Quantum nondemolition measurements of a flux qubit coupled to a noisy detector*

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We theoretically study the quantum nondemolition measurements of a flux qubit coupled to a noisy superconducting quantum interference device (SQUID). The obtained analytical results indicate that the measurement probability is frequency-dependent in a short time scale and has a close relationship with the measurement-induced dephasing. Furthermore, when the detuning between the driven and bare resonator equals the coupling strength, we can obtain the maximum measurement rate that is determined by the character of the noise in the SQUID. Finally, we analysed the mixed effect caused by coupling between the non-diagonal term and the external variable. It is found that the initial information of the qubit is destroyed due to quantum tunneling between the qubit states.

Keywords: Josephson device, quantum nondemolition measurements, quantum noise

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1. Introduction

The concept of quantum nondemolition (QND) measurement, which can be used for the detection of weak forces acting on systems such as gravitational waves, was introduced to allow for repeatable detection.[1] The basic requirement of a QND measurement is that a series of measurements of the variable of system should give predictable results.[2] It means that the observable $\hat{A}$ must commute with Hamiltonian $\hat{H}$ that describes the interacting system and the detector, i.e., $[\hat{A}, \hat{H}] = 0$. A large number of works associated with QND measurements have been undertaken in quantum optics,[3−5] cavity quantum electrodynamics (QED)[6,7] and circuit QED.[8,9]

Recently, QND measurements have been achieved experimentally on a superconducting flux qubit system by investigating the correlation between the results of two consecutive measurements.[10] However, in general, the measuring apparatus is a mesoscopic system (e.g., a direct current superconducting quantum interference device (DC-SQUID)), which is coupled to the environment. Therefore, noise can affect the discrimination of the signal, which reflects the information of the qubit via coupling between the meter and the qubit (see Fig. 1(b)). There are a large number of works regarding a mesoscopic system where one is forced to think about the quantum mechanics of the detection process[11−13] and about the fundamental quantum limits constraining the performance of the detector.[14,15] In practice, noise plays an important role in quantum measurement: quantum noise from the meter acts back on the system, such as the qubit; at the same time, the information about the variable conjugate to the measured variable is destroyed. This phenomenon is omnipresent, because information about the system is carried away into the surroundings due to indirect environmental coupling via coupling to the detector.[16] For a weak measurement, the detection may be quantum limited so that the signal-to-noise of measurement, defined as the ratio of the amplitude of the oscillation line in the output spectrum to background noise, is no more than 4.[15]

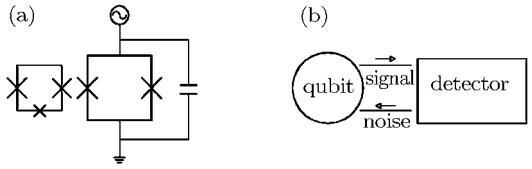


Fig. 1. (a) Schematic diagram of a flux qubit coupled to an SQUID, which is used for qubit readout as a detector. (b) Schematic diagram of the process of information detraction from the qubit and noise injection from a one-port detector.

In this article, we study the role of noise in QND

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measurements. Our model, which is used to analyse the interaction between the qubit and the detector, is based on the theory of QND measurement of flux qubit dynamics in a short time.[17] The conclusion shows that the easier we gain information about one variable, the faster we lose information about its conjugate variable. It means that the DC-SQUID can be used as a quantum-limited mesoscopic detector.[18] We also study the deviation of QND measurement due to the coupling between the non-diagonal term of the qubit and the variable of detector. This article is organized as follows. In Section 2 the basic models are introduced, from which we use a master equation to discuss the dephasing caused by quantum noise and its correlation with measurement probability distribution. In Section 3, we discuss the effect of the non-diagnose term on QND measurement. It shows that the mixed effect caused by coupling between the non-diagnose term and the detector can destroy the qubit information.

