have the same meanings as those in Eq. (1), and the following two terms account for the gain and loss in $a_k(t)$ due to the natural enemy-driven pest-monomer death processes $A_{k+1} + C \xrightarrow{J_2(k+1)} A_k + C + C$ and $A_k + C \xrightarrow{J_2(k)} A_{k-1} + C + C$, respectively. In Eq. (35), this accounts for the gain in $c(t)$ due to the process $A_l + C \xrightarrow{J_2(l)} A_{l-1} + C + C$.

We assume that there exist only monomer aggregates of pests at $t = 0$, i.e., $a_k(0) = A_0\delta_{k1}$ and the initial concentration of natural enemy aggregates C is C_0.

Utilizing the *Ansatz* (3), we find that the rate equation (34) becomes the differential equations for $a(t)$ and $a_1(t)$,

$$\frac{\mathrm{d}a}{\mathrm{d}t} = J_2ca^2 - \left(I + \frac{L}{2} + J_2c\right)a + I, \quad (36)$$

$$\frac{\mathrm{d}a_1}{\mathrm{d}t} = -Ia_1 + LaM_0^A + 2J_2ca_1a - J_2ca_1. \quad (37)$$

The evolution equation of $M_1^A(t)$ can be derived from Eqs. (36) and (37),

$$\frac{\mathrm{d}\ln M_1^A}{\mathrm{d}t} = \frac{2}{1-a}\frac{\mathrm{d}a}{\mathrm{d}t} + \frac{1}{a_1}\frac{\mathrm{d}a_1}{\mathrm{d}t} = I - J_2c. \quad (38)$$

Equation (35) can be rewritten as

$$\frac{\mathrm{d}\ln c}{\mathrm{d}t} = J_2M_1^A. \quad (39)$$

Next, we will study the kinetic evolution behaviour of pest control with a natural enemy.

3.2.2. Kinetic evolution behaviour of pest control with a natural enemy

From Eqs. (38) and (39), we find that $M_1^A(t)$ and $c(t)$ cannot be solved exactly under general conditions. By analysing Eqs. (38) and (39), we find that $M_1^A(t)$ will tend to zero while $c(t)$ tends to a certain value in a long time limit. Let

$$c(t) = \tau_\infty - \tau(t), \tag{40}$$

where $\tau_\infty = \lim_{t\to\infty} c(t)$ is a nonzero constant, $\tau(t)$ is a time-dependent decreasing function and $\tau(t) \to 0$ as $t \to \infty$. From Eqs. (38) and (39), we can determine that τ_∞ obeys the following equation under the initial condition ($M_1^A(0) = A_0$ and $c(0) = C_0$):

$$\tau_\infty - \frac{I}{J_2} \ln \frac{\tau_\infty}{C_0} = C_0 + A_0. \tag{41}$$

Substituting Eq. (40) into Eqs. (38) and (39), we can derive the asymptotical differential equations in a long time,

$$\frac{\mathrm{d} \ln M_1^A}{\mathrm{d}t} \simeq I - J_2\tau_\infty < 0, \tag{42}$$

$$-\frac{\mathrm{d}\tau}{\mathrm{d}t} \simeq J_2\tau_\infty M_1^A. \tag{43}$$

Then the asymptotical solutions of $M_1^A(t)$ and $c(t)$ can be obtained in a long time as

$$M_1^A(t) \simeq C_3 \ \mathrm{e}^{-(J_2\tau_\infty - I)t}, \tag{44}$$

$$c(t) \simeq \tau_\infty - \frac{J_2\tau_\infty C_3}{J_2\tau_\infty - I} \ \mathrm{e}^{-(J_2\tau_\infty - I)t}, \tag{45}$$

where C_3 is an integration constant. The results show that the total mass of pest aggregates decays to zero, while the concentration of the natural enemy increases to a certain value at the end. Thus, pests will be eliminated by a natural enemy eventually.

