Attenuation characteristics of a light attenuator combined by polarizers with different extinction ratios*

Huang Chong(黄 翀), Deng Peng(邓 鹏),
Zhao Shuang(赵 爽), and Chen Hai-Qing(陈海清)†

College of Optoelectronics Science and Engineering, Wuhan National Laboratory for Optoelectronics, Huazhong University of Science and Technology, Wuhan 430074, *China*

(Received 5 January 2011; revised manuscript received 13 February 2011)

This paper deals with a systematical analysis and an algorithm of attenuation characteristics of a light attenuator combined by n pieces of polarizers (n-LACP) whose extinction ratios are different from each other. The attenuation ratio expression of a two-LACP is deduced. We find that the monotonic attenuation interval depends on the first polarizer and that the attenuation range depends on the second one. For the three-LACP, a method for obtaining a monotonic attenuation interval is proposed. Moreover, the attenuation ratio expression is demonstrated. Analysis and experiment show that when the initial status of the three-LACP is at the maximum output, if the second or third polarizer rotates alone, the minimum attenuation ratios can reach K_2^{-1} and K_3^{-1}, respectively, and if the first polarizer rotates, a minimum attenuation ratio of $K_2^{-1}K_3^{-1}$ can be obtained (K_1, K_2 and K_3 represent the extinction ratios of the three polarizers in turn). Furthermore, the attenuation ratio expression of n-LACP and the relevant attenuation characteristics are proposed. The minimum attenuation ratio of an n-LACP is $(K_2K_3\cdots K_n)^{-1}$.

Keywords: light attenuator, polarizer, extinction ratio, Jones vector

PACS: 42.79.Ci, 42.79.Pw, 42.79.Hp **DOI:** 10.1088/1674-1056/20/8/084209

1. Introduction

An attenuator of polarizer combination is widely used for continuous light attenuation in modern optical measurement or simulation systems.[1−4] A dynamic attenuation range is an important indicator of an attenuator of polarizer combination.[3−5] The attenuation range greatly depends on the extinction ratio of the combined polarizer. Kim[6] studied the polarization characteristics of a rotating wire grid polarizer and demonstrated the extinction ratio with a rotation angle around the optical axis. Nee *et al.*[7−10] made a series of significant studies of polarization characteristics under imperfect incident conditions. However, in all of this research, only light modulation with a single polarizer, but no attenuation characteristics, were considered. Mielenz and Eckerie[11] presented their research about the transmittance precision of a polarizer combination attenuator. Thus, in our previous work, the features of two-polarizer and three-polarizer attenuators, including the monotonic attenuation range, the monotonic attenuation interval and the expression of attenuation ratio (which is defined as the ratio of output intensity in arbitrary state to the maximum output intensity), were studied.[12]

In the present paper, we further study a light attenuator combined by n pieces of polarizers (n-LACP) whose extinction ratios are different from each other. It exhibits some good properties that an attenuator combined by polarizers with the same extinction ratio does not have (in this paper, by default, LACP refers to the attenuator that is combined by polarizers with different extinction ratios). For an optoelectronic instrument whose output signal is of a large dynamic variation and high precision, the attenuation ratio range, monotonic interval and expression of a specified combination are essential for determining the angle between the transmission axes of the polarizers.[13,14] Generally, if an attenuator combined by polarizers with a much larger attenuation range is used in the case where smaller attenuation is needed, it has more obvious disadvantages in variation precision than those in the case where its attenuation range just covers what is needed. This work offers theoretical guidance for combining different polarizers into a suitable attenuator because an LACP can make dif-

*Project supported by the Technological Innovation Project of Air-to-Air Missile Institute of China (Grant No. 5S85000FS).
†Corresponding author. E-mail: hqchen@mail.hust.edu.cn
© 2011 Chinese Physical Society and IOP Publishing Ltd

ferent attenuation ranges by rotating different polarizers, which is convenient for controlling the output light signal. In addition, we develop a concise and usable presentation method of expressing the attenuation ratio of an LACP by introducing the definition of critical angle. The method of expressing the attenuation ratio can generally be used in an LACP with either the same polarizers or different ones, as well as with combinations of random pieces.

The rest of the present paper is organized as follows. In Section 2, we analyse the general attenuation characteristics of linearly polarized light by one polarizer. In Section 3, the attenuation ratio expression and the monotonic attenuation interval of a two-LACP are deduced. In Section 4 we present the attenuation properties of a three-LACP and the critical status attenuation ratio. The numerical analysis and experimental results are shown in Section 5. Based on all the above results, some inferences about the attenuation properties of n-LACP (n is more than three) are discussed in Section 6, which is followed by a conclusion in Section 7.

2. General attenuation characteristics of linearly polarized light

Natural light becomes linearly polarized after passing through the first polarizer of an attenuator and it is linearly polarized all along after passing through the following polarizers but with polarization state variation.

Figure 1(a) shows a schematic diagram of linearly polarized light projecting onto a single polarizer. The extinction ratio of the polarizer is assumed to be K. The electric vector amplitude ratio of the polarizer is set to be ε. In detail, ε means that when natural light passes through the polarizer, the ratio of the electric vector amplitude of the component perpendicular to the transmission axis of the polarizer to that parallel to the transmission axis direction. Therefore, according to the definitions of K and ε, the relation can be acquired as $\varepsilon = K^{-1/2}$. Figure 1(b) depicts the coordination system of the incident linearly polarized light and the transmission axis of the polarizer. In the general case, linearly polarized light can be regarded as two orthogonal components. Let the directions of the two components of the incident light coincide with X_1OY_1 coordination and the major component is in the OX_1 direction. The transmission axis direction of the polarizer coincides with OX_2, thus, the extinction axis direction coincides with OY_2. Assume the angle enclosed by OX_1 and OX_2 to be θ.

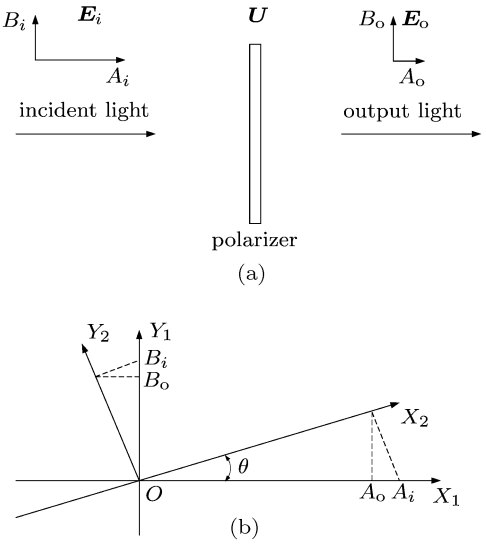


Fig. 1. (a) Schematic diagram of linearly polarized light attenuation by a single polarizer. The Jones vector of the incident light is composed of two components, A_i and B_i, the Jones vector of the output light is composed of A_o and B_o. (b) Coordination system of the Jones vectors. OX_1 and OY_1 represent two electric component directions of incident linearly polarized light. OX_2 denotes the transmission axis of the polarizer.

