Complete synchronization of double-delayed Rössler systems with uncertain parameters*

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In this paper, we investigate complete synchronization of double-delayed Rössler systems with uncertain parameters as the master system is in chaotic synchronization. The uncertain parameters can be nonlinearly expressed in the system. The analysis and proof are given by means of the Lyapunov stability theorem. Based on theoretical analysis, some sufficient conditions of complete synchronization are proved. In order to validate the proposed scheme, numerical simulations are performed and the numerical results show that our scheme is very effective.

Keywords: complete synchronization, double-delayed Rössler system, uncertain parameters

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1. Introduction

In recent years, the synchronization of chaotic systems has become an area of active research, partly due to its potential application in secure communication. Since the synchronization of chaotic dynamical systems was observed by Pecora and Carroll[1,2] in 1990, different methods of chaotic synchronization have been proposed in interacting chaotic systems both theoretically and experimentally, such as the drive-response method, the variable feedback method,[3] the adaptive method,[4,5] the pulse method,[6,7] and the linear observer based method.[8,9] However, most of the research has focused on chaotic systems with certain parameters. In fact, we can hardly ever obtain parameters of chaotic dynamical systems in communication, so the study on the synchronization of chaotic systems with uncertain parameters has some practical value.[10,11] Recently, chaotic time-delayed systems have been suggested as good candidates for secure communication and they have been paid more attention because the dimensions of their chaotic dynamics can be raised by increasing the delay time sufficiently. Otherwise, chaotic time-delayed systems can be considered as a special cases of spatiotemporal systems. It extends complete synchronization to infinite-dimensional systems.[12−16]

In the present paper, we consider the complete synchronization in double-delayed Rössler systems with uncertain parameters. The rest of this paper is organized as follows. In Section 2, we describe a double-delayed Rössler system. In Section 3, we theoretically analyse the complete synchronization of double-delayed Rössler systems with uncertain parameters on the basis of the Lyapunov stability theory. In Section 4, numerical simulations are presented to confirm the validity of the proposed theoretical approach in Section 3. Finally, conclusions are drawn in Section 5.

2. Description of the system

A double-delayed Rössler system is written as[17]

$$\begin{cases} \dot{x}_1 = -x_2 - x_3 + ax_1(t-\tau_1) + bx_1(t-\tau_2), \\ \dot{x}_2 = x_1 + ax_2, \\ \dot{x}_3 = 0.2 + x_1x_3 - cx_3. \end{cases} \tag{1}$$

In Eq. (1), x_1, x_2, and x_3 are state variables; a, b, and c are the related parameters; τ_1 and τ_2 are delay parameters. The system is chaotic over a wide parameter range. The chaotic attractor for $a = 0.2$, $b = 0.5$, $c = 5.7$, and $\tau_1 = 1$, $\tau_2 = 2$ is given in Fig. 1.