Equation (36) is a Riccati equation. It cannot be solved exactly under general conditions. Substituting Eq. (40) into it, we can obtain the asymptotical differential equation in a long time,

$$\frac{\mathrm{d}a}{\mathrm{d}t} \simeq J_2\tau_\infty \left(a^2 - \frac{I + L/2 + J_2\tau_\infty}{J_2\tau_\infty} a + \frac{I}{J_2\tau_\infty} \right). \tag{46}$$

The $\Delta = (I + L/2 + J_2\tau_\infty)^2/(J_2\tau_\infty)^2 - 4I/(J_2\tau_\infty)$ of the quadratic polynomial, in the bracket of the right-hand side of Eq. (46), can be proved to be greater than zero. The two roots of the quadratic polynomial can be solved as

$$C_4 = \frac{I + L/2 + J_2\tau_\infty - \sqrt{(I + L/2 + J_2\tau_\infty)^2 - 4IJ_2\tau_\infty}}{2J_2\tau_\infty},$$

$$C_5 = \frac{I + L/2 + J_2\tau_\infty + \sqrt{(I + L/2 + J_2\tau_\infty)^2 - 4IJ_2\tau_\infty}}{2J_2\tau_\infty}. \tag{47}$$

We can see that $0 < C_4 < 1$ and $C_5 > C_4$. Substituting the above expressions into Eq. (46), we can derive the asymptotical solution for $a(t)$ in a long time,

$$a(t) \simeq C_4 \left[1 - \frac{C_5 - C_4}{C_4C_6 \ \mathrm{e}^{J_2\tau_\infty(C_5 - C_4)t} - C_4} \right], \tag{48}$$

where C_6 is an integration constant. Using the relation of $a_1(t) = M_1^A(t)[1-a(t)]^2$, Eq. (44) and the *Ansatz* (3), we can obtain the asymptotical solution of the pest aggregate size distribution $a_k(t)$,

$$a_k(t) \simeq C_3 \ \mathrm{e}^{-(J_2\tau_\infty - I)t} \left\{ 1 - C_4 \left[1 - \frac{C_5 - C_4}{C_4C_6 \ \mathrm{e}^{J_2\tau_\infty(C_5 - C_4)t} - C_4} \right] \right\}^2 \times \left\{ C_4 \left[1 - \frac{C_5 - C_4}{C_4C_6 \, \mathrm{e}^{J_2\tau_\infty(C_5 - C_4)t} - C_4} \right] \right\}^{k-1}. \tag{49}$$

This can be rewritten in the scaling region of $k \gg 1$ and $t \gg 1$ as

$$a_k(t) \simeq C_3(1 - C_4)^2 C_4^k \, \mathrm{e}^{-(J_2\tau_\infty - I)t} \exp\left[-\frac{k}{S(t)} \right], \tag{50}$$

where the characteristic size of the pest aggregates $S(t) = C_4C_6 \ \mathrm{e}^{J_2\tau_\infty(C_5 - C_4)t}/(C_5 - C_4)$, which grows with time as $S(t) \propto \mathrm{e}^{J_2\tau_\infty(C_5 - C_4)t}$. Obviously, it approaches the modified scaling form as Eq. (15).

Using the relation of $M_0^A(t) = M_1^A(t)[1 - a(t)]$, we can obtain the asymptotical solution of $M_0^A(t)$ as

$$M_0^A(t) \simeq C_3 \ \mathrm{e}^{-(J_2\tau_\infty - I)t} \left\{ 1 - C_4 \left[1 - \frac{C_5 - C_4}{C_4 C_6 \ \mathrm{e}^{J_2\tau_\infty (C_5 - C_4)t} - C_4} \right] \right\}. \tag{51}$$

In conclusion, from Eqs. (44), (45), and (51), we can see that no matter how many natural enemies we introduce at first, $M_0^A(t)$ and $M_1^A(t)$ will both decay to zero while the concentration of the natural enemy increases to a certain quantity. Thus, pests will be eliminated by their natural enemy eventually.

