A novel mixed-synchronization phenomenon in coupled Chua's circuits via non-fragile linear control*

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Dynamical variables of coupled nonlinear oscillators can exhibit different synchronization patterns depending on the designed coupling scheme. In this paper, a non-fragile linear feedback control strategy with multiplicative controller gain uncertainties is proposed for realizing the mixed-synchronization of Chua's circuits connected in a drive-response configuration. In particular, in the mixed-synchronization regime, different state variables of the response system can evolve into complete synchronization, anti-synchronization and even amplitude death simultaneously with the drive variables for an appropriate choice of scaling matrix. Using Lyapunov stability theory, we derive some sufficient criteria for achieving global mixed-synchronization. It is shown that the desired non-fragile state feedback controller can be constructed by solving a set of linear matrix inequalities (LMIs). Numerical simulations are also provided to demonstrate the effectiveness of the proposed control approach.

Keywords: mix-synchronization, Chua's circuit, non-fragile control, Lyapunov stability, linear matrix inequality

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1. Introduction

Chua's circuit is a nonlinear electronic circuit, consisting of four linear elements (two capacitors, one resistor and one inductor) and a piecewise-linear nonlinear resistor.[1] Its chaotic nature was first observed through computer simulations by Matsumoto *et al.*,[2] and later confirmed experimentally by others.[3,4] The rigorous theoretical proofs of the chaotic behaviour in Chua's circuit have been given in Ref. [5], showing that the system can exhibit a double-scroll strange attractor.

Because of its simplicity, robustness and low cost, Chua's circuit has become a favourite tool for many scientific research directions of chaos. Among them, the synchronization of Chua's circuits has received a tremendous increasing interest in recent years.[6−21] These and other earlier studies on the synchronization of Chua's circuits are mainly concerned with conventional synchronization (including complete synchronization (CS),[6−8] anti-synchronization (AS),[10,11] adaptive synchronization,[13−15] phase synchronization,[16] robust fault-sensitive synchronization,[17] and projective synchronization (PS)[18,19]) with either linear or nonlinear, unidirectional or bidirectional coupling schemes. In addition, there is another novel synchronization phenomenon, i.e., mixed-synchronization, a regime where different dynamical variables of the coupled nonlinear oscillators can achieve different synchronization pattern.[22] This type of synchronization may have potential applications in the processing industry or in a catalytic reaction where there is a need to control the concentrations of reacting chemical components in different proportions for obtaining a desired product output. Therefore, it is significant to investigate the mixed-synchronization of coupled Chua's circuits with an appropriate designed controller. This strongly motivates the research of this paper.

Another motivation comes from concerns relating to controller gain variations. A common implicit assumption made in the above publications is that the control methodologies can be implemented precisely. In practice, however, uncertainties or inaccuracies do occur in the implementation stage of the designed controllers due to round-off errors in numerical arithmetic, finite word length in digital systems, inherent imprecision in analog systems, etc.[23] Consequently,

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even though the designed controllers are robust with respect to system uncertainties, they may be very fragile due to their own uncertainties. In fact, it has been demonstrated by Keel and Bhattacharyya[24] that controllers designed by modern robust control techniques can be very fragile to relatively small perturbations in the controller coefficients. This brings about the so-called non-fragile controller design problem, i.e., how to design a controller such that it is insensitive (or resilient) to some uncertainties in its coefficients.

Following on from this idea, there have been many relevant studies tackling the non-fragile controller design problem (see for example Refs. [25]–[31] and references therein). Haddad and Corrado[25] proposed a non-fragile controller design method via quadratic Lyapunov bounds. For linear time-invariant systems, a non-fragile H_∞ controller is given in Ref. [27] with respect to multiplicative controller gain variations in terms of symmetric positive-definite solutions of algebraic Riccati inequalities. In Ref. [28], Park constructed a robust non-fragile decentralized controller for discrete-delay large-scale systems with parametric uncertainties and multiplicative controller gain variations. In the work of Zhang *et al.*,[29] a new approach to the problem of robust and non-fragile H_∞ state feedback controller design for uncertain T–S fuzzy systems with linear fractional parametric uncertainties has been provided. The design problem of non-fragile controller for discrete descriptor systems with time-invariant and norm-bounded controller gain variations was addressed in Ref. [31]. These efforts allow us to consider the controller fragility problem for the control and synchronization of coupled chaotic systems. To the best of our knowledge, non-fragile control for mixed-synchronization phenomenon in coupled Chua's circuits has never been investigated in the past. The above situation is exactly what concerns and interests us.