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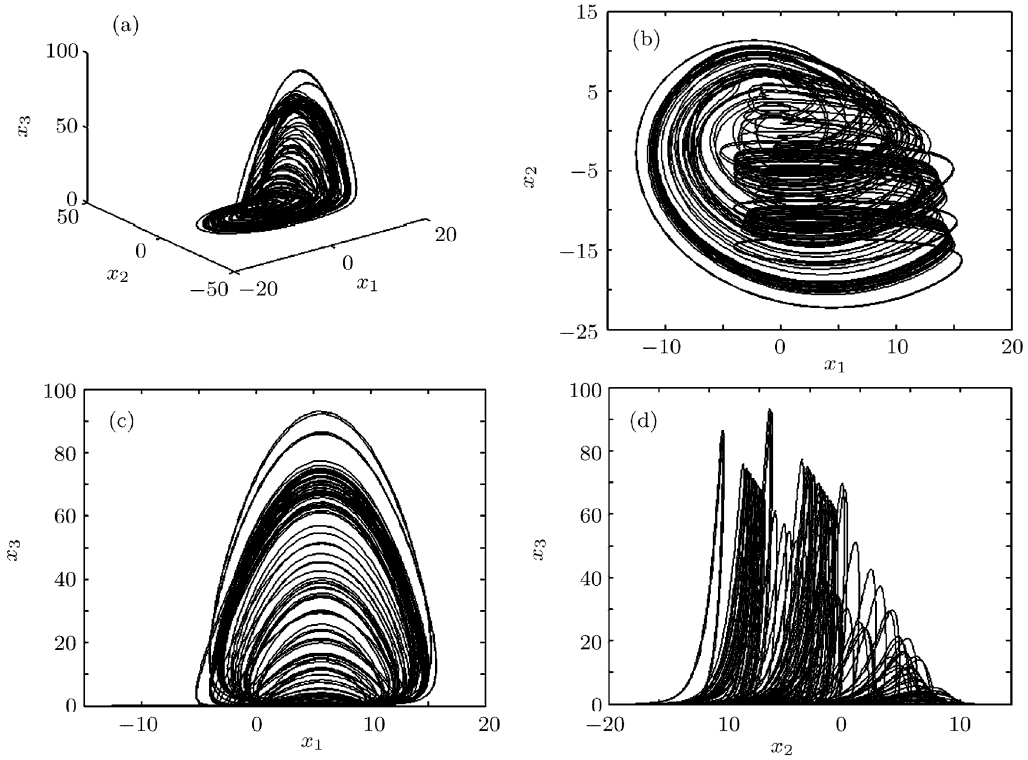


Fig. 1. Chaotic attractor of a double-delayed Rössler system.

3. Synchronization of double-delayed Rössler systems with uncertain parameters

Assume that we have two double-delayed Rössler systems where the master (or drive) system is denoted as $\boldsymbol{x}$ and the slave (or response) system denoted as $\boldsymbol{y}$. For system (1), the master (or drive) and the slave (or response) systems can be described as

$$\begin{cases} \dot{x}_1 = -x_2 - x_3 + ax_1(t-\tau_1) + bx_1(t-\tau_2), \\ \dot{x}_2 = x_1 + ax_2, \\ \dot{x}_3 = 0.2 + x_1x_3 - cx_3, \end{cases} \tag{2}$$

and

$$\begin{cases} \dot{y}_1 = -y_2 - y_3 + \tilde{a}y_1(t-\tau_1) + \tilde{b}y_1(t-\tau_2) + u_1, \\ \dot{y}_2 = y_1 + \tilde{a}y_2 + u_2, \\ \dot{y}_3 = 0.2 + y_1y_3 - \tilde{c}y_3 + u_3, \end{cases} \tag{3}$$

where $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ are the parameters of the slave system which need to be estimated, and u_1, u_2, and u_3 are the nonlinear controller such that the two double-delayed Rössler systems can be synchronized, i.e.

$$\begin{cases} \lim_{t\to\infty} \|y_1 - x_1\| = 0, \\ \lim_{t\to\infty} \|y_2 - x_2\| = 0, \\ \lim_{t\to\infty} \|y_3 - x_3\| = 0. \end{cases} \tag{4}$$

The error states are

$$\begin{cases} \dot{e}_1 = \dot{y}_1 - \dot{x}_1, \\ \dot{e}_2 = \dot{y}_2 - \dot{x}_2, \\ \dot{e}_3 = \dot{y}_3 - \dot{x}_3. \end{cases} \tag{5}$$

The error dynamical system can be written as

$$\begin{cases} \dot{e}_1 = -y_2 - y_3 + \tilde{a}y_1(t-\tau_1) + \tilde{b}y_1(t-\tau_2) + u_1 \\ \qquad - (-x_2 - x_3 + ax_1(t-\tau_1) + bx_1(t-\tau_2)), \\ \dot{e}_2 = y_1 + \tilde{a}y_2 + u_2 - (x_1 + ax_2), \\ \dot{e}_3 = 0.2 + y_1y_3 - \tilde{c}y_3 + u_3 \\ \qquad - (0.2 + x_1x_3 - cx_3). \end{cases} \tag{6}$$