The transmission characteristics can be modeled in Jones space. The Jones vector of incident linearly polarized light is assumed to be $\boldsymbol{E}_i$ and the Jones matrix of the polarizer is $\boldsymbol{U}$. The Jones vector of the output light is obtained to be $\boldsymbol{E}_o$ as follows:

$$\boldsymbol{E}_i = \begin{bmatrix} A_i \\ B_i \end{bmatrix}, \tag{1}$$

$$\begin{aligned} \boldsymbol{U} &= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2\theta + \varepsilon\sin^2\theta & (1-\varepsilon)\sin\theta\cos\theta \\ (1-\varepsilon)\sin\theta\cos\theta & \sin^2\theta + \varepsilon\cos^2\theta \end{bmatrix}, \end{aligned} \tag{2}$$

$$\begin{aligned} \boldsymbol{E}_o &= \begin{bmatrix} A_o \\ B_o \end{bmatrix} = \boldsymbol{U} \times \begin{bmatrix} A_i \\ B_i \end{bmatrix} \\ &= \begin{bmatrix} A_i - (A_i\sin\theta - B_i\cos\theta)(1-\varepsilon)\sin\theta \\ B_i + (A_i\sin\theta - B_i\cos\theta)(1-\varepsilon)\cos\theta \end{bmatrix}. \end{aligned} \tag{3}$$

The output light intensity is I_o and expressed as

$$\begin{aligned} I_o &= |E_o|^2 = A_o^2 + B_o^2 \\ &= [A_i - (A_i\sin\theta - B_i\cos\theta)(1-\varepsilon)\sin\theta]^2 \end{aligned}$$

$$
\begin{aligned}
&+[B_i+(A_i\sin\theta-B_i\cos\theta)(1-\varepsilon)\cos\theta]^2\\
&=A_i^2+B_i^2+(A_i\sin\theta-B_i\cos\theta)^2(1-\varepsilon)^2\\
&\quad-2(A_i\sin\theta-B_i\cos\theta)^2(1-\varepsilon)\\
&=A_i^2+B_i^2-(1-\varepsilon^2)(A_i\sin\theta-B_i\cos\theta)^2. \quad (4)
\end{aligned}
$$

Since $I_i=|\boldsymbol{E}_i|^2=A_i^2+B_i^2$, $I_{\rm o}$ can be depicted as

$$I_{\rm o}=I_i-(1-\varepsilon^2)(A_i\sin\theta-B_i\cos\theta)^2. \quad (5)$$

For the purpose of obtaining the extrema of $I_{\rm o}$, let partial derivative $\partial I_{\rm o}/\partial\theta=0$, i.e.,

$$
\begin{aligned}
\partial I_{\rm o}/\partial\theta=&-2(1-\varepsilon^2)(A_i\sin\theta-B_i\cos\theta)\\
&\times(A_i\cos\theta+B_i\sin\theta)=0. \quad (6)
\end{aligned}
$$

The solutions of θ value in the interval $[0,\pi]$ can be obtained by solving Eq. (6)

$$\theta=\arctan B_i/A_i \quad \text{or} \quad \theta=B_i/A_i+\pi/2. \quad (7)$$

Substituting the two θ values into Eq. (5), when $\theta=\arctan B_i/A_i$, $I_{\rm o}$ attains the maximum $I_{\rm o\,max}=I_i$ and when $\theta=\arctan B_i/A_i+\pi/2$, $I_{\rm o}$ attains the minimum $I_{\rm o\,min}=\varepsilon^2 I_i$. Assume $\varphi=\arctan B_i/A_i$, $I_{\rm o}$ can be written as

$$I_{\rm o}=I_i[1-(1-\varepsilon^2)\sin^2(\theta-\varphi)]. \quad (8)$$

Since $\varepsilon\ll 1$ in general case, $1-\varepsilon_2^2>0$. In the interval $\theta\in[\arctan B_i/A_i,\ \arctan B_i/A_i+\pi/2]$, the value domain of $I_{\rm o}$ is $[\varepsilon^2 I_i, I_i]$.

Accordingly, instead of the monotonic attenuation interval $[0,\ \pi/2]$ presented by Malus's law, a deviation angle $\arctan B_i/A_i$ exists in the real monotonic attenuation interval. In addition, the deviation angle only correlates to the polarization state of the incident light and the attenuation range is from 1 to $1/K$ times.

3. Two-LACP

Figure 2 depicts a model of the LACP. In a general case, the incident light is natural light. After transmitting through the first polarizer, it becomes linearly polarized. In this section, we discuss the two-polarizer combination.

For a two-LACP, if the amplitude ratios of the polarizers are assumed to be ε_1 and ε_2, respectively, $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are described below:

$$\boldsymbol{E}_1=\begin{bmatrix}A_1\\B_1\end{bmatrix}=\begin{bmatrix}1\\\varepsilon_1\end{bmatrix}, \quad (9)$$

$$
\begin{aligned}
\boldsymbol{U}_1&=\begin{bmatrix}\cos\theta_1&-\sin\theta_1\\\sin\theta_1&\cos\theta_1\end{bmatrix}\begin{bmatrix}1&0\\0&\varepsilon_2\end{bmatrix}\begin{bmatrix}\cos\theta_1&\sin\theta_1\\-\sin\theta_1&\cos\theta_1\end{bmatrix}\\
&=\begin{bmatrix}\cos^2\theta_1+\varepsilon_2\sin^2\theta_1&(1-\varepsilon_2)\sin\theta_1\cos\theta_1\\(1-\varepsilon_2)\sin\theta_1\cos\theta_1&\sin^2\theta_1+\varepsilon_2\cos^2\theta_1\end{bmatrix}, \quad (10)
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{E}_2&=\begin{bmatrix}A_2\\B_2\end{bmatrix}=\boldsymbol{U}_1\boldsymbol{E}_1\\
&=\begin{bmatrix}\varepsilon_2\sin^2\theta_1+\cos^2\theta_1+\varepsilon_1(1-\varepsilon_2)\sin\theta_1\cos\theta_1\\(1-\varepsilon_2)\sin\theta_1\cos\theta_1+\varepsilon_1(\sin^2\theta_1+\varepsilon_2\cos^2\theta_1)\end{bmatrix}\\
&=\begin{bmatrix}1-(1-\varepsilon_2)(\sin\theta_1-\varepsilon_1\cos\theta_1)\sin\theta_1\\\varepsilon_1+(1-\varepsilon_2)(\sin\theta_1-\varepsilon_1\cos\theta_1)\cos\theta_1\end{bmatrix}, \quad (11)
\end{aligned}
$$

$$
\begin{aligned}
I_2&=|\boldsymbol{E}_2|^2\\
&=[1-(1-\varepsilon_2)(\sin\theta_1-\varepsilon_1\cos\theta_1)\sin\theta_1]^2\\
&\quad+[\varepsilon_1+(1-\varepsilon_2)(\sin\theta_1-\varepsilon_1\cos\theta_1)\cos\theta_1]^2\\
&=1+\varepsilon_1^2-(1-\varepsilon_2^2)(\sin\theta_1-\varepsilon_1\cos\theta_1)^2. \quad (12)
\end{aligned}
$$