2. Probability distribution and measurement-induced dephasing

The Hamiltonian of the system is given by[17]

$$\hat{H}(t) = \hat{H}_{\rm s} + \hat{H}_{\rm meter} + \hat{H}_{\rm int} + \hat{H}_{\rm drive}(t), \tag{1}$$

where $\hat{H}_{\rm s}$ is the Hamiltonian of the qubit, which can be considered as a two-level system

$$\hat{H}_{\rm s} = \frac{\epsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x, \tag{2}$$

where $\hat{\sigma}_z$ and $\hat{\sigma}_x$ are Pauli matrices, ϵ and Δ are the energy difference and tunnel splitting of the qubit. The Hamiltonian of the detector, which acts as an oscillator, is

$$\hat{H}_{\rm meter} = \hbar\omega_{\rm os}\hat{a}^\dagger\hat{a}. \tag{3}$$

The Hamiltonian that describes coupling between the qubit and the oscillator is

$$\hat{H}_{\rm int} = \hbar g\hat{\sigma}_z\hat{a}^\dagger\hat{a}, \tag{4}$$

from which we know that the variable that we want to measure is $\hat{\sigma}_z$. Variable g is the coupling strength. In other words, we can conclude the state of the qubit from the states of the detector according to the value of $\hat{\sigma}_z$. We assign $|0\rangle$ to $\sigma_z = 1$ and $|1\rangle$ to $\sigma_z = -1$. The Hamiltonian of the external drive of the harmonic oscillator is

$$\hat{H}_{\rm drive}(t) = f(t)(\hat{a} + \hat{a}^\dagger), \tag{5}$$

here $f(t) = 2f\cos(\omega_{\rm d}t)$ is the driving force with driving frequency $\omega_{\rm d}$. For an ideal QND measurement, it is necessary that the condition $[\hat{\sigma}_z, \hat{H}] = 0$ must be satisfied. However, for superconducting flux qubits, due to the off-diagonal term $\Delta \neq 0$, the measurement is not perfect QND. In cases where $\Delta \ll \epsilon$ is satisfied, $\hat{\sigma}_z$ still would be treated as a conserved quantity on the time scale determined by $1/\epsilon$ and one expects small deviations from an ideal QND case.[17] After neglecting the off-diagonal term and moving in the rotating frame with frequency $\omega_{\rm d}$, we obtain the new Hamiltonian

$$\hat{H} = \frac{\epsilon}{2}\hat{\sigma}_z + \hbar(\delta\omega + g\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + f(\hat{a} + \hat{a}^\dagger) \tag{6}$$

with $\delta\omega = \omega_{\rm os} - \omega_{\rm d}$. Then we can obtain a QND measurement from the form of the Hamiltonian. In order to find the relationship between measurement-induced dephasing and probability distribution, we analyse the impact of noise on QND measurement in subsection 2.1 and make a more quantitative analysis in subsection 2.2.

2.1. The impact of noise on QND measurement

Based on the model mentioned in Ref. [17], we substitute Gaussian white noise for zero-point fluctuation due to the fact that noise contributes a large part to the actual measurement. The distribution of qubit signal I conditioned on the qubit state $\sigma_z = \pm 1$ is[18,19]

$$p(I|\hat{\sigma}_z) = \frac{1}{\sqrt{2\pi S_{\rm II}t}}\exp\left[\frac{-(x - x_i)}{2S_{\rm II}t}\right], \tag{7}$$

here $x_i(t) = \sqrt{2}{\rm Re}[\alpha_i(t)]$, $i = \pm$, $S_{\rm II}$ is noise spectra density. After turning on the interaction with time t, the coherent state qubit-dependent amplitude is found to be[17]

$$\alpha_i(t) = A_i\,{\rm e}^{{\rm i}\phi_i}[1 - {\rm e}^{-{\rm i}\delta\omega_i t - \kappa t/2}] \tag{8}$$

with qubit-dependent detuning $\delta\omega_i = \delta\omega - g\sigma_z$, the qubit-dependent amplitudes and phase are given by

