The time-dependent Ginzburg–Landau equation for the two-velocity difference model*

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A thermodynamic theory is formulated to describe the phase transition and critical phenomenon in traffic flow. Based on the two-velocity difference model, the time-dependent Ginzburg–Landau (TDGL) equation under certain condition is derived to describe the traffic flow near the critical point through the nonlinear analytical method. The corresponding two solutions, the uniform and the kink solutions, are given. The coexisting curve, spinodal line and critical point are obtained by the first and second derivatives of the thermodynamic potential. The modified Korteweg de Vries (mKdV) equation around the critical point is derived by using the reductive perturbation method and its kink–antikink solution is also obtained. The relation between the TDGL equation and the mKdV equation is shown. The simulation result is consistent with the nonlinear analytical result.

Keywords: traffic flow, two-velocity difference model, TDGL equation, mKdV equation

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1. Introduction

Generally speaking, traffic flow is divided into three regions: the stable region out of the coexisting curve, the metastable region between the coexisting and neutral stability lines, and the unstable region within the neutral stability line.[1,2]

To our knowledge, thermodynamic theory can be applied to describe the phase transition and critical phenomenon of traffic flow. It is well known that the time-dependent Ginzburg–Landau (TDGL) equation can describe the nonequilibrium phase transition including metastability.[3−7] It performs the jamming transition and critical phenomenon in traffic flow. Until now, there has been little analytical work on the TDGL equation in traffic flow. So it is necessary to derive the TDGL equation, which consists of the thermodynamic formulation, for describing the jamming transition from traffic flow models. It is also interesting to connect the traffic metastability with the spinodal decomposition in the first-order phase transition.[3,4]

In 1998, Nagatani[3,4] firstly presented the thermodynamic theory in traffic flow and derived the TDGL equation from two kinds of the car-following models:

$$\mathrm{d}x_n(t+\tau)/\mathrm{d}t = V(\Delta x_n(t)), \tag{1}$$

$$x_n(t+2\tau) = x_n(t+\tau) + V(\Delta x_n(t))\tau, \tag{2}$$

where $\Delta x_n(= x_{n+1} - x_n)$ is the headway, τ is the delay time, $x_n(t)$ is the position of the car n at the time t and $V(\Delta x)$ is the optimal velocity. Equation (1) reveals that a driver decides his car velocity relating to the headway $\Delta x_n(t)$. Equation (2) is the difference equation of Eq. (1). Equation (2) shows that the position of the car n at time $t+2\tau$ is determined by the position at time $t+\tau$ and the headway at time t. Then, Nagatani[7] proposed two new lattice versions of the continuum models in 1999 and derived the TDGL equations for them. Later, based on the lattice hydrodynamic models proposed by Nagatani, a lot of further work has been done.[8−10] However, unfortunately, not all traffic flow models can obtain the TDGL equation. Therefore, we will investigate the existence of the TDGL equation for the two-velocity difference model (TVDM), which is presented by Ge *et al.*[11]:

$$\mathrm{d}^2x_j(t)/\mathrm{d}t^2 = a[V(\Delta x_j(t)) - \mathrm{d}x_j(t)/\mathrm{d}t]$$

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$$+ \lambda(p\Delta v_n + (1-p)\Delta v_{n+1}), \quad (3)$$

where a is the driver's sensitivity, which is the inverse of delay time τ, λ ($\lambda > 0$) is a parameter, and p is the weighting value. It is obvious that the TVDM is more theoretically advanced than previous ones, because on the basis of the optimal velocity model (OVM)[12] and the TVDM,[13] it considers more aspects, the velocity difference $\Delta V_n (\equiv V_{n+1} - V_n)$ and ΔV_{n+1}, in the car-following process. Moreover, compared with other traffic flow models, TDVM will not happen on an unrealistcally high deceleration.

According to the TVDM, the TDGL equation, which has been seldom studied in recent years, is derived by using the nonlinear analytical method. The corresponding uniform and kink–antikink solutions are given, respectively. The coexisting curve, the spinodal line and the critical point are obtained from the thermodynamic potential. The modified Korteweg-de Vries (mKdV) equation near the critical point is derived following a similar method and the kink–antikink solution is given.