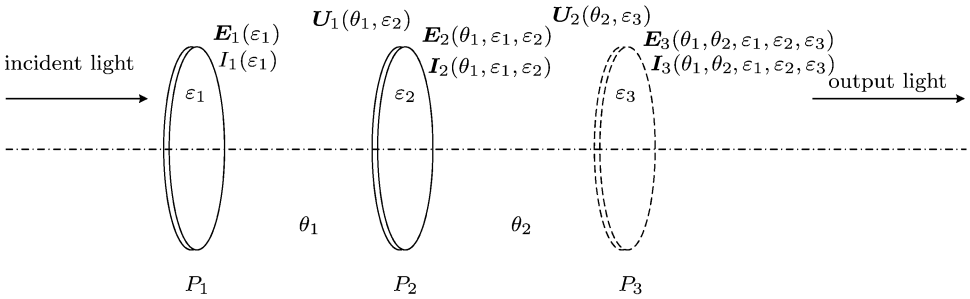


Fig. 2. Two- and three-LACP models. P_1, P_2, and P_3 represent the polarizers aligned along the optical axis. $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ are the Jones matrix of P_2 and P_3. $\boldsymbol{E}_1$, $\boldsymbol{E}_2$, and $\boldsymbol{E}_3$ represent The Jones vectors after light has passed through P_1, P_2, and P_3. Light intensities after P_2 and P_3 are denoted as I_2 and I_3, whose values are determined by the variables enclosed in parentheses. θ_1 and θ_2 represent the angles enclosed by the transmission axes of the two polarizers.

$I_1 = 1 + \varepsilon_1^2$ is kept unchanged. According to Section 2, the critical angle when the output light intensity obtains the maximum is $\varphi_1 = \arctan\varepsilon_1$, so $\cos\varphi_1 = 1/\sqrt{1+\varepsilon_1^2}$ and $\sin\varphi_1 = \varepsilon_1/\sqrt{1+\varepsilon_1^2}$. I_2 can be depicted as

$$\begin{aligned} I_2 &= 1 + \varepsilon_1^2 - (1-\varepsilon_2^2)(1+\varepsilon_2^2) \\ &\quad \times \left(\frac{1}{\sqrt{1+\varepsilon_2^2}}\sin\theta_1 - \frac{\varepsilon_1}{\sqrt{1+\varepsilon_2^2}}\cos\theta_1\right)^2 \\ &= 1 + \varepsilon_1^2 - (1-\varepsilon_2^2)(1+\varepsilon_1^2)\sin^2(\theta_1-\varphi_1) \\ &= I_1[1-(1-\varepsilon_2^2)\sin^2(\theta_1-\varphi_1)]. \end{aligned} \tag{13}$$

When $\theta_1 = \varphi_1$, $\boldsymbol{E}_2(\arctan\varepsilon_1, \varepsilon_1, \varepsilon_2) = \begin{bmatrix} 1 \\ \varepsilon_1 \end{bmatrix}$, I_2 attains the maximum $I_{2\max} = I_1 = 1 + \varepsilon_1^2$; When $\theta_1 = \varphi_1 + \pi/2$, $\boldsymbol{E}_2(\arctan\varepsilon_1 + \pi/2, \varepsilon_1, \varepsilon_2) = \begin{bmatrix} \varepsilon_2 \\ \varepsilon_1\varepsilon_2 \end{bmatrix}$, I_2 reaches the minimum $I_{2\min} = \varepsilon_2^2 I_1 = \varepsilon_2^2\left(1+\varepsilon_1^2\right)$. In the interval $[\arctan\varepsilon_1, \arctan\varepsilon_1 + \pi/2]$, I_2 monotonically decreases from $1+\varepsilon_1^2$ to $\varepsilon_2^2\left(1+\varepsilon_1^2\right)$.

According to the definition of attenuation ratio, the attenuation ratio of a two-LACP is written as

$$T_2 = \frac{I_2}{I_{2\max}} = 1 - (1-\varepsilon_2^2)\sin^2(\theta_1-\varphi_1). \tag{14}$$

From Eq. (14), the attenuation ratio of a two-LACP attains a maximum of 1 and a minimum of ε_2^2 separately. The extremum values do not correlate to ε_1. From the above analyses, it can be concluded that the attenuation ratio range of a two-LACP, which is from 1 to ε_2^2 ($1/K^2$ times), depends on the second polarizer. The angle φ_1, which is the monotonic attenuation interval deviation angle relative to $[0, \pi/2]$, depends on the extinction ratio of the first polarizer.

4. Three-LACP

4.1. Critical value of $\boldsymbol{\theta_2}$

The attenuator of the three-polarizer model depicted in Fig. 2 represents the general case. The three polarizers have different extinction ratios. The amplitude ratios of the polarizers P_1, P_2, P_3 are ε_1, ε_2, ε_3, respectively.

In Eq. (2), U is obtained by multiplication of three matrixes. As $\boldsymbol{E}_1$ is a vector in X_1OY_1 coordination, it can be projected into X_2OY_2 coordination by multiplying matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta\cos\theta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}$ indicates the attenuation effect of the polarizer. In a two-LACP, it indicates the attenuation effect of the second polarizer. $\begin{bmatrix} \cos\theta - \sin\theta \\ \sin\theta \quad \cos\theta \end{bmatrix}$ converts $\boldsymbol{E}_2$ into the one in the X_1OY_1 coordination.