This paper is concerned with the non-fragile controller design problem for mixed-synchronization of coupled Chua's circuits. It is assumed that the feedback gains to be designed have multiplicative controller gain variations. The main contribution of this paper lies in the following two aspects: Firstly, a novel non-fragile feedback controller is designed for two identical Chua's circuits connected in a drive-response configuration. Through both theoretical analysis and numerical simulations, we prove that the designed controllers are efficient for realizing a mixed-synchronization pattern. Secondly, by exploring a Schur complement lemma, the desired non-fragile state feedback controllers for coupled Chua's circuits can be constructed by solving a set of linear matrix inequalities (LMIs), which can be easily tested by using commercially available software.

The rest of this paper is organized as follows. In Section 2, problem formulation and preliminaries are given. Section 3 is devoted to deriving some sufficient criteria for the mixed-synchronization. In Section 4, numerical simulations are given to show the effectiveness of the proposed control method. Finally, conclusions in Section 5 close the paper.

2. Model description and preliminaries

2.1. The Chua's circuit

Chua's circuit in a dimensionless form is as follows:

$$\begin{cases} \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = p(x_2 - x_1 - f(x_1)), \\ \dfrac{\mathrm{d}x_2}{\mathrm{d}t} = x_1 - x_2 + x_3, \\ \dfrac{\mathrm{d}x_3}{\mathrm{d}t} = -qx_2, \end{cases} \tag{1}$$

where $p > 0$, $q > 0$, $f(\cdot)$ is a piecewise-linear function described by

$$f(x_1) = bx_1 + \frac{1}{2}(a-b)(|x_1+1| - |x_1-1|) \tag{2}$$

with $a < b < 0$.

In compact form, we express Eq. (1) as

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{x} + \boldsymbol{F}(\boldsymbol{x}), \tag{3}$$

where $\boldsymbol{x} = (x_1, x_2, x_3)^{\mathrm{T}}$, and

$$\boldsymbol{A} = \begin{pmatrix} -p-pb & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{B} = \begin{pmatrix} 0 & p & 0 \\ 1 & 0 & 1 \\ 0 & -q & 0 \end{pmatrix},$$

$$\boldsymbol{F}(\boldsymbol{x}) = \begin{pmatrix} \frac{p(b-a)}{2}(|x_1+1| - |x_1-1|) \\ 0 \\ 0 \end{pmatrix}. \tag{4}$$

2.2. The mixed-synchronization scheme

To realize mix-synchronization of two coupled Chua's circuits via linear feedback control in a drive-response configuration, we consider system (3) as the master system. The corresponding slave system with the same model equations is designed as

$$\frac{\mathrm{d}\boldsymbol{y}}{\mathrm{d}t} = \boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}\boldsymbol{y} + \boldsymbol{F}(\boldsymbol{y}) + \boldsymbol{u}(t), \tag{5}$$

where $\boldsymbol{y} = (y_1, y_2, y_3)^{\mathrm{T}}$ is the state vector of the response system, $\boldsymbol{u}(t) = (u_1(t), u_2(t), u_3(t))^{\mathrm{T}}$ denotes the linear controller to be designed later.

Let constant matrix $\boldsymbol{H} = \mathrm{diag}\{\alpha_1, \alpha_2, \alpha_3\}$, where $\alpha_i \in \{1, -1, 0\}$ $(i = 1, 2, 3)$, we define the mixed-synchronization phenomenon as follows.

Definition 1 For drive system (3) and response system (5), we say that they achieve global mixed-synchronization with respect to the scaling matrix $\boldsymbol{H}$ if

$$\lim_{t\to\infty} \|\boldsymbol{y}(t, \boldsymbol{y}_0, \boldsymbol{u}) - \boldsymbol{H}\boldsymbol{x}(t, \boldsymbol{x}_0)\| = 0 \tag{6}$$

holds for any initial conditions $\boldsymbol{x}_0$, $\boldsymbol{y}_0 \in \boldsymbol{R}^3$, where $\boldsymbol{x}(t, \boldsymbol{x}_0)$ and $\boldsymbol{y}(t, \boldsymbol{y}_0, \boldsymbol{u})$ represent the solutions of systems (3) and (5), respectively.

Remark 1 It is easy to see that Definition 1 includes the known notions of CS and AS as its special cases. When all $\alpha_i = 1$ $(i = 1, 2, 3)$, i.e., $\boldsymbol{H} = \mathrm{diag}\{1, 1, 1\}$, mixed-synchronization phenomenon reduces to CS. When all $\alpha_i = -1$ $(i = 1, 2, 3)$, i.e., $\boldsymbol{H} = \mathrm{diag}\{-1, -1, -1\}$, mixed-synchronization phenomenon reduces to AS, which has been investigated much more.

2.3. Some lemmas

In this subsection, we shall present some lemmas that will be required during the proof of our main results.

Lemma 1[11] The nonlinear function $\boldsymbol{F}(\cdot)$ of Chua's circuit defined in Eq. (4) is a continuous nonlinear function satisfying Lipschitz condition, namely,

$$\|\boldsymbol{F}(\boldsymbol{y}) - \boldsymbol{F}(\boldsymbol{x})\| \leq \lambda\|\boldsymbol{y} - \boldsymbol{x}\|, \tag{7}$$

with the Lipschitz constant $\lambda = p(b - a)$ for any $\boldsymbol{x}$, $\boldsymbol{y} \in \boldsymbol{R}^3$.