$$A_i = \frac{f}{\sqrt{(\delta\omega_i)^2 + \kappa^2/4}}, \tag{9}$$

$$\phi_i = \arctan\left(\frac{\delta\omega_i}{\kappa/2}\right) - \frac{\pi}{2}, \tag{10}$$

where κ is the decay rate of the SQUID. If we choose a time $t \approx 1/\epsilon$, we obtain the outcome probability distribution

$$P(I,t) = \frac{1}{2}\left[1 + I\,{\rm erf}\left(\frac{\delta x(t)}{\sqrt{2S_{\rm II}t}}\right)\langle\hat{\sigma}_z\rangle_0\right], \tag{11}$$

where the signal difference is

$$\delta x(t) = \sqrt{2}\mathrm{Re}[\alpha_+ - \alpha_-] = \frac{ft}{2\pi}\sqrt{(\cos 2\phi_+ - \cos 2\phi_-)^2 + (\sin 2\phi_+ - \sin 2\phi_-)^2}. \quad (12)$$

From Eq. (11) we can find that the signal difference δx plays a crucial role in probability distribution and the only thing that noise can affect is measurement time. From Fig. 2, we can find that the signal difference is frequency-dependent. The measurement rate can be reflected by the slope of the curves, which reaches the maximum when the detuning between the driven and the bare resonator equals coupling strength, i.e. the white region is an invalid region where we cannot obtain such a huge measurement rate. In the yellow region, the strong project measurement is possible due to its large measurement rate. In the gray (the weak measurement) region the measurement rate is too weak to dispersively resolve the signal, but a projected measurement can still be realized by increasing measurement time.

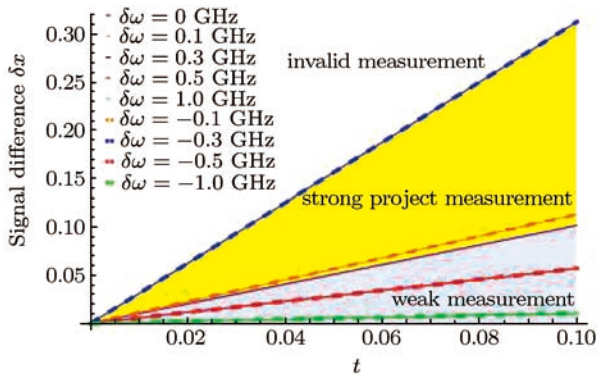


Fig. 2. (colour online) Numerable calculation of signal difference with detuning $\delta\omega = 0$ GHz (purple straight line), $\delta\omega = 0.1$ GHz (orange straight line), $\delta\omega = -0.1$ GHz (orange dashed line), $\delta\omega = 0.3$ GHz (blue straight line), $\delta\omega = -0.3$ GHz (blue dashed line), $\delta\omega = 0.5$ GHz (red straight line), $\delta\omega = -0.5$ GHz (red dashed line), $\delta\omega = 1$ GHz (green straight line), $\delta\omega = -1$ GHz (green dashed line). The adopted parameters are coupling strength $g = 0.3$ GHz, driving force $f = 1$ GHz, decay rate $\kappa = 0.1$ GHz.

In Fig. 3 we plot the probability of measuring the 0 state on the condition that we consider the contribution of zero point fluctuation (Fig. 3(a)). We also plot the probability of measuring the 0 state under the condition that we consider the contribution of back action noise instead of the zero point fluctuation during the measurement process (Fig. 3(b)). Due to the influence of back action noise, we can find that the strong measurement region is reduced, in comparison with the one only considered in the zero point fluctuation. It means that we need more time to reach the same measurement strength as the situation where we only consider the contribution of the zero point fluctuation. Another interesting character in Fig. 3 is the detuning symmetry around the resonance point, which is also reflected by Fig. 2. All of these prove that noise effectively reduces the intensity of measurement.