For an attenuator of three polarizers, the Jones matrix of P_3 is

$$\boldsymbol{U}_2 = \begin{bmatrix} \varepsilon_3\sin^2\theta_{1,3} + \cos^2\theta_{1,3} & (1-\varepsilon_3)\sin\theta_{1,3}\cos\theta_{1,3} \\ (1-\varepsilon_3)\sin\theta_{1,3}\cos\theta_{1,3} & \sin^2\theta_{1,3} + \varepsilon_3\cos^2\theta_{1,3} \end{bmatrix}, \tag{15}$$

where $\theta_{1,3}$ represents the angle between the transmission axes of P_3 and P_1. The Jones vector after P_3 is

$$\boldsymbol{E}_3 = \begin{bmatrix} A_3 \\ B_3 \end{bmatrix} = U_2E_2 = \begin{bmatrix} A_2 - (1-\varepsilon_3)(A_2\sin\theta_{1,3} - B_2\cos\theta_{1,3})\sin\theta_{1,3} \\ B_2 + (1-\varepsilon_3)(A_2\sin\theta_{1,3} - B_2\cos\theta_{1,3})\cos\theta_{1,3} \end{bmatrix}. \tag{16}$$

$\boldsymbol{E}_3$ is a vector in X_1OY_1 coordination and it depends on θ_1 and $\theta_{1,3}$. Set $\varphi_{1,3}$ to be the critical angle of $I_{3\max}$. According to the conclusion in Section 3, $\varphi_{1,3}$ is depicted as

$$\varphi_{1,3} = \arctan\frac{B_2}{A_2} = \arctan\frac{\varepsilon_1 + (1-\varepsilon_2)(\sin\theta_1 - \varepsilon_1\cos\theta_1)\cos\theta_1}{1-(1-\varepsilon_2)(\sin\theta_1 - \varepsilon_1\cos\theta_1)\sin\theta_1}, \tag{17}$$

where $\varphi_{1,3}$ is the critical angle in the X_1OY_1 coordination. Since it is hard to make sure its value from Eq. (17), we find a simple presentation in X_2OY_2 coordination as described below.

Note that $\boldsymbol{E}_2$ can be converted into the one in the X_2OY_2 coordination and is expressed as $\boldsymbol{E}_2'$. A schematic diagram of the conversion is shown in Fig. 3.

$$\begin{aligned} \boldsymbol{E}_2' &= \begin{bmatrix} A_2' \\ B_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta_1 + \varepsilon_1\sin\theta_1 \\ -\varepsilon_2(\sin\theta_1 - \varepsilon_1\cos\theta_1), \end{bmatrix}, \end{aligned} \tag{18}$$

$$\varphi_2 = \arctan\frac{B_2'}{A_2'} = -\arctan\left(\varepsilon_2\frac{\tan\theta_1-\varepsilon_1}{1+\varepsilon_1\tan\theta_1}\right). \quad (19)$$

Angle φ_2 is the critical angle of θ_2, which is the angle between the transmission axis directions of P_3 and P_2 when I_3 attains a maximum. Since $\varepsilon_1 = \tan\varphi_1$, equation (19) can be written as

$$\varphi_2 = -\arctan[\varepsilon_2\tan(\theta_1-\varphi_1)]. \quad (20)$$

From Eq. (20), it can be concluded that φ_2 depends on θ_1, ε_1 (which correlates with φ_1) and ε_2. Meanwhile, the relation between $\varphi_{1,3}$ and φ_2 is

$$\varphi_{1,3} = \theta_1+\varphi_2 = \theta_1-\arctan[\varepsilon_2\tan(\theta_1-\varphi_1)]. \quad (21)$$

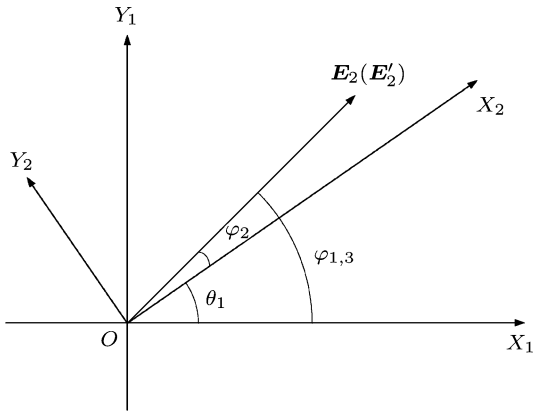


Fig. 3. Location relation between the coordinations X_1OY_1 and X_2OY_2. OX_1 and OX_2 enclose an angle of θ_1. When I_3 attains the maximum, the angle between the vector directions of $\boldsymbol{E}_2(\boldsymbol{E}_2')$ and OX_2 is denoted as φ_2 and the angle between the vector directions of $\boldsymbol{E_2}(\boldsymbol{E}_2')$ and OX_1 is represented as $\varphi_{1,3}$.

The light intensity after P_3 is

$$\begin{aligned} I_3 &= I_2[1-(1-\varepsilon_3^2)\sin^2(\theta_2-\varphi_2)] \\ &= I_1[1-(1-\varepsilon_2^2)\sin^2(\theta_1-\varphi_1)] \\ &\quad\times[1-(1-\varepsilon_3^2)\sin^2(\theta_2-\varphi_2)]. \end{aligned} \quad (22)$$

Equation (22) shows the expression of I_3 versus ε_2, ε_3, θ_1, θ_2. For a specified three-LACP, whose ε_2 and ε_3 are not changed, I_3 is determined only by θ_1 and θ_2 according to Eq. (22). We study I_3 in special cases as well as general cases in the following context of this section.

4.2. The case of θ_1 attaining a critical value

Based on the result obtained in Subsection 4.1, some critical cases where the former two polarizers attain critical states are analysed below.

4.2.1. $\theta_1 = \varphi_1$

According to the conclusion in Section 3, when the output intensity of P_2 attains the maximum, $\boldsymbol{E}_2(\arctan\varepsilon_1,\varepsilon_1,\varepsilon_2)$ equals $\begin{bmatrix}1\\ \varepsilon_1\end{bmatrix}$ in the X_1OY_1 coordination. Thus, in the X_2OY_2 coordination,

$$\begin{aligned} \boldsymbol{E}_2'(\varphi_1,\varepsilon_1,\varepsilon_2) &= \begin{bmatrix}1&0\\0&\varepsilon_2\end{bmatrix}\begin{bmatrix}\cos\varphi_1 & \sin\varphi_1\\ -\sin\varphi_1&\cos\varphi_1\end{bmatrix}, \\ E_1 &= \begin{bmatrix}\sqrt{1+\varepsilon_1^2}\\0\end{bmatrix}. \end{aligned} \quad (23)$$

In this case, φ_2 is 0 according to Eq. (19). E_3' in the X_2OY_2 coordination is expressed as

$$\begin{aligned} \boldsymbol{E}_3'(\varphi_1,0,\varepsilon_1,\varepsilon_2,\varepsilon_3) &= \boldsymbol{U}(\boldsymbol{0},\varepsilon_3)\times E_2'(\varphi_1,\varepsilon_1,\varepsilon_2) \\ &= \begin{bmatrix}\sqrt{1+\varepsilon_1^2}\\0\end{bmatrix}. \end{aligned} \quad (24)$$

Accordingly, $I_3(\varphi_1,0,\varepsilon_1,\varepsilon_2,\varepsilon_3)$ is obtained as

$$I_3(\varphi_1,0,\varepsilon_1,\varepsilon_2,\varepsilon_3) = 1+\varepsilon_1^2 = I_1, \quad (25)$$

where I_3 is the maximum output intensity of the three-LACP. When $\varphi_2 = \pi/2$, $\boldsymbol{E}_3'$ in the X_2OY_2 coordination and I_3 are respectively as follows:

$$\begin{aligned} \boldsymbol{E}_3'(\varphi_1,\pi/2,\varepsilon_1,\varepsilon_2,\varepsilon_3) &= \boldsymbol{U}(\pi/2,\varepsilon_3)\times\boldsymbol{E}_2'(\varphi_1,\varepsilon_1,\varepsilon_2) \\ &= \begin{bmatrix}\varepsilon_3\sqrt{1+\varepsilon_1^2}\\0\end{bmatrix}, \end{aligned} \quad (26)$$

$$I_3(\varphi_1,\pi/2,\varepsilon_1,\varepsilon_2,\varepsilon_3) = \varepsilon_3^2\left(1+\varepsilon_1^2\right) = \varepsilon_3^2 I_1, \quad (27)$$

it is shown that I_3 decreases ε_3^2 times from $\varphi_2 = 0$ to $\varphi_2 = \pi/2$. Since $\boldsymbol{E}_2(\arctan\varepsilon_1,\varepsilon_1,\varepsilon_2) = E_1$, this rule is similar to that of a two-LACP.