Lemma 2 Let $\boldsymbol{\xi}$, $\boldsymbol{\eta}$ be real column vectors of appropriate dimensions, $\boldsymbol{P}$ is a real matrix with appropriate dimension, then we have the following inequality

$$2\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{\eta} \leq \sigma\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\xi} + \frac{1}{\sigma}\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\eta} \tag{8}$$

for any scalar $\sigma > 0$.

3. Global mixed-synchronization in coupled Chua's circuits

Define the mix-synchronization error signal as

$$\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{H}\boldsymbol{x}(t),$$

where $\boldsymbol{e} = (e_1, e_2, e_3)^{\mathrm{T}} \in \boldsymbol{R}^3$. Then the error dynamics between the master system (3) and the slave system (5) can be expressed as

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{e} + \boldsymbol{B}\boldsymbol{y} - \boldsymbol{H}\boldsymbol{B}\boldsymbol{x} + \boldsymbol{F}(\boldsymbol{y}) - \boldsymbol{H}\boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{u}(t). \tag{9}$$

Here, we design a non-fragile state feedback controller with multiplicative controller gain variations of the following form

$$\boldsymbol{u}(t) = \boldsymbol{H}\boldsymbol{B}\boldsymbol{x} - \boldsymbol{B}\boldsymbol{y} - (\boldsymbol{K} + \Delta\boldsymbol{K})\boldsymbol{e}, \tag{10}$$

where $\boldsymbol{K} \in \boldsymbol{R}^{3\times 3}$ is the nominal controller gain to be designed, $\Delta\boldsymbol{K}$ represents the following multiplicative gain perturbations:

$$\Delta\boldsymbol{K} = \boldsymbol{M}_m\Phi_m(t)\boldsymbol{N}_m\boldsymbol{K}, \tag{11}$$

with $\boldsymbol{M}_m$ and $\boldsymbol{N}_m$ being known constant matrices, and the uncertain matrix $\Phi_m(t)$ satisfying

$$\Phi_m^{\mathrm{T}}(t)\Phi_m(t) \leq \boldsymbol{I}. \tag{12}$$

Remark 2 In designed controller (10), only multiplicative uncertainties in the controller gain matrix $\boldsymbol{K}$ are considered. However, from a practical point of view one expects the matrices $\boldsymbol{H}\boldsymbol{B}$ and $\boldsymbol{B}$ (that also appears in the controller) not to be exactly known (for instance, due to parametric uncertainties). Indeed, we can also investigate the non-fragile property of controller (10) with respect to uncertainties in $\boldsymbol{H}\boldsymbol{B}$ and $\boldsymbol{B}$ through replacing $\boldsymbol{H}\boldsymbol{B}$ and $\boldsymbol{B}$ by $\boldsymbol{H}\boldsymbol{B} + \Delta\boldsymbol{K}_1$ and $\boldsymbol{B} + \Delta\boldsymbol{K}_2$ ($\Delta\boldsymbol{K}_1$ and $\Delta\boldsymbol{K}_2$ represent the gain perturbations), respectively. The procedure is similar to the present work and thus is omitted here for simplicity.

With control laws (10)–(12), the error dynamics can be expressed by the following form,

$$\begin{aligned}\dot{\boldsymbol{e}} = {} & \boldsymbol{A}\boldsymbol{e} + \boldsymbol{F}(\boldsymbol{y}) - \boldsymbol{H}\boldsymbol{F}(\boldsymbol{x}) \\ & - (\boldsymbol{K} + \boldsymbol{M}_m\Phi_m(t)\boldsymbol{N}_m\boldsymbol{K})\boldsymbol{e}.\end{aligned} \tag{13}$$

Now we will establish an LMI criterion for the stability of error system (13) using the Lyapunov stability theory.

Theorem 1 For master-slave Chua's circuits (3) and (5) with linear controllers (10)–(12), if there exist a symmetric positive-definite matrix $\boldsymbol{X}$, a matrix $\boldsymbol{Y}$, and positive constants μ and ε, such that the following LMI holds

$$\begin{pmatrix} \boldsymbol{XA}+\boldsymbol{AX}-(\boldsymbol{Y}^{\mathrm{T}}+\boldsymbol{Y})+\mu\boldsymbol{M}_m\boldsymbol{M}_m^{\mathrm{T}}+\varepsilon\boldsymbol{I} & \boldsymbol{Y}^{\mathrm{T}}\boldsymbol{N}_m^{\mathrm{T}} & \lambda\boldsymbol{X} \\ \boldsymbol{N}_m\boldsymbol{Y} & -\mu\boldsymbol{I} & 0 \\ \lambda\boldsymbol{X} & 0 & -\varepsilon\boldsymbol{I} \end{pmatrix} < 0, \tag{14}$$

then error system (13) is globally exponentially stable, implying that coupled systems (3) and (5) achieve global mix-synchronization. In this case, a desired controller can be obtained in the form of expressions (10)–(12) with the controller gain

$$\boldsymbol{K} = \boldsymbol{Y}\boldsymbol{X}^{-1}. \tag{15}$$

Proof We choose the Lyapunov function as

$$V(t) = \boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}, \tag{16}$$

where $\boldsymbol{P} > 0$ is to be selected later.