2. Model and linear stability method

We rewrite Eq. (3) as the difference equation:

$$\begin{aligned}&\frac{\mathrm{d}^2\Delta x_j(t)}{\mathrm{d}t^2}\\ &= a\left[V(\Delta x_{j+1}(t)) - V(\Delta x_j(t)) - \frac{\mathrm{d}\Delta x_j(t)}{\mathrm{d}t}\right]\\ &\quad + \lambda\left[p\frac{\mathrm{d}(\Delta x_{j+1}(t) - \Delta x_j(t))}{\mathrm{d}t}\right.\\ &\quad \left. + (1-p)\frac{\mathrm{d}(\Delta x_{j+2}(t) - \Delta x_{j+1}(t))}{\mathrm{d}t}\right]. \quad (4)\end{aligned}$$

We choose the optimal velocity $V(\Delta x_j)$ as that proposed by Bando *et al.*[12]

$$V(\Delta x_j(t)) = \tanh(\Delta x_j(t) - h_\mathrm{c}) + \tanh(h_\mathrm{c}), \quad (5)$$

where h_c is the safety distance. This optimal function has the turning point at $\Delta x_j(t) = h_\mathrm{c}$. Equation (5) increases monotonously with $\Delta x_j(t)$.

We use the linear stability method to describe the traffic flow model. Obviously, with constant headway h and the optimal velocity $V(h)$ the traffic flow keeps steady, given as

$$x_{j,0} = hj + V(h)t \quad \text{with} \quad h = L/N, \quad (6)$$

where L is the system size, N is the number of cars.

Give $y_j(t)$, a small deviation, to the uniform steady state flow:

$$x_j(t) = x_{j,0}(t) + y_j(t). \quad (7)$$

Substituting Eqs. (6) and(7) into Eq. (4), then we obtain the linearized equation,

$$\begin{aligned}\frac{\mathrm{d}^2 y_j(t)}{\mathrm{d}t^2} &= a\left[V'(h)\Delta y_j(t) - \frac{\mathrm{d}y_j(t)}{\mathrm{d}t}\right] + \lambda p\frac{\mathrm{d}\Delta y_j(t)}{\mathrm{d}t}\\ &\quad + \lambda(1-p)\frac{\mathrm{d}\Delta y_{j+1}(t)}{\mathrm{d}t}, \quad (8)\end{aligned}$$

where $V'(h)$ is the derivative of the optimal velocity $V(\Delta x_j)$ at $\Delta x_j = h$ and $\Delta y_j(t) \equiv y_{j+1}(t) - y_j(t)$.

Expanding $y_j(t) = \exp(\mathrm{i}kj + zt)$, and where $z = z_1(\mathrm{i}k) + z_2(\mathrm{i}k)^2 + \cdots$, then inserting them into Eq. (8) and keeping the first- and second-order terms of $\mathrm{i}k$, we obtain the coefficients of z, respectively,

$$z_1 = V'(h) \text{ and } z_2 = \frac{V'(h)}{2} + \frac{\lambda V'(h) - V'(h)^2}{a}. \quad (9)$$

If $z_2 > 0$, the uniform traffic flow is always stable. If $z_2 < 0$, the uniform traffic flow is unstable for long wavelength modes. Therefore we have the unstable condition:

$$a < 2V'(h) - 2\lambda. \quad (10)$$

The neutral stability condition is given as

$$a = 2V'(h) - 2\lambda. \quad (11)$$

The result is relevant with the parameter λ.

Figure 1 shows that the neutral stability lines and the critical points decline with increasing value of the parameter λ ($0 < \lambda < 1$). As the limit of $\lambda = 0$, the neutral stability line and the critical point in the TVDM are consistent with those in the OVM, which was proposed by Bando *et al.*[12] Obviously, from Fig. 1 we can see that the TVDM is more stable than the OVM, especially once the value λ becomes bigger, the stable region also becomes larger.