4.2.2. $\theta_1 = \pi/2 + \varphi_1$

When $\theta_1 = \pi/2+\varphi_1 = \pi/2+\arctan\varepsilon_1$, the light intensity output from P_2 attains a minimum. In the X_2OY_2 coordination

$$\begin{aligned} &\boldsymbol{E}_2'(\pi/2+\varphi_1,\varepsilon_1,\varepsilon_2) \\ &= \begin{bmatrix}1&0\\0&\varepsilon_2\end{bmatrix}\begin{bmatrix}\cos(\pi/2+\varphi_1) & \sin(\pi/2+\varphi_1)\\ -\sin(\pi/2+\varphi_1)&\cos(\pi/2+\varphi_1)\end{bmatrix}\boldsymbol{E}_1 \\ &= \begin{bmatrix}0\\-\varepsilon_2\sqrt{1+\varepsilon_1^2}\end{bmatrix}. \end{aligned} \quad (28)$$

The value of φ_2 here is $-\pi/2$. In the X_2OY_2 coordination, when $I_2(\theta_1,\varepsilon_1,\varepsilon_2)$ obtains the maximum and

the minimum separately, the Jones vector E_3' is respectively as follows:

$$\begin{aligned}&\boldsymbol{E}_3'(\pi/2+\varphi_1,-\pi/2,\varepsilon_1,\varepsilon_2,\varepsilon_3)\\&=\boldsymbol{U}(-\pi/2,\varepsilon_3)\times\boldsymbol{E}_2'(\pi/2+\varphi_1,\varepsilon_1,\varepsilon_2)\\&=\begin{bmatrix}0\\-\varepsilon_2\sqrt{1+\varepsilon_1^2}\end{bmatrix},\end{aligned}\tag{29}$$

$$\begin{aligned}&\boldsymbol{E}_3'(\pi/2+\varphi_1,0,\varepsilon_1,\varepsilon_2,\varepsilon_3)\\&=\boldsymbol{U}(0,\varepsilon_3)\times\boldsymbol{E}_2'(\pi/2+\varphi_1,\varepsilon_1,\varepsilon_2)\\&=\begin{bmatrix}0\\-\varepsilon_3\varepsilon_2\sqrt{1+\varepsilon_1^2}\end{bmatrix},\end{aligned}\tag{30}$$

The minimum of I_3 is

$$I_3(\pi/2+\varphi_1,0,\varepsilon_1,\varepsilon_2,\varepsilon_3)=\varepsilon_2^2\varepsilon_3^2\left(1+\varepsilon_1^2\right).\tag{31}$$

According to Eqs. (25) and (31), the minimum value of the attenuation ratio of a three-LACP is expressed as

$$\begin{aligned}T_{3,\min}&=\frac{I_3(\pi/2+\varphi_1,0,\varepsilon_1,\varepsilon_2,\varepsilon_3)}{I_3(\varphi_1,0,\varepsilon_1,\varepsilon_2,\varepsilon_3)}=\varepsilon_2^2\varepsilon_3^2\\&=K_2^{-1}K_3^{-1}.\end{aligned}\tag{32}$$

4.3. The case of θ_1 being a random value

When P_2 is fixed at a certain position where θ_1 does not attain a critical angle and only P_3 rotates, I_3 can reach a maximum and a minimum in the specified condition, while φ_2, which is presented by Eq. (20), is the key factor that makes $I_3(\theta_1,\theta_2,\varepsilon_1,\varepsilon_2,\varepsilon_3)$ attain a maximum. Under such circumstances, according to Eq. (22)

$$I_{3\max}=I_1[1-(1-\varepsilon_2^2)\sin^2(\theta_1-\arctan\varepsilon_1)].\tag{33}$$

$I_3(\theta_1,\theta_2,\varepsilon_1,\varepsilon_2,\varepsilon_3)$ reaches a minimum when $\theta_2=\varphi_2+\pi/2$. Under this circumstances,

$$I_{3\min}=\varepsilon_3^2 I_1[1-(1-\varepsilon_2^2)\sin^2(\theta_1-\arctan\varepsilon_1)].\tag{34}$$

$I_3(\theta_1,\theta_2,\varepsilon_1,\varepsilon_2,\varepsilon_3)$ monotonically decreases in the interval $[-\arctan[\varepsilon_2\tan(\theta_1-\varphi_1)],\ \pi/2-\arctan[\varepsilon_2\tan(\theta_1-\varphi_1)]]$. And the attenuation ratio can be expressed as

$$\begin{aligned}T_3&=\frac{I_3}{I_{3\max}}=1-(1-\varepsilon_3^2)\\&\times\sin^2[\theta_2+\arctan[\varepsilon_2\tan(\theta_1-\varphi_1)]].\end{aligned}\tag{35}$$

It can be found in the equation that the minimum attenuation ratio is ε_3^2, which is expressed as K_3^{-1} as well. Referring to the above process, when P_2 and P_3 are rotating, attenuation ratio should be expressed as

$$\begin{aligned}T_{2,3}&=[1-(1-\varepsilon_2^2)\sin^2(\theta_1-\varphi_1)]\\&\times[1-(1-\varepsilon_3^2)\sin^2(\theta_2-\varphi_2)].\end{aligned}\tag{36}$$

Since φ_2 depends on θ_1, $T_{2,3}$ can be obtained by Eq. (36).

5. Numerical and experimental analysis of LACP

5.1. Two-LACP

A two-LACP model with extinction ratios $K_1=1000$ and $K_2=4000$ (corresponding $\varepsilon_1=10^{-3/2}$ and $\varepsilon_2=0.5\times10^{-3/2}$) respectively is cited here as a case. The curve of $I_2(\theta_1,\varepsilon_1,\varepsilon_2)$ is shown in Fig. 4. A minimum value of I_2 is obtained near the point $\theta_1=90^\circ$ and is approximate to 2.5×10^{-4}. It is seen from the curve that the monotonic interval is still π while a right shift with a tiny deviation angle of 1.81°, is approximate to $\arctan 10^{-3/2}$.