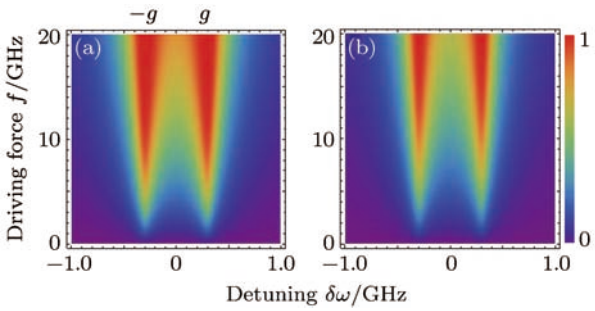


Fig. 3. (colour online) (a) Probability distribution considering zero point fluctuation for obtaining the measurement result about 0 and the initial state $|0\rangle\langle 0|$, plotted as a function of detuning $\delta\omega$. (b) Probability distribution considering the back action noise instead of zero point fluctuation with the same parameter. The damping rate of resonator $\kappa = 0.1$ GHz, the couple strength between the qubit and the oscillator $g = 0.3$ GHz and the back action noise spectra density $S_{\mathrm{II}} = 2/\kappa$.

2.2. Correlation between measurement probability and the dephasing rate

We choose the initial state of the system to be $\rho(0) = \rho_0 \otimes |0\rangle\langle 0|$, i.e., at time $t = 0$ the harmonic oscillator is in the vacuum state. Then, we turn on the qubit and oscillator interaction and, after a period of time, the qubit and oscillator entangle with each other. At the same time, we perform a strong measurement of the flux quadrature $\hat{x} = \hat{a} + \hat{a}^\dagger$. As a result, the pointer state (the oscillator state) is projected to the state $|x\rangle\langle x|$. If the measurement is strong enough that the states of oscillator $|x_\pm\rangle\langle x_\pm|$, which correspond to the qubit states $|0\rangle$ and $|1\rangle$ respectively are orthogonal, the coherence of qubit is destroyed by the measurement. To describe this phenomenon, we must find the equation of motion that can describe the evolution of the system state. The master equation used in a circuit QED system is valid here.[20] The qubit density matrix obeys

$$\dot{\rho} = \mathscr{L}\rho = -\frac{\mathrm{i}}{\hbar}[\hat{H}, \rho] + \kappa\mathscr{D}[\hat{a}]\rho + \gamma_1\mathscr{D}[\hat{\sigma}_-]\rho + \frac{\gamma_2}{2}\mathscr{D}[\hat{\sigma}_z]\rho, \quad (13)$$

where $\mathscr{D}[\hat{L}]$ is a damping superoperator defined by $\mathscr{D}[\hat{L}] = (2\hat{L}\rho\hat{L}^{\dagger} - \hat{L}^{\dagger}\hat{L}\rho - \rho\hat{L}^{\dagger}\hat{L})/2$. In the above expression, κ is the decay rate of the resonator, γ_1 is the qubit relaxation rate and γ_2 is the dephasing rate. To figure out the effect induced by measurement due to the coupling of the qubit to the oscillator, we do not consider the coupling between the qubit and the environment. Therefore, we neglect relaxation due to coupling with the environment and set $\gamma_1 = 0$ (i.e., for an ideal QND measurement no quantum tunneling occurs between qubit states $|0\rangle$ and $|1\rangle$ during measurement). Equation (13) can be solved under the condition of the density matrix with an expansion in positive P representation. In Ref. [20] the solution of the master equation was found to be

$$a_{10}(t) = a_{10}(0)\exp\Big[-\mathrm{i}(\epsilon - \mathrm{i}\gamma_2)t - \mathrm{i}2g\int_0^t \alpha_+(t)\alpha_-^*(t)\,\mathrm{d}t'\Big] \tag{14}$$

and $a_{01}(t) = a_{10}^*(t)$. The amplitudes of the coherence states $\alpha_{\pm}(t)$ are determined by

$$\alpha_+(t) = \alpha_+^{\mathrm{s}} + \exp[-(\kappa/2 + \mathrm{i}g + \mathrm{i}\delta\omega)t] \times (\alpha_+(0) - \alpha_+^{\mathrm{s}}) \tag{15}$$