4. Summary

We have studied the dynamics of pest propagation and pest control on the basis of the mean-field theory. In the pest propagation model, the pest aggregates evolve according to driving by self-birth and fragmentation processes. The rate kernels of self-birth and fragmentation are $I(k) = Ik$ and $L(i, j) = L$, respectively. By solving the rate equations, we find that I) if $L = 0$, $a_k(t)$ obeys a generalized scaling form, $M_0^A(t)$ is conservative, and $M_1^A(t)$ increases rapidly as $M_1^A(t) \propto \mathrm{e}^{It}$; II) if $L \neq 0$, $a_k(t)$ obeys a modified scaling form, and $M_0^A(t)$ and $M_1^A(t)$ both increase exponentially with time as $M_0^A(t) \propto \mathrm{e}^{It}$ and $M_1^A(t) \propto \mathrm{e}^{It}$. Thus, the pests will increase rapidly in this system, and they would generate tremendous damage to the agricultural production, so we have studied the dynamics of pest control with a pesticide and a natural enemy, respectively.

In the model of pest control with a pesticide, the evolution of the pest aggregates is driven by self-birth, fragmentation, and pesticide-driven pest-monomer death processes. The rate kernels of self-birth and fragmentation are the same as those in the first model, and the rate kernel of pesticide-driven pest-monomer death is $J_1(k) = J_1 k$. By solving the rate equations, we obtain the evolution behaviours of the pest aggregates: (a) $L = 0$ and $I = J_1 B_0$, $a_k(t)$ obeys a generalized scaling form, $M_0^A(t)$ decreases with time as $M_0^A(t) \propto t^{-1}$ and $M_1^A(t) \equiv A_0$, thus all pests will concentrate together eventually; (b) $L = 0$ and $I > J_1 B_0$, $a_k(t)$ obeys a generalized scaling form, $M_1^A(t)$ increases with the time-exponential function rapidly as $M_1^A(t) \propto \mathrm{e}^{(I - J_1 B_0)t}$; however, $M_0^A(t)$ decreases with time and approaches a constant value $A_0(I - J_1 B_0)/I$ as $t \to \infty$, thus the density of the pest aggregates increases rapidly; (c) $L \neq 0$ and $I = J_1 B_0$, $a_k(t)$ obeys a modified scaling form, $M_1^A(t)$ is equal ever to A_0, and $M_0^A(t)$ will decrease with time and approach a constant value $A_0(1 - C_1)$ as $t \to \infty$; (d) $L \neq 0$ and $I > J_1 B_0$, $a_k(t)$ obeys a modified scaling form, $M_1^A(t)$ and $M_0^A(t)$ both increase rapidly as $M_1^A(t) \propto \mathrm{e}^{(I - J_1 B_0)t}$ and $M_0^A(t) \propto \mathrm{e}^{(I - J_1 B_0)t}$; (e) $I < J_1 B_0$, $a_k(t)$ obeys a modified scaling form, $M_1^A(t)$ and $M_0^A(t)$ both decrease exponentially with time as $M_1^A(t) \propto \mathrm{e}^{(I - J_1 B_0)t}$ and $M_0^A(t) \propto \mathrm{e}^{(I - J_1 B_0)t}$, thus the pests can be killed off thoroughly.

In the model of pest control with a natural enemy, the evolution of the pest aggregates is driven by self-birth, fragmentation and natural enemy-driven pest-monomer death processes, and the evolution of the concentration of natural enemies is driven by a pest-driven natural enemy-monomer birth process. The rate kernels of self-birth and fragmentation are still the same as the first model, and the rate kernel of the natural enemy-driven pest-monomer death is $J_2(k) = J_2 k$. By solving the rate equations, we find that no matter how many natural enemies we put in at the beginning, pests will be completely eliminated by their natural enemy eventually. The concentration of natural enemy will approach to a certain constant τ_∞, and the pest aggregate size distribution $a_k(t)$ obeys a modified scaling form.

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