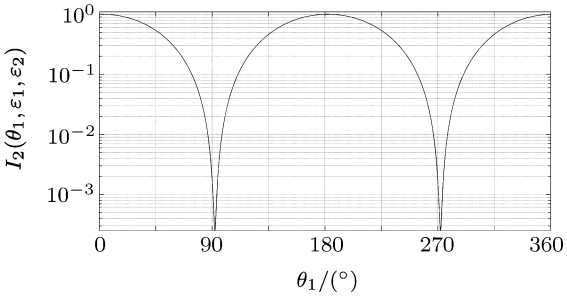


Fig. 4. Output light intensity $I_2(\theta_1,\varepsilon_1,\varepsilon_2)$ when $K_1=1000$ and $K_2=4000$.

5.2. Three-LACP

To analyse the three-LACP, a third polarizer with $K_3=10^5$ is added at the offside of the two-LACP discussed in Subsection 5.1. The attenuation characteristics, when one of the three polarizers rotates alone, are discussed respectively in detail.

5.2.1. The case of only P_3 rotating

As shown in Fig. 5, when $\theta_1=\varphi_1=\arctan\varepsilon_1$ and $\theta_2=0$, the three-LACP reaches the maximum output. The anti-clockwise rotation of P_3 from $\theta_2=0$ to $\pi/2$ will cause an attenuation of K_3^{-1} times according to the discussion in Subsubsection 4.2.1. This process is graphically given by the curve $I_3(\varphi_1,\theta_2,\varepsilon_1,\varepsilon_2,\varepsilon_3)$ versus θ_2 in Fig. 6.

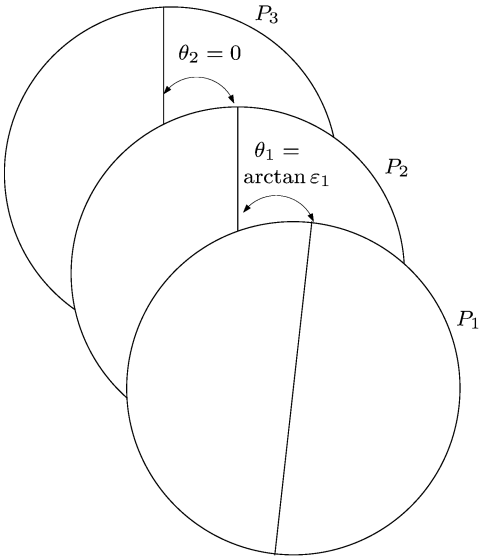


Fig. 5. Schematic diagram of the three-LACP.

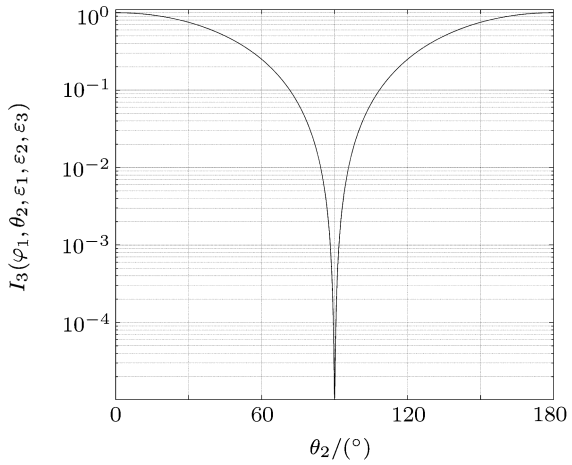


Fig. 6. Curve of $I_3(\varphi_1, \theta_2, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ versus θ_2.

It is seen from Fig. 6 that $I_3(\varphi_1, \theta_2, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ decreases monotonically in the interval $[0, \pi/2]$ and $I_{3\max}$ is 10^{-5}, which meets the theoretical result in Subsubsection 4.2.1. In the interval $[\pi/2, \pi]$, $I_3(\varphi_1, \theta_2, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ increases monotonically in axially symmetric rule to the first part. A comparison between Fig. 6 and Fig. 4 shows that the attenuation range of P_3 is larger than that of P_2 and that the curve near the minimum point in Fig. 6 is steeper than that in Fig. 4. Thus, it is more difficult to achieve the minimum point in the higher extinction ratio attenuator.

5.2.2. The case of only P_1 rotating

As shown in Fig. 5, to analyse the case of only P_1 rotating, we rotate P_1 clockwise and fix P_2 and P_3. In this case, θ_1 is the only variable. Set the initialized state to be $\theta_1 = \varphi_1$ and $\theta_2 = 0$. P_1 is rotated in the direction where θ_1 decreases, from $\theta_1 = \varphi_1$ to $\varphi_1 + \pi/2$, which is a monotonically decreasing interval. In the minimum output point $\theta_1 = \varphi_1 + \pi/2$, $\boldsymbol{E}_2'$ is expressed as Eq. (28) and $\boldsymbol{E}_3'$ is expressed as Eq. (30). The attenuation ratio is $\varepsilon_3^2\varepsilon_2^2$.

Figure 7 shows the curve of $I_3(\theta_1, 0, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ versus θ_1. From simulated data, $I_3(\theta_1, 0, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ attains a maximum at $\theta_1 = \varphi_1 = 1.81^\circ$, then decreases monotonically by 8 orders of magnitude in $\theta_1 \in [\varphi_1, \varphi_1 + \pi/2]$ and these can be observed from the curve. We may also find from the curve that near the minimum output point $\theta_1 = \varphi_1 + \pi/2 = 91.81^\circ$, the curve is steeper than the case in Subsubsection 5.2.1. In experiment, we can attain only a minimum output intensity of 10^{-5}.

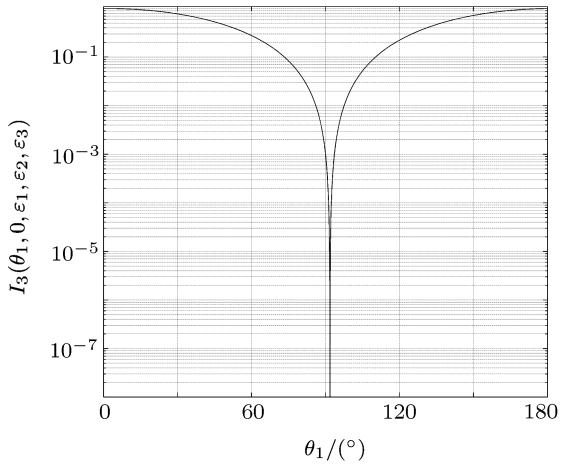


Fig. 7. Curve of $I_3(\theta_1, 0, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ versus θ_1.