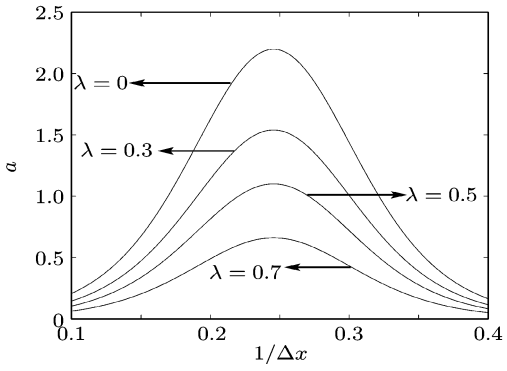


Fig. 1. The phase diagram of the model according to different values of parameter λ, ($v_\mathrm{max} = 2, \rho_\mathrm{c} = 1/h_\mathrm{c} = 0.25$).

3. Nonlinear analysis

3.1. The TDGL equation

We use the long wavelength modes to derive the TDGL equation for describing the collective motion on a coarse-grained scale. We analyse the slowly varying behaviour at long waves near the critical point. Firstly, we give slow scales for space variable j and time variable t, and define the slow variables X and T

$$X = \varepsilon(j + bt), \quad T = \varepsilon^3 t, \quad 0 < \varepsilon \ll 1, \tag{12}$$

where b is a constant to be determined later. Give

$$\Delta x_j(t) = h_c + \varepsilon R(X, T). \tag{13}$$

Next, substituting Eqs. (12) and (13) into Eq. (4) and making the Taylor expansion to the fifth-order of ε, we obtain the following nonlinear partial differential equation

$$\begin{aligned}&\varepsilon^2 f_1 \partial_X R + \varepsilon^3 f_2 \partial_X^2 R \\ &+ \varepsilon^4(\partial_T R + f_3 \partial_X^3 R + f_4 \partial_X R^3) \\ &+ \varepsilon^5(f_5 \partial_X \partial_T R + f_6 \partial_X^4 R + f_7 \partial_X^2 R^3) = 0.\end{aligned} \tag{14}$$

Here the coefficients f_i are given in Table 1.

Table 1. The coefficients f_i of the model.

f_1	f_2	f_3	
$b - V'$	$b^2/a - V'/2 - \lambda b/a$	$-V'/6 - \lambda b/2a - \lambda(1-p)b/a$	
f_4	f_5	f_6	f_7
$V'''/6$	$(2b - \lambda)/a$	$-V'/24 - \lambda b/6a - \lambda(1-p)b/a$	$V'''/12$

For convenience, in Table 1 and mentioned later we denote

$$V'(h_c) = \frac{\mathrm{d}V(\Delta x_j)}{\mathrm{d}\Delta x_j}\Big|_{\Delta x_j = h_c},$$

$$V'''(h_c) = \frac{\mathrm{d}^3 V(\Delta x_j)}{\mathrm{d}\Delta x_j^3}\Big|_{\Delta x_j = h_c}.$$

Now, we study the traffic flow near critical point $a = (1 - \varepsilon^2)a_c$, $a_c = 2V'(h_c) - 2\lambda$. In order to derive the TDGL equation, we should define

$$\lambda = V'/(6 - 6p)(0 \le p < 5/6),$$

the parameter λ is related with the weighting parameter p. Then, taking $b = V'(h_c)$, we eliminate the second-order terms of ε from Eq. (14). Equation (14) is rewritten as

$$\begin{aligned}\varepsilon^4 \partial_T R =\ & \varepsilon^4 \frac{(19 - 18p)V'}{12(5 - 6p)} \partial_X^3 R + \varepsilon^4 \frac{V'''}{6} \partial_X R^3 \\ & - \varepsilon^3 \left(\frac{5 - 6p}{6 - 6p} V'^2 \tau - V'/2\right) \partial_X^2 R \\ & + \varepsilon^5 \frac{(p-1)(19 - 18p)V'}{4(5 - 6p)^2} \partial_X^4 R \\ & + \varepsilon^5 \frac{(p-1)V'''}{2(5 - 6p)} \partial_X^2 R^3.\end{aligned} \tag{15}$$

Transforming variables X and T to variables $x = \varepsilon^{-1} X$ and $t = \varepsilon^{-3} T$, and making $S(x, t) = \varepsilon R(X, T)$, we rewrite Eq. (15) as follows:

$$\begin{aligned}&\partial_t S \\ &= \frac{(19 - 18p)V'}{12(5 - 6p)} \partial_x^3 S - \frac{|V'''|}{6} \partial_x S^3 \\ &- \left(\frac{5 - 6p}{6 - 6p} V'^2 \tau - V'/2\right) \partial_x^2 S \\ &+ \frac{(p-1)(19 - 18p)V'}{4(5 - 6p)^2} \partial_x^4 S \\ &- \frac{(p-1)|V'''|}{2(5 - 6p)} \partial_x^2 S^3,\end{aligned} \tag{16}$$

where $V''' < 0$.