with $\alpha_+^{\mathrm{s}} = -\mathrm{i}f/(\mathrm{i}\kappa/2 + \mathrm{i}g + \delta\omega)$, and

$$\alpha_-(t) = \alpha_-^{\mathrm{s}} + \exp[-(\kappa/2 - \mathrm{i}g + \mathrm{i}\delta\omega)t] \times (\alpha_-(0) - \alpha_-^{\mathrm{s}}) \tag{16}$$

with $\alpha_-^{\mathrm{s}} = -\mathrm{i}f/(k/2 - \mathrm{i}g + \mathrm{i}\delta\omega)$, where $\alpha_{\pm}^{\mathrm{s}}$ are the steady coherence state values of the oscillator, α_+ and α_- are the amplitudes of the coherent state $|\alpha_+\rangle$ and $|\alpha_-\rangle$, respectively, which depend on the qubit states, a_{01} is the coherence term.

We can find that γ_2 is the intrinsic dephasing rate from Eq. (14) and Γ_{m} is the measurement-induced dephasing rate, which is determined by $\alpha_{\pm}^{\mathrm{s}}$ for a project measurement and expressed as follows:[20]

$$\Gamma_{\mathrm{m}} = 2g\mathrm{Im}(\alpha_+^{\mathrm{s}}\alpha_-^{\mathrm{s}}) = \frac{(n_+ + n_-)\kappa g^2}{\kappa^2/4 + g^2 + \delta\omega^2}, \tag{17}$$

where $n_{\pm} = |\alpha_{\pm}^{\mathrm{s}}|^2 = f^2/[\kappa^2/4 + (\delta\omega \pm g)^2]$ are the average numbers of photons in the resonator. We find that for $\delta\omega = \pm\sqrt{g^2 + \kappa^2/4}$ we will obtain a large probability if we measure the amplitude of the signal. In other words, when matching one of the frequencies of the qubit (i.e., $\delta\omega = \pm g$), the information of the qubit is encoded in the amplitude rather than in the phase, which is the conjugate variable with amplitude. At the same time, the measurement-induced dephasing rate will obtain the maximum value. In Fig. 4, we plot the probability distribution that measuring 0 state conditioned on initial state is prepared on $|0\rangle\langle 0|$ and measurement-induced dephasing as a function of detuning. When matching qubit frequency, both the measurement-induced dephasing rate and the measurement probability obtains a maximum value. In the region where $\mathrm{erf}(|A|t) \approx 1$, it corresponds to a strong projective measurement case. In addition, the two coherent states of the oscillator are well separated in phase space due to the decay of the coherence term at a measurement-induce dephasing rate Γ_{m}. In the general case, at time $t = 0$, the system is in a product state

$$|\psi(t=0)\rangle = \frac{1}{2}(|0\rangle + |1\rangle) \otimes |\alpha\rangle, \tag{18}$$

where $|\alpha\rangle$ is the initial state of oscillator. At time t, the state of the system can be written as

$$|\psi(t)\rangle = \frac{1}{2}(|0\rangle|\alpha_+\rangle + |1\rangle|\alpha_-\rangle). \tag{19}$$

Then, if we know that the oscillator state is α_i, the qubit state can be determined exactly. From Eq. (19) we find that measurement-induced dephasing depends on the overlap of both two oscillator states $\alpha_{\pm}$.[21] In case where the overlap of the oscillator states $|\alpha_-\rangle$ and $|\alpha_+\rangle$ is negligible, the measurement can be considered as a strong projective measurement, i.e., in the region $\mathrm{erf}(|A|t) = 1$. For the maximum dephasing value, the oscillator states decay to a steady state significantly, which explains the feature of Fig. 4 that both measurement-induced dephasing rate and measurement probability obtain a high value for matching the qubit frequency. In this situation, the qubit state is encoded in the amplitude of the coherence rather than in the phase. It means that the easier we gain information about amplitude, the faster we lose information about phase. The measurement-induced dephasing rate can also be expressed as

$$|\langle\alpha_-(t)|\alpha_+(t)\rangle| = \mathrm{e}^{-\Gamma_{\mathrm{m}}t}. \tag{20}$$

In addition, as shown in Fig. 4, with the increase of the value of damping rate κ, both structures of probability distribution and measurement-induced dephasing rate become flat with synchronous tendency, which is consistent with the expectation. The increase of driving force f only changes the overall scale of Figs. 4(e)–4(h) rather than the structure, which explains the trend that with the increase of the value of driving force, the strong measurement region in Figs. 4(a)–(h) becomes wider and the measurement probability increases significantly, especially for matching the qubit frequency.