Differentiating Eq. (16) with respect to t along the trajectory of system (13) gives

$$\begin{aligned} \dot{V}(t) &= \dot{\boldsymbol{e}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}\dot{\boldsymbol{e}} \\ &= (\boldsymbol{A}\boldsymbol{e} + \boldsymbol{F}(\boldsymbol{y}) - \boldsymbol{F}(\boldsymbol{H}\boldsymbol{x}) - \boldsymbol{K}\boldsymbol{e} \\ &\quad - \boldsymbol{M}_m\Phi_m(t)\boldsymbol{N}_m\boldsymbol{K}\boldsymbol{e})^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e} \\ &\quad + \boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{A}\boldsymbol{e} + \boldsymbol{F}(\boldsymbol{y}) - \boldsymbol{F}(\boldsymbol{H}\boldsymbol{x}) \\ &\quad - \boldsymbol{K}\boldsymbol{e} - \boldsymbol{M}_m\Phi_m(t)\boldsymbol{N}_m\boldsymbol{K}\boldsymbol{e}) \\ &= \boldsymbol{e}^{\mathrm{T}}\left[\boldsymbol{A}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - (\boldsymbol{K}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{K})\right]\boldsymbol{e} \\ &\quad - 2\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{M}_m\Phi_m(t)\boldsymbol{N}_m\boldsymbol{K}\boldsymbol{e} \\ &\quad + 2\left(\boldsymbol{F}(\boldsymbol{y}) - \boldsymbol{F}(\boldsymbol{H}\boldsymbol{x})\right)^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}. \end{aligned} \tag{17}$$

According to Lemma 1 and Lemma 2, we have a new bound of the time-derivative of $V(t)$ as follows:

$$\begin{aligned} \dot{V}(t) \le \boldsymbol{e}^{\mathrm{T}}\Bigg[&\boldsymbol{A}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - (\boldsymbol{K}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{K}) \\ &+ \mu\boldsymbol{P}\boldsymbol{M}_m\boldsymbol{M}_m^{\mathrm{T}}\boldsymbol{P}^{\mathrm{T}} \\ &+ \frac{1}{\mu}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{N}_m^{\mathrm{T}}\boldsymbol{N}_m\boldsymbol{K} + \varepsilon\boldsymbol{P}^2 + \frac{1}{\varepsilon}\lambda^2\boldsymbol{I}\Bigg]\boldsymbol{e}. \end{aligned} \tag{18}$$

If the following inequality holds,

$$\begin{aligned} \boldsymbol{A}\boldsymbol{P} &+ \boldsymbol{P}\boldsymbol{A} - (\boldsymbol{K}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{K}) + \mu\boldsymbol{P}\boldsymbol{M}_m\boldsymbol{M}_m^{\mathrm{T}}\boldsymbol{P}^{\mathrm{T}} \\ &+ \frac{1}{\mu}\boldsymbol{K}^{\mathrm{T}}\boldsymbol{N}_m^{\mathrm{T}}\boldsymbol{N}_m\boldsymbol{K} + \varepsilon\boldsymbol{P}^2 + \frac{1}{\varepsilon}\lambda^2\boldsymbol{I} < 0, \end{aligned} \tag{19}$$

then we have

$$\dot{V}(t) < 0. \tag{20}$$

Based on the Lyapunov stability theory, error dynamical system (13) is globally exponentially stable. Thus, global mix-synchronization between systems (3) and (5) has been achieved.

By defining $\boldsymbol{X} = \boldsymbol{P}^{-1}$, $\boldsymbol{Y} = \boldsymbol{K}\boldsymbol{X}$, and pre-multiplying and post-multiplying inequality (19) by the matrix $\boldsymbol{X}$, this inequality is modified as

$$\begin{aligned} \boldsymbol{X}\boldsymbol{A} &+ \boldsymbol{A}\boldsymbol{X} - (\boldsymbol{Y}^{\mathrm{T}} + \boldsymbol{Y}) + \mu\boldsymbol{M}_m\boldsymbol{M}_m^{\mathrm{T}} \\ &+ \frac{1}{\mu}\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{N}_m^{\mathrm{T}}\boldsymbol{N}_m\boldsymbol{Y} + \varepsilon\boldsymbol{I} + \frac{1}{\varepsilon}\lambda^2\boldsymbol{X}^2 < 0. \end{aligned} \tag{21}$$

The equivalence of Eq. (21) and Eq. (14) follows from the Schur complement lemma.[32] Moreover, it is easy to obtain the feedback gain matrix $\boldsymbol{K} = \boldsymbol{Y}\boldsymbol{X}^{-1}$. This completes the proof.