Adding term

$$\frac{5 - 6p}{3 - 3p} \left(\frac{5 - 6p}{6 - 6p} V'^2 \tau - V'/2\right) \partial_x S$$

to both the left- and right-hand sides and performing Galilean transformation $t_1 = t$ and

$$x_1 = x - \frac{5 - 6p}{3 - 3p} \left(\frac{5 - 6p}{6 - 6p} V'^2 \tau - V'/2\right) t$$

in Eq. (16), we obtain

$$\begin{aligned}&\partial_{t_1} S \\ &= \left(\partial_{x_1} - \frac{3 - 3p}{5 - 6p} \partial_{x_1}^2\right) \left[\frac{(19 - 18p)}{12(5 - 6p)} V' \partial_{x_1}^2 S \right. \\ &\left. + \frac{5 - 6p}{3 - 3p} \left(\frac{5 - 6p}{6 - 6p} V'^2 \tau - V'/2\right) S - \frac{|V'''|}{6} S^3\right].\end{aligned} \tag{17}$$

We define the thermodynamic potential:

$$\begin{aligned}\phi(S) \equiv & -\frac{5 - 6p}{6 - 6p} \left(\frac{5 - 6p}{6 - 6p} V'^2 \tau - V'/2\right) S^2 \\ & + \frac{|V'''|}{24} S^4.\end{aligned} \tag{18}$$

By taking Eq. (18) into Eq. (17), we derive the TDGL equation as

$$\partial_{t_1} S = -\left(\partial_{x_1} - \frac{3-3p}{5-6p}\partial_{x_1}^2\right)\frac{\delta \Phi(S)}{\delta S}$$

with

$$\Phi(S) \equiv \int \mathrm{d}x_1 \left[\frac{(19-18p)}{24(5-6p)}V'(\partial_{x_1}S)^2 + \phi(S)\right], \quad (19)$$

where $\phi(S)$ is given by Eq. (18).

Except for the trivial solution $S = 0$, the TDGL Eq. (19) has two steady-state solutions: the one is the uniform solution

$$S(x_1, t_1) = \pm\left[\frac{\frac{2(5-6p)}{1-p}\left(\frac{5-6p}{6-6p}V'^2\tau - V'/2\right)}{|V'''|}\right]^{1/2}, \quad (20)$$

and the other is the kink solution

$$S(x_1, t_1) = \pm\left[\frac{\frac{2(5-6p)}{1-p}\left(\frac{5-6p}{6-6p}V'^2\tau - V'/2\right)}{|V'''|}\right]^{1/2} \times \tanh\left[\left\{\frac{(5-6p)^2}{(1-p)(19-18p)} \times \left(\frac{5-6p}{3-3p}V'\tau - 1\right)\right\}^{1/2}(x_1 - x_0)\right], \quad (21)$$

where x_0 is a constant. Equation (21) represents the coexisting phase.

We can have the coexisting curve, the spinodal line and the critical point according to the thermodynamic potential Eq. (18). The condition of the coexisting curve is given as

$$\frac{\partial \phi}{\partial S} = 0 \quad \text{and} \quad \frac{\partial^2 \phi}{\partial S^2} > 0. \quad (22)$$

Inserting Eq. (18) into Eq. (22), we have the coexisting curve in terms of the original parameters:

$$(\Delta x)_{\mathrm{co}} = h_{\mathrm{c}} + \left[\frac{\frac{5-6p}{2(1-p)}V'\left(\frac{5-6p}{3-3p}V'\tau - 1\right)}{|V'''|}\right]^{1/2}. \quad (23)$$