5.2.3. The case of only P_2 rotating

The case where only the middle polarizer P_2 is rotating while P_1 and P_3 are fixed is discussed. In this case, θ_1 and θ_2 vary simultaneously while the angle $\theta_{1,3}$ is kept unchanged, and they are related through $\theta_2 = \theta_{1,3} - \theta_1$. Let P_2 in Fig. 5 rotate anticlockwise from $\theta_1 = 0$ to $\theta_1 = \varphi_1 + \pi$, therefore, θ_2 changes from $-\varphi_1$ to $-\pi$. When $\theta_1 = \varphi_1 + \pi/2$ and $\theta_2 = -\pi/2$, $\boldsymbol{E}_3'$ is expressed as Eq. (29) and $I_3(\theta_1, \varphi_1 - \theta_1, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ attains the minimum in this case. The curve of $I_3(\theta_1, \varphi_1 - \theta_1, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ versus θ_1 is shown in Fig. 8. The simulated data indicate that $I_3(\theta_1, \varphi_1 - \theta_1, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ rises by a tiny margin from $\theta_1 = 0$ to $\theta_1 = \varphi_1$, then declines in the interval of $\theta_1 \in [\varphi_1, \varphi_1 + \pi/2]$ and rises monotonically in $\theta_1 \in [\varphi_1 + \pi/2, \varphi_1 + \pi]$. These can be observed from the curve in Fig. 8. The curve in $\theta_1 \in [\varphi_1 + \pi/2]$ is axially symmetrical to the curve in $\theta_1 \in [\varphi_1, \varphi_1 + \pi/2]$. The minimum attenuation ratio is 2.5×10^{-4} at $\theta_1 =$

91.81°, which equals ε_2^2. It validates that when P_2 is rotating, the minimum is determined by the extinction ratio of P_2. Compared with the case of the two-LACP in Subsection 5.1, the attenuation ability is not improved. However, the curve near the minimum point is less steep than that in Fig. 4, which indicates that the minimum point in the three-LACP is more acquirable than that in the two-LACP discussed in Subsection 5.1.

According to the above analysis, P_3 has little effect on the minimum attenuation ratio when only P_2 rotates, but reduces the difficulty in obtaining it. We simulate other four cases of P_3 with different extinction ratios, i.e., 10^4, 10^3, 10^2 and 10. (P_1, P_2 and the initializing condition are the same as those in Subsubsection 5.2.3.) The attenuation ratio curves of $I_3(\theta_1, \varphi_1 - \theta_1, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ are shown in Fig. 9, showing that they are little difference in their maxima and the minima, but some tiny variations in shape, especially near the minimum points. The larger the ER of P_3, the smoother the curve appears near the minimum point.

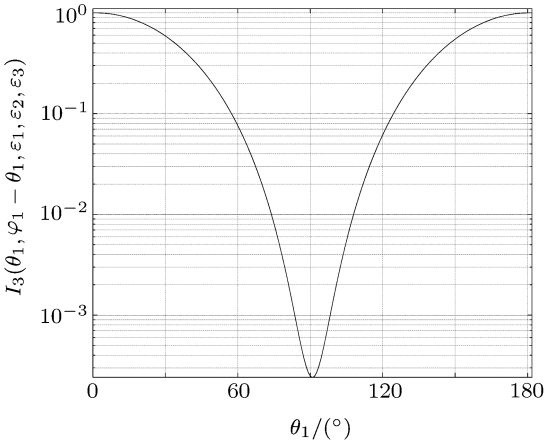


Fig. 8. Curve of $I_3(\theta_1, \varphi_1 - \theta_1, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ versus θ_1.

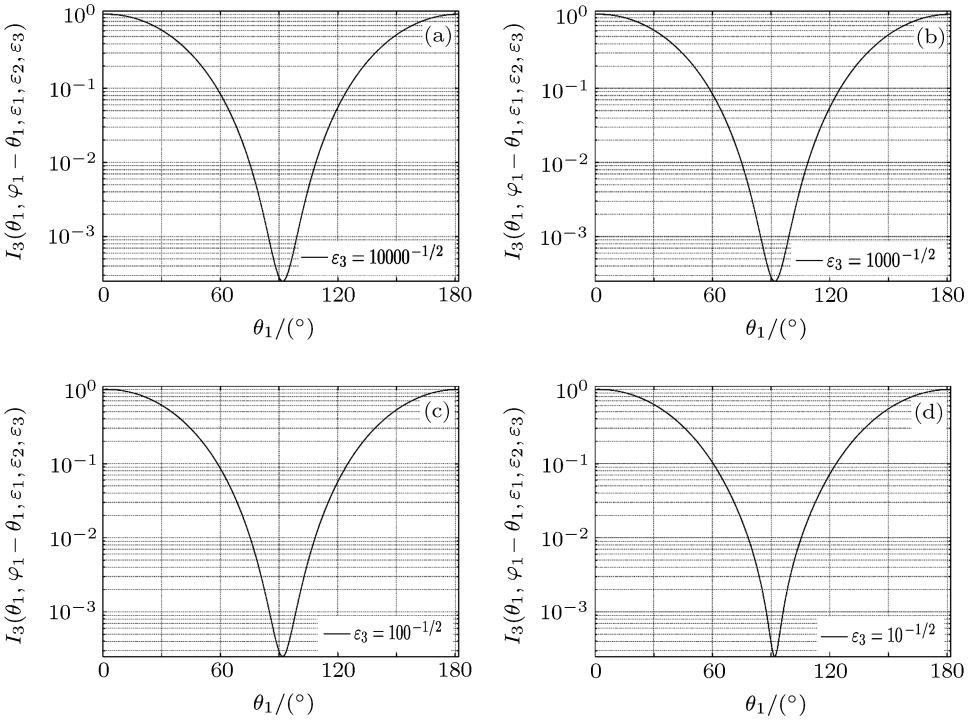


Fig. 9. Curves of $I_3(\theta_1, \varphi_1 - \theta_1, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ versus θ_1 with different values of ε_3. (a) $\varepsilon_3 = 10000^{-1/2}$, (b) $\varepsilon_3 = 1000^{-1/2}$, (c) $\varepsilon_3 = 100^{-1/2}$, (d) $\varepsilon_3 = 10^{-1/2}$.

Table 1. Angle and attenuation ratio values of the critical points when $K_1 = 1000$, $K_2 = 4000$ and $K_3 = 10^5$.