Remark 3 It should be observed that matrix inequality (14) is linear in variables $\boldsymbol{X}$, $\boldsymbol{Y}$, μ, and ε, which can be easily solved using the Matlab LMI Control Toolbox.

4. Numerical simulations

In this section, numerical simulations are given to illustrate the correctness and effectiveness of our theoretical analysis. In all of the simulations, we always choose the system parameters of the Chua's circuit as $p = 10$, $q = 14.87$, $a = -1.27$ and $b = -0.68$, which causes the Chua's circuit to exhibit a double-scroll chaotic attractor, as shown in Fig. 1 with the initial states $x_1(0) = 0.6$, $x_2(0) = -0.2$, $x_3(0) = 0.3$. In addition, the initial conditions $x_1(0) = 0.6$, $x_2(0) = -0.2$, $x_3(0) = 0.3$ for drive system (3) and $y_1(0) = -1$, $y_2(0) = 0.5$, $y_3(0) = 2$ for response system (5) are chosen for the mixed-synchronization problem.

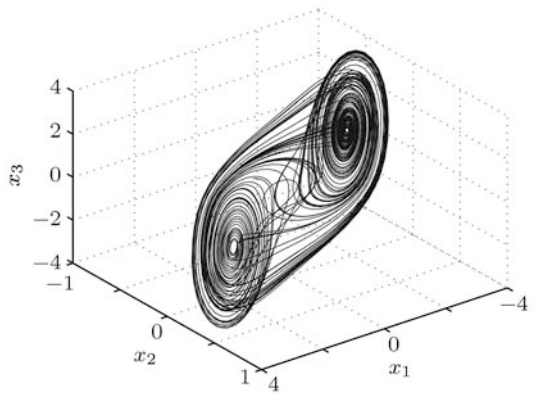


Fig. 1. The double-scroll chaotic attractor for the Chua's circuit in Eq. (1) for $p = 10$, $q = 14.87$, $a = -1.27$, and $b = -0.68$.

The multiplicative controller uncertainties in Eq. (11) are considered through

$$\boldsymbol{M}_m = \begin{pmatrix} 0.13 & 0.13 & 0.13 \\ 0.13 & 0.13 & 0.13 \\ 0.13 & 0.13 & 0.13 \end{pmatrix},$$

$$\boldsymbol{N}_m = \begin{pmatrix} 0.05 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 \end{pmatrix},$$

$$\boldsymbol{\Phi}_m(t) = \sin(3t)\boldsymbol{I}. \tag{22}$$

By using the Matlab LMI Control Toolbox, the feasible solution of the LMI (14) is attainted as

$$\boldsymbol{X} = \begin{pmatrix} 4.3540 & -0.1990 & -0.2073 \\ -0.1990 & 4.3172 & -0.2259 \\ -0.2073 & -0.2259 & 4.3004 \end{pmatrix},$$

$$\boldsymbol{Y} = \begin{pmatrix} 37.5843 & -28.6732 & 20.6987 \\ 28.0819 & 47.0456 & -27.4611 \\ -21.5321 & 26.0336 & 51.2918 \end{pmatrix},$$

$$\mu = 39.2151,\ \varepsilon = 49.9540.$$

Therefore, a non-fragile linear state feedback controller in the form of Eqs. (10)–(12) is obtained with controller gain

$$\boldsymbol{K} = \boldsymbol{Y}\boldsymbol{X}^{-1} = \begin{pmatrix} 8.5923 & -5.9884 & 4.9129 \\ 6.6873 & 10.9181 & -5.4897 \\ -4.0746 & 6.4740 & 12.0710 \end{pmatrix}. \tag{23}$$

According to Theorem 1, we know that the synchronization error $\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{H}\boldsymbol{x}(t)$ converges to zero, thus the mixed-synchronization of linearly coupled systems (3) and (5) is achieved. Again, the scaling matrix $\boldsymbol{H}$ allows choices of different goals set by $\alpha_i \in \{1, -1, 0\}$ for different state variables of the response system. Figures 2 and 3 exhibit the CS phenomenon with $\boldsymbol{H} = \boldsymbol{I}$ and the AS phenomenon with $\boldsymbol{H} = -\boldsymbol{I}$, respectively, which are the special cases of the mixed-synchronization phenomenon. Figure 4 shows the mixed-synchronization phenomenon with $\boldsymbol{H} = \mathrm{diag}\{-1, 1, 0\}$, from which we observe that the AS state is realized in response variable y_1 when CS in y_2 and AD in y_3. In the figures, one can see that the system is well synchronized irrespective of controller gain variations.