Our aim is to find control law u_1, u_2, and u_3 for stabilizing the error variables of the systems at the origin. At the end of this part we propose the following control law and update the rule for the system

$$\begin{cases} u_1 = k_1(y_1 - x_1), \\ u_2 = k_2(y_2 - x_2), \\ u_3 = x_1x_3 - y_1y_3. \end{cases} \tag{7}$$

The update rule for the three unknown parameters $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ are

$$\begin{cases} \dot{\tilde{a}} = -e_1y_1(t-\tau_1) - e_2y_2, \\ \dot{\tilde{b}} = -e_1y_1(t-\tau_2), \\ \dot{\tilde{c}} = e_3y_3. \end{cases} \tag{8}$$

Theorem 1 For the given $k_1 \leq -5/2 - (a^2+b^2)/4$, $k_2 \leq -2-a$, the synchronization between drive system (2) and response system (3) will occur by control law (7) and update rule (8).

Proof Choose the following function

$$\begin{aligned} & V_{(e_1,e_2,e_3)} \\ =& \frac{1}{2}(e_1^2+e_2^2+e_3^2)+\frac{1}{2}(e_a^2+e_b^2+e_c^2) \\ &+ \int_{-\tau_1}^{0}(e_1^2(t+\theta)+e_2^2(t+\theta)+e_3^2(t+\theta))\,\mathrm{d}\theta \\ &+ \int_{-\tau_2}^{0}(e_1^2(t+\beta)+e_2^2(t+\beta)+e_2^2(t+\beta))\,\mathrm{d}\beta, \end{aligned} \tag{9}$$

where $e_a = \tilde{a}-a, e_b = \tilde{b}-b, e_c = \tilde{c}-c$. The time derivative of V along the trajectory of error system (5) is

$$\begin{aligned} \dot{V} =& (e_1\dot{e}_1+e_2\dot{e}_2+e_3\dot{e}_3+e_a\dot{e}_a+e_b\dot{e}_b+e_c\dot{e}_c) \\ &+(e_1^2-e_1^2(t-\tau_1))+(e_2^2-e_2^2(t-\tau_1)) \\ &+(e_3^2-e_3^2(t-\tau_1))+(e_1^2-e_1^2(t-\tau_2)) \\ &+(e_2^2-e_2^2(t-\tau_2))+(e_3^2-e_3^2(t-\tau_2)), \\ \dot{V} =& e_1(-e_2-e_3+ae_1(t-\tau_1)+be_1(t-\tau_2) \\ &+e_ay_1(t-\tau_1)+e_by_1(t-\tau_2)+k_1e_1) \\ &+e_2(e_1+ae_2+e_ay_2+k_2e_2) \\ &+e_3(-ce_3-e_cy_3)+e_a\dot{e}_a+e_b\dot{e}_b+e_c\dot{e}_c \\ &+(e_1^2-e_1^2(t-\tau_1))+(e_2^2-e_2^2(t-\tau_1)) \\ &+(e_3^2-e_3^2(t-\tau_1))+(e_1^2-e_1^2(t-\tau_2)) \\ &+(e_2^2-e_2^2(t-\tau_2))+(e_3^2-e_3^2(t-\tau_2)) \\ =& -e_1e_3+ae_1e_1(t-\tau_1)+be_1e_1(t-\tau_2) \\ &+e_1(e_ay_1(t-\tau_1)+e_by_1(t-\tau_2))+k_1e_1^2 \\ &+ae_2^2+e_2e_ay_2+k_2e_2^2-ce_3^2-e_3e_cy_3+e_a\dot{e}_a \\ &+e_b\dot{e}_b+e_c\dot{e}_c+(e_1^2-e_1^2(t-\tau_1)) \\ &+(e_2^2-e_2^2(t-\tau_1))+(e_3^2-e_3^2(t-\tau_1)) \\ &+(e_1^2-e_1^2(t-\tau_2))+(e_2^2-e_2^2(t-\tau_2)) \\ &+(e_3^2-e_3^2(t-\tau_2)) \\ \leq& \left(\frac{5}{2}+\frac{a^2}{4}+\frac{b^2}{4}+k_1\right)e_1^2+(a+2+k_2)e_2^2 \\ &+\left(\frac{5}{2}-c\right)e_3^2+e_a(\dot{e}_a+e_1y_1(t-\tau_1)+e_2y_2) \\ &+e_b(\dot{e}_b+e_1y_1(t-\tau_2))+e_c(\dot{e}_c-e_3y_3). \end{aligned}$$