The condition of the spinodal line is given as

$$\frac{\partial^2 \phi}{\partial S^2} = 0. \quad (24)$$

Inserting Eq. (18) into Eq. (24), we have the spinodal line in terms of the original parameters:

$$(\Delta x)_{\mathrm{sp}} = h_{\mathrm{c}} + \left[\frac{\frac{5-6p}{3(1-p)}V'\left(\frac{5-6p}{3-3p}V'\tau - 1\right)}{|V'''|}\right]^{1/2}. \quad (25)$$

The condition of the critical point is given as

$$\frac{\partial \phi}{\partial S} = 0 \quad \text{and} \quad \frac{\partial^2 \phi}{\partial S^2} = 0. \quad (26)$$

Inserting Eq. (18) into Eq. (26), we have the critical point in terms of the original parameters:

$$(\Delta x)_{\mathrm{c}} = h_{\mathrm{c}} \quad \text{and} \quad \tau_{\mathrm{c}} = \frac{3(1-p)}{(5-6p)V'}. \quad (27)$$

Substituting $\lambda = V'/(6-6p)$ defined before into Eq. (27), we find that the result is consistent with Eq. (11).

3.2. The mKdV equation

We use the reductive perturbation method to obtain the mKdV equation. We analyse the traffic jam around the critical point $a = (1-\varepsilon^2)a_{\mathrm{c}}$. We also give slow scales for space variable x and time variable t. In Eq. (14), substituting $a_{\mathrm{c}} = 2V'(h_{\mathrm{c}}) - 2\lambda$, making $b = V'(h_{\mathrm{c}})$ and eliminating the second- and third-order terms of ε, we obtain

$$\varepsilon^4[\partial_T R - g_1\partial_X^3 R + g_2\partial_X R^3] + \varepsilon^5[g_3\partial_X^2 R + g_4\partial_X^4 R + g_5\partial_X^2 R^3] = 0. \quad (28)$$

Making the following transformations to obtain the standard mKdV equation from Eq. (28):

$$T' = g_1 T, \quad R = \sqrt{\frac{g_1}{g_2}}R', \quad (29)$$

We obtain the standard mKdV equation as follows:

$$\partial_{T'}R' = \partial_X^3 R' - \partial_{X'}R'^3 - \varepsilon\left[\frac{g_3}{g_1}\partial_X^2 R' + \frac{g_4}{g_1}\partial_X^4 R' + \frac{g_5}{g_2}\partial_X^2 R'^3\right]. \quad (30)$$

If we ignore the $O(\varepsilon)$, then we obtain the mKdV equation with a kink–antikink soliton solution

$$R'_o(X, T') = \sqrt{c}\tanh\sqrt{c/2}(X - cT'). \quad (31)$$

Assuming $R'(X, T') = R'_o(X, T') + \varepsilon R'_1(X, T')$, we consider the $O(\varepsilon)$ correction. For the purpose of determining the selected value of the propagation velocity c for the kink solution, it is necessary to agree with the solvability condition:

$$(R'_0, M[R'_0]) \equiv \int_{-\infty}^{\infty} \mathrm{d}X R'_0 M[R'_0] = 0, \quad (32)$$

where

$$M[R'_o] = \frac{g_3}{g_1}\partial_X^2 R' + \frac{g_4}{g_1}\partial_X^4 R' + \frac{g_5}{g_2}\partial_X^2 R'^3.$$

Performing the integration, we obtain the general velocity c,[14]

$$c = \frac{5g_2g_3}{2g_2g_4 - 3g_1g_5}. \tag{33}$$

Substituting the coefficients in Table 2 into Eq. (33), we have ($V' = 1$)

$$c = \frac{20(-1+\lambda)^2}{-4 - 4\lambda^2 + 4p\lambda^2 - 7\lambda + 6p\lambda}. \tag{34}$$

Taking the original variables, we obtain the general kink–antikink soliton solution for the model[11] ($V' = 1, V''' = -2$)

$$\Delta x_j(t) = h_c + \sqrt{\frac{g_1 c}{g_2}\left(\frac{\tau}{\tau_c} - 1\right)} \tanh\sqrt{\frac{c}{2}\left(\frac{\tau}{\tau_c} - 1\right)} \times \left[j + \left(1 - cg_1\left(\frac{\tau}{\tau_c} - 1\right)\right)t\right]. \tag{35}$$