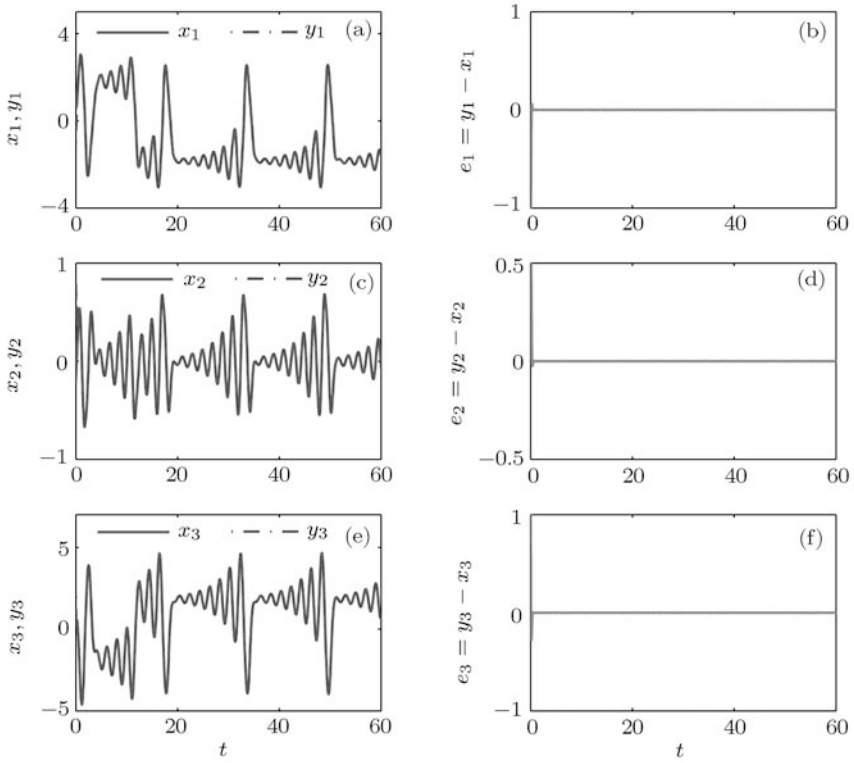


Fig. 2. Complete synchronization in coupled Chua's circuits (3) and (5) with non-fragile control laws (10)–(12) for $\boldsymbol{H} = \mathrm{diag}\{1, 1, 1\}$: (a) x_1 and y_1, (b) $e_1 = y_1 - x_1$, (c) x_2 and y_2, (d) $e_2 = y_2 - x_2$, (e) x_3 and y_3, (f) $e_3 = y_3 - x_3$.

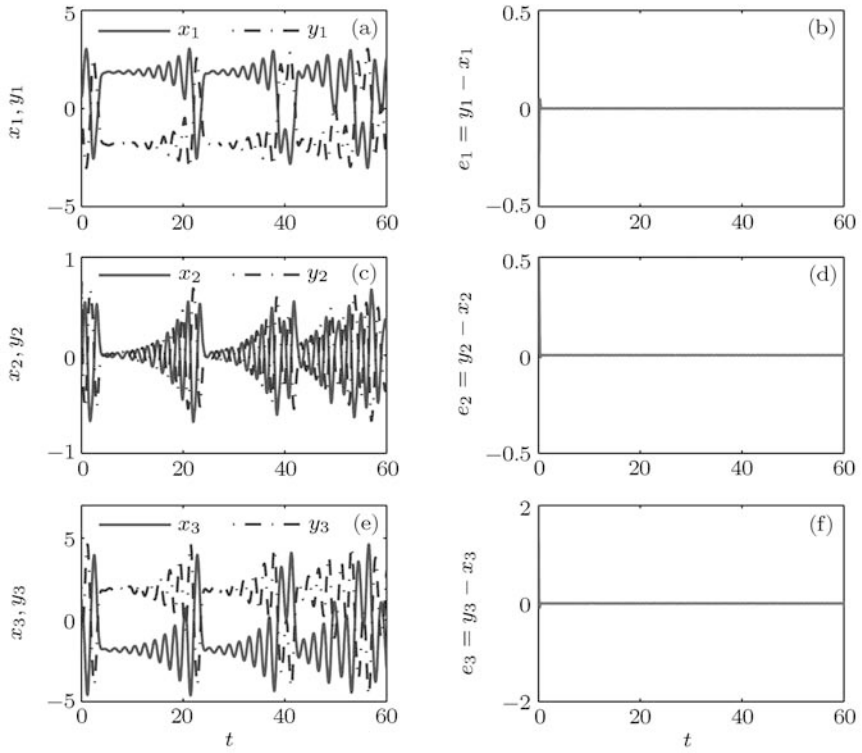


Fig. 3. Anti-synchronization in coupled Chua's circuits (3) and (5) with non-fragile control laws (10)–(12) for $\boldsymbol{H} = \mathrm{diag}\{-1, -1, -1\}$: (a) x_1 and y_1, (b) $e_1 = y_1 + x_1$, (c) x_2 and y_2, (d) $e_2 = y_2 + x_2$, (e) x_3 and y_3, (f) $e_3 = y_3 + x_3$.