Inserting Eq. (8) into the above equation yields the following expression

$$\dot{V} = -\,\mathrm{e}^{\mathrm{T}}Ke,$$

where $\mathrm{e}^{\mathrm{T}} = (e_1,e_2,e_3)^{\mathrm{T}}$, and

$$K = \begin{bmatrix} \left(-\frac{5}{2}-\frac{a^2}{4}-\frac{b^2}{4}-k_1\right) & 0 & 0 \\ 0 & (-a-2-k_2) & 0 \\ 0 & 0 & \left(c-\frac{5}{2}\right) \end{bmatrix} \geq 0.$$

Since $\dot{V} \leq 0$, we have $e_i \to 0, (i=1,2,3)$ i.e., $\lim_{t\to\infty}\|e_i\| = 0$. This completes the proof.

4. Numerical simulations

In this section, numerical simulations are presented to verify the effectiveness of the proposed synchronization controller. The parameters are chosen to be $a = 0.2$, $b = 0.5$, $c = 5.7$ and delay parameters are chosen as $\tau_1 = 1, \tau_2 = 2$. So that the double-delayed Rössler system exhibits chaotic behaviour if no control is applied to it. The initial conditions of the drive system are $x_{1(0)} = -4$, $x_{2(0)} = 4$, and $x_{3(0)} = 0$, and those of the response system are $y_{1(0)} = 1$, $y_{2(0)} = 1$, and $y_{3(0)} = 1$. Moreover, the initial values of the estimated parameters are chosen to be $\tilde{a}_{(0)} = 0.1$, $\tilde{b}_{(0)} = 0.1$, and $\tilde{c}_{(0)} = 0.1$. Furthermore, the control gains are chosen as $k_1 = -3$, $k_2 = -2.5$.

The simulation results are shown in Figs. 2–4. The evolution of the errors is depicted in Fig. 2, which displays the time response of the errors $e \to 0$ with $t \to \infty$. Figure 3 shows the portrait of state variables between the drive and response systems. Figure 4 depicts the evolution of the estimated parameters of the double-delayed Rössler system, which shows that the estimates of the unknown parameters adapt themselves to the true values. These results show that the required synchronization has been achieved by our designed control law (7) and parameter update law (8).

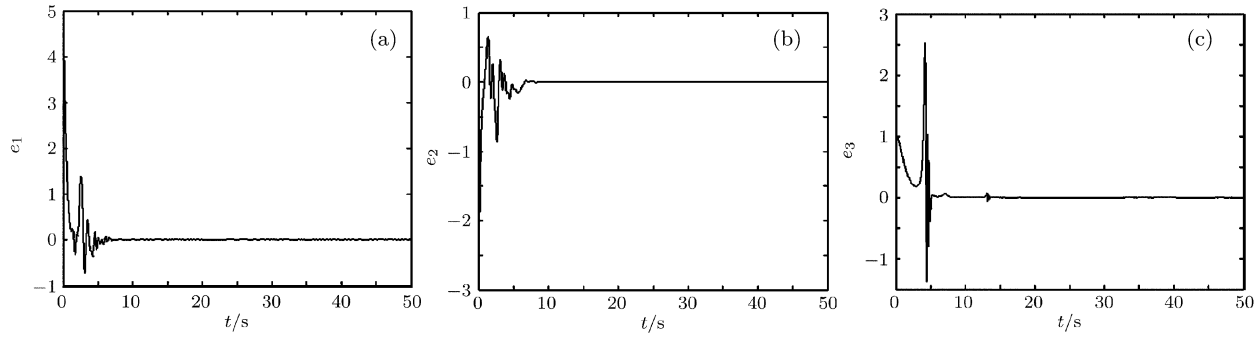


Fig. 2. The time evolution of errors between the drive and response systems.