Two-LACP		Three-LACP		
θ_1	Attenuation ratio	θ_1	θ_2	Attenuation ratio
0.12°	1	0.14°	0.04°	1
		0.16°	90.02°	1.185×10^{-5}
90.16°	2.515×10^{-4}	90.14°	0.10°	1.026×10^{-7}
		90.14°	–90.21°	2.537×10^{-4}

5.3. Experimental results of a two- and three-LACP

Experiment is carried out with two and three-LACP models demonstrated in Subsections 5.1 and 5.2. Servomotor linked by harmonic drive gearing, which has a low backlash of 59 s, rotates the transmission gear of each polarizer. Backlash of the transmission gear is eliminated by small springs. Movement accuracy is 0.01° per step. Using 100-mW laser with a central wavelength of 532.8 nm as the incident light, the measuring range of the PD covers 9 orders of magnitude. We carry out the attenuation process based on the analyses in Subsections 5.1 and 5.2. Rotate the polarizers with a tiny variation of the angle to find out extremum points of the output light intensity. The angle where each extremum point of the output light intensity occurs is measured. The critical attenuation ratios and the corresponding angles are calculated and shown in Table 1. The data verify the theoretical attenuation ratio values in Subsection 4.2. The variation between the maximum output point and a random minimum point is monotonic. However, extremely smaller attenuation ratio is difficult to obtain due to its requirement for location accuracy and the absolute sensitivity of the light power meter.

6. Inference about n-LACP

Based on the theoretical analyses and experiments in the above sections, we can carry out some inferences on n-LACP ($n > 3$). According to the analysis in Subsection 4.1, we can deduce that

$$\begin{aligned}\boldsymbol{E}'_{n-1} &= \begin{bmatrix} A'_{n-1} \\ B'_{n-1} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_{n-1} \end{bmatrix} \begin{bmatrix} \cos\theta_{n-2} & \sin\theta_{n-2} \\ -\sin\theta_{n-2}\cos\theta_{n-2} \end{bmatrix} \begin{bmatrix} A'_{n-2} \\ B'_{n-2} \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta_{n-2} + \varepsilon_{n-2}\sin\theta_{n-2} \\ -\varepsilon_{n-1}(\sin\theta_{n-2} - \varepsilon_{n-2}\cos\theta_{n-2}) \end{bmatrix}, \end{aligned} \quad (37)$$

where $\begin{bmatrix} A'_{n-2} \\ B'_{n-2} \end{bmatrix}$ represents the Jones vector after the $(n-2)$-th polarizer in the $(n-2)$-th coordination. E'_{n-1} represents the Jones vector of the light after the $(n-1)$-th polarizer in $(n-1)$-th coordination, if the angle enclosed by the transmission axes of $(n-1)$-th and $(n-2)$-th polarizers is θ_{n-2}. Thus, according to the analysis in Subsection 4.1, the critical angle of the n-th polarizer is

$$\begin{aligned}\varphi_{n-1} &= \arctan\frac{B'_{n-1}}{A'_{n-1}} \\ &= -\arctan\left(\varepsilon_{n-1}\frac{\tan\theta_{n-2} - \varepsilon_{n-2}}{1 + \varepsilon_{n-2}\tan\theta_{n-2}}\right) \\ &= -\arctan[\varepsilon_{n-1}\tan(\theta_{n-2} - \varphi_{n-2})]. \end{aligned} \quad (38)$$

Obviously, φ_{n-1} is determined by ε_{n-1}, θ_{n-2} and φ_{n-2} simultaneously. It is easy to conclude that φ_{n-1} is related to all values of ε_1 to ε_{n-1} and θ_1 to θ_{n-2}. In a monotonic interval from $\theta_{n-1} = \varphi_{n-1}$ to $\theta_{n-1} = \varphi_{n-1} + \pi/2$, the minimum attenuation ratio of the n-th polarizer of the attenuator is ε_n^2. By referring the deducing method adopted for the two-LACP, the attenuation ratio expression of n-LACP can be obtained as

$$\begin{aligned} I_n = I_1[1 - (1-\varepsilon_2^2)\sin^2(\theta_1 - \varphi_1)] \\ \times [1 - (1-\varepsilon_3^2)\sin^2(\theta_2 - \varphi_2)]\cdots \\ \times [1 - (1-\varepsilon_n^2)\sin^2(\theta_{n-1} - \varphi_{n-1})]. \end{aligned} \quad (39)$$

The maximum output intensity is $1 + \varepsilon_1^2$ at the point $\theta_1 = \varphi_1$ and $\theta_2 = \theta_3 = \cdots = \theta_{n-1} = 0$. The minimum attenuation ratio of n-LACP is $(\varepsilon_2\varepsilon_3\cdots\varepsilon_n)^2$, at the point $\theta_1 = \varphi_1 + \pi/2$ and $\theta_2 = \theta_3 = \cdots = \theta_{n-1} = 0$.

7. Conclusion

In this paper we proposed two basic principles of a two-LACP which differ from that of an attenuator combined by the same polarizer pair. One principle is that the minimum attenuation ratio, K_2^{-1}, depends on the extinction ratio of the second polarizer. The other is the deviation angle between the monotonic attenuation interval of the two-LACP and $[0, \pi/2]$ depends on the extinction ratio of the first polarizer, which is $\varphi_1 = \arctan\varepsilon_1$. Moreover, the attenuation ratio expression of the two-LACP is given as Eq. (14). Based on the demonstration of the two-LACP, the method of deducing critical angle φ_2 of the three-LACP is further presented and the deducing output intensity expression is also given, which is shown in Eq. (22). Furthermore, through theoretical calculation and experimental verification, it is found that when the initial state is the maximum output circumstance, rotating P_3 can cause an minimum attenuation ratio of ε_3^2 and, likewise, rotating P_2 causes an attenuation ratio of ε_2^2. Rotating P_1 causes an attenuation ratio of $\varepsilon_2^2\varepsilon_3^2$. Some inferences about φ_{n-1} and the minimum attenuation ratio of an n-LACP are also presented in this paper.

References

[1] Hsiao V K S, Li Z, Chen Z, Peng P C and Tang J Y 2009 *Opt. Express* **17** 19988

[2] Ferrari J A, Frins E M, Garbusi E and Korn M 2006 *Opt. Eng.* **45** 065001

[3] Lehmann S, Wulfmeyer V and Bosenberg J 1997 *Appl. Opt.* **36** 3469

[4] Zhao Y, Jia Y, Yang J and Zhu X 2007 *Opt. Eng.* **46** 044301

[5] Zhang X G and Zheng Y 2008 *Chin. Phys.* B **17** 2509

[6] Kim D 2005 *Appl. Opt.* **44** 1366

[7] Nee S M F 2006 *Appl. Opt.* **45** 6497

[8] Nee S M F, Cole T L, Yoo C and Burge D K 1997 *Proc. SPIE* **3121** 213

[9] Nee S M F 2003 *J. Opt. Soc. Am.* A **20** 1651

[10] Nee S M F, Yoo C, Cole T and Burge D 1998 *Appl. Opt.* **37** 54

[11] Mielenz K D and Eckerle K L 1972 *Appl. Opt.* **11** 594

[12] Huang C, Chen H Q, Liao Z S and Zhao S 2010 *Acta Phys. Sin.* **59** 1756 (in Chinese)

[13] Sun Y, Zhang C M, Du J A and Zhao B C 2010 *Acta Phys. Sin.* **59** 3863 (in Chinese)

[14] Liu B Y and Li C Y 2008 *Chin. Phys.* B **17** 3720