If we take $\lambda = V'/(6-6p)$ ($V' = -1, 0 \le p < 5/6$) defined before into Eq. (35), we will obtain a new kink soltion solution:

$$\Delta x_j(t) = h_c \pm \left[\frac{\frac{2(5-6p)}{1-p}\left(\frac{5-6p}{6-6p}V'^2\tau - V'/2\right)}{|V'''|}\right]^{1/2} \tanh\left[\left\{\frac{(5-6p)^2}{(1-p)(19-18p)}\left(\frac{5-6p}{3-3p}V'\tau - 1\right)\right\}^{1/2} \times \left(j + \frac{5-6p}{6-6p}V'\left(1 - \frac{5-6p}{3-3p}V'\tau\right)t\right]. \tag{36}$$

Table 2. The coefficients g_i of the model.

g_1	g_2	g_3
$V'(-2V' - 7\lambda + 6p\lambda)/12(-V' + \lambda)$	$\lvert V'''\rvert/6$	$V'/2$
g_4	g_5	
$-V'(-V'^2 - 2\lambda^2 + 2p\lambda^2)/8(-V' + \lambda)^2$	$V'\lvert V'''\rvert/12(-V' + \lambda)$	

This kink soliton solution also represents the coexisting phase. The kink soliton solution is obtained from the mKdV equation, which agrees with Eq. (21) from the TDGL equation. Therefore, the result reveals that the jamming transition can be described by both the TDGL equation with a nontravelling solution and the mKdV equation with a propagating solution.[3,4]

The simulation is carried out by Eq. (8). The boundary condition is periodic. The initial conditions are selected as follows: $\Delta x_j(0) = \Delta x_0 = 4$, $\Delta x_j(1) = \Delta x_0 = 4.0$ for $j \neq 100, 101$, $\Delta x_j(1) = 4.0 - 0.5$ for $j = 100$, and $\Delta x_j(1) = 4.0 + 0.5$ for $j = 101$, where the sum of cars is $N = 200$ and the safety distance is $h_c = 4.0$. Figure 2 shows the space–time evolution of the headway for $a = 1\ \mathrm{s}^{-1}$, $h = 4.0$ m, $V_{\max} = 2.0$ m/s, and $p = 0.667$. The kink–antikink soliton near the critical point is depicted clearly by Fig. 2 and the density wave propagates backward with time t. From Fig. 2, we can see that the simulation result is in agreement with the analytical consequence.

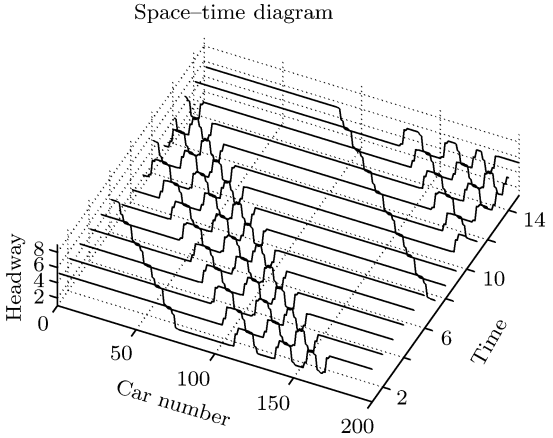


Fig. 2. Space–time evolution of the headway after $t = 10000$ for $a = 1\ \mathrm{s}^{-1}$, $h = 4.0$ m, and $v_{\max} = 2.0$ m/s.

4. Conclusion

We have applied the thermodynamic theory to describe the jamming transition in the TVDM. By constructing a certain parameter relation between λ and p in the TVDM, we can derive the TDGL equation through the perturbation method. The coexisting curve, the spinodal line and the critical point are given by the derivatives of the thermodynamic poten-

tial that is related to the delay time. Also we have obtained the mKdV equation near the critical point from the TVDM. The connection between the TDGL equation and the mKdV equation has been shown. The simulation result is consistent with the analytical one.

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