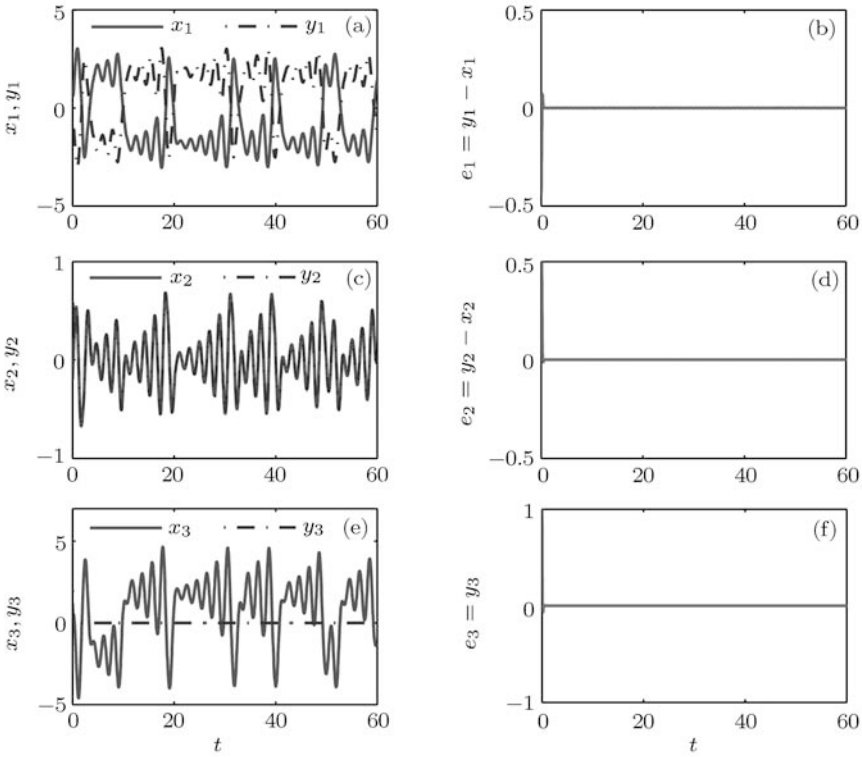


Fig. 4. Mixed-synchronization in coupled Chua's circuits (3) and (5) with non-fragile control laws (10)–(12) for $\boldsymbol{H} = \mathrm{diag}\{-1, 1, 0\}$: (a) x_1 and y_1, (b) $e_1 = y_1 + x_1$, (c) x_2 and y_2, (d) $e_2 = y_2 - x_2$, (e) x_3 and y_3, (f) $e_3 = y_3$.

5. Conclusions and discussions

While many studies have studied the synchronization problem of Chua's circuits,[6−21] this study presents a novel mixed-synchronization phenomenon. The mixed-synchronization not only enables the CS and AS to be achieved in a systematic way, but also can exhibit CS, AS and AD simultaneously in different state variables. Unlike previous non-feedback and nonlinear feedback approaches, which are inevitably much less flexible, we have presented the non-fragile linear state feedback control method for realizing mixed-synchronization behaviour. Our designed linear controller is not only easy to implement, but also non-fragile to controller gain variations. By the Lyapunov stability theorem, we have derived Theorem 1 for the global mixed-synchronization with multiplicative controller gain variations, which are arranged in the form of linear matrix inequality (LMI). Thus, they are easy to verify through the LMI Control Toolbox in Matlab.

In view of similar structures of the generalizations of Chua's cirucit (e.g., modified Chua's circuit,[33−36] fourth-order Chua's circuit,[37,38] Chua's circuit family,[39] and Chua's oscillator,[40]) with original Chua's circuit in Eq. (1), the proposed non-fragile control method here can be further extended to the mixed-synchronization of such systems. In addition, dynamical units in nature and society are often interconnected to form various complex networks, ranging from the microscopic level of genetic, proteins, metabolic and signal transduction networks to the macroscopic human level of the Internet, food webs and power grids.[41] Therefore, realizing the mixed-synchronization behaviour of largely networked systems will be an interesting and challenging subject, and this will be our future work.