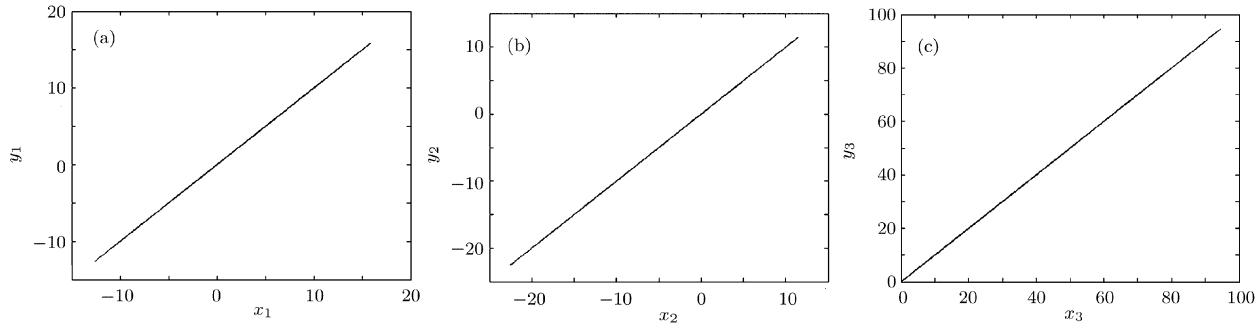


Fig. 3. Plots of state variables between drive and response systems.

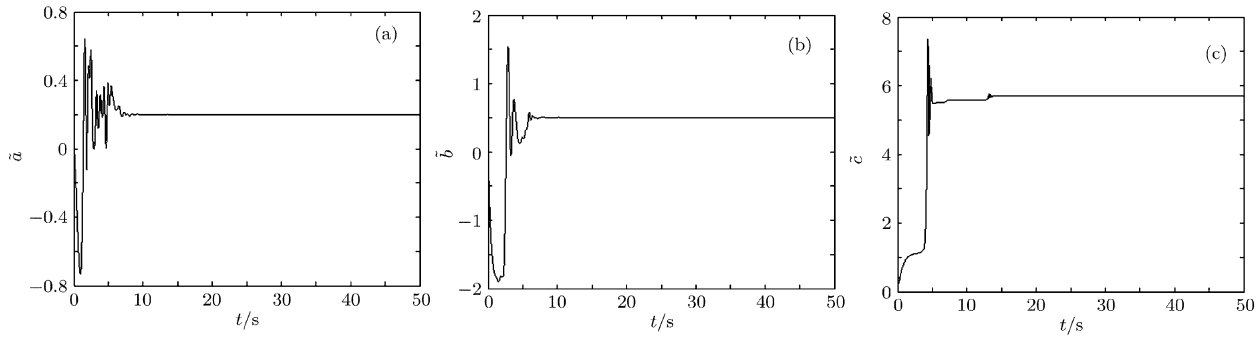


Fig. 4. The time evolution of the estimated unknown parameters.

5. Conclusion

This paper investigated the synchronization of double-delayed Rössler systems. A simple but efficient controller was designed to synchronize the chaotic dynamics between two coupled identical double-delayed Rössler chaotic systems, which could be proved by the Lyapunov stability theorem. By virtue of the synchronization-based method, the unknown parameters of the Rössler system were exactly estimated. The strengths k_1 and k_2 have less effect on the time of synchronization. The synchronization can be almost realized in 8 s if k_1 and k_2 had satisfied the theorem. The synchronization scheme we proposed, as it realized the synchronization of double-delayed Rössler systems with uncertain parameters, is also adaptable to multiple-delayed chaotic system. It will be contribute to synchronization research in the area of time-delayed chaotic systems and chaotic systems with uncertain parameters.

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