References

[1] Chua L O, Komuro M and Matsumoto T 1986 *IEEE Trans. Circuits Syst.* **CAS-33** 1073

[2] Matsumoto T 1984 *IEEE Trans. Circuits Syst.* **CAS-31** 1055

[3] Zhong G Q and Ayrom F 1985 *Int. J. Circuit Theor. Appl.* **13** 93

[4] Matsumoto T, Chua L O and Komuro M 1985 *IEEE Trans. Circuits Syst.* **CAS-32** 797

[5] Shil'nikov L P 1994 *Int. J. Bifur. Chaos* **4** 489

[6] Chua L O, Itoh M, Kocarev L and Eckert K 1993 *J. Circuits Syst. Comput.* **3** 93

[7] Zhang H B, Xia J W, Yu Y B and Dang C Y 2010 *Chin. Phys.* B **19** 030505

[8] Fu S H and Pei L J 2010 *Acta Phys. Sin.* **59** 5985 (in Chinese)

[9] Kocarev L M and Janjić P A 1995 *IEEE Trans. Circuits Syst.* I: *Fund. Theor. Appl.* **42** 779

[10] Miller D A, Kowalski K L and Lozowski A 1999 *Proc. IEEE ISCAS '99* **5** 458

[11] Li C D and Liao X F 2006 *Int. J. Bifur. Chaos* **16** 1041

[12] Liao X X, Chen G, Xu B J and Shen Y 2005 *Int. J. Bifur. Chaos* **15** 2227

[13] Zhang J, Xu H B and Wang H J 2006 *Chin. Phys.* **15** 953

[14] Li J F, Li N, Cai L and Zhang B 2008 *Acta Phys. Sin.* **57** 7500 (in Chinese)

[15] Feng Y F and Zhang Q L 2010 *Chin. Phys.* B **19** 120504

[16] Wang S H, Liu W Q, Ma B J, Xiao J H and Jiang D Y 2005 *Chin. Phys.* **14** 55

[17] Xu S Y, Yang Y, Liu X, Tang Y and Sun H D 2011 *Chin. Phys.* B **20** 020509

[18] Hung Y C, Yan J J and Liao T L 2008 *Math. Comput. Simul.* **77** 374

[19] Meng J and Wang X Y 2009 *Acta Phys. Sin.* **58** 819 (in Chinese)

[20] Buscarino A, Fortuna L, Frasca M and Sciuto G 2009 *Int. J. Bifur. Chaos* **19** 3101

[21] Porfiri M and Fiorilli F 2010 *Physica* D **239** 454

[22] Grosu I, Banerjee R, Roy P K and Dana S K 2009 *Phys. Rev.* E **80** 016212

[23] Jadbabaie A, Abdallah C T, Famularo D and Dorato P 1998 *Proc. American Control Conference* June 21–26, 1998, Philadelphia, USA, pp. 2842–2846

[24] Keel L H and Bhattacharyya S P 1997 *IEEE Trans. Automat. Control* **42** 1098

[25] Haddad W M and Corrado J 1997 *Proc. 35th IEEE Conf. Decis. Control* December 10–12, 1997, San Diego, USA, pp. 2678–2683

[26] Dorato P 1998 *Proc. American Control Conference* June 21–26, 1998, Philadelphia, USA, pp. 2829–2831

[27] Yang G H and Wang J L 2001 *Automatica* **37** 727

[28] Park J H 2004 *Appl. Math. Comput.* **149** 147

[29] Zhang B Y, Zhou S S and Li T 2007 *Informat. Sci.* **177** 5118

[30] Yang G H and Che W W 2008 *Automatica* **44** 2849

[31] Wo S L, Zou Y, Chen Q W and Xu S Y 2009 *J. Franklin Inst.* **346** 914

[32] Boyd S, EI Ghaoui L, Feron E and Balakrishnan V 1994 *Linear Matrix Inequalities in System and Control Theory* (Philadelphia: SIAM)

[33] Zhong G Q 1994 *IEEE Trans. Circuits Syst.* I: *Fund. Theor. Appl.* **41** 934

[34] Huang A, Pivka L, Wu C W and Franz M 1996 *Int. J. Bifur. Chaos* **6** 2175

[35] Tang K S, Man K F, Zhong G Q and Chen G 2003 *Contr. Theor. Appl.* **20** 223

[36] Stegemann C, Albuquerque H A and Rech P C 2010 *Chaos* **20** 023103

[37] Thamilmaran K, Lakshmanan M and Venkatesan A 2004 *Int. J. Bifur. Chaos* **14** 221

[38] Liu X, Wang J Z and Huang L 2007 *Int. J. Bifur. Chaos* **17** 2705

[39] Chua L O and Lin G N 1990 *IEEE Trans. Circuits Syst.* **CAS-37** 885

[40] Chua L O, Wu C W, Huang A and Zhong G Q 1993 *IEEE Trans. Circuits Syst.* I: *Fund. Theor. Appl.* **CAS-40** 732

[41] Wang X F and Chen G 2003 *IEEE Circuits Syst. Mag.* **3** 6