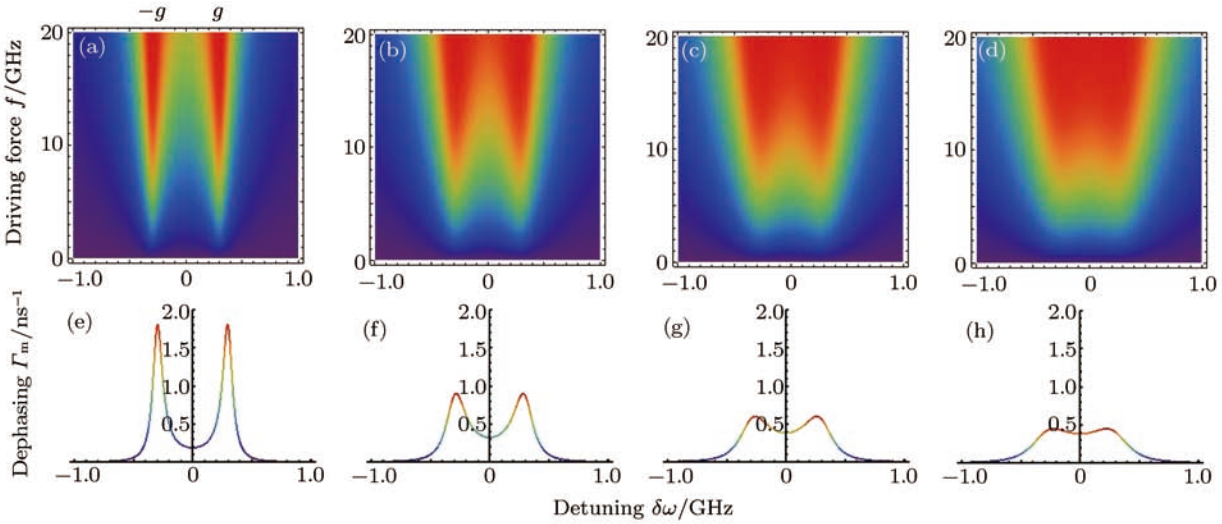


Fig. 4. (colour online) (a)–(d) Probability of measuring the 0 state for the initial state $|0\rangle\langle 0|$ with damping rate of oscillator $\kappa = 0.1$ GHz, $\kappa = 0.2$ GHz, $\kappa = 0.3$ GHz, and $\kappa = 0.4$ GHz, respectively, and qubit coupling strength $g = 0.3$ GHz, $t = 0.1$ ns, noise spectra density $S_{\rm II} = 2/\kappa$. (e)–(h) Measurement-induced dephasing rate $\Gamma_{\rm m}$ as a function of detuning $\delta\omega$ between resonator and drive frequency with driving force $f = 1$ GHz. The parameters of panels (e)–(h) are the same as those in panels (a)–(d), respectively.

3. Deviation of a perfect QND measurement due to measurement-induced tunneling

Taking into account the impact of a non-diagonal term of qubit, we obtain the system Hamiltonian

$$\hat{H}_{\rm s} = \frac{\epsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x. \tag{21}$$

Then we can diagonalize the qubit Hamiltonian $\hat{H}_{\rm s}$. In new eigenvector space, the system Hamiltonian can be written as

$$\hat{H}_{\rm s} = \frac{E}{2}\hat{\sigma}_z, \tag{22}$$

meanwhile, the interacting Hamiltonian can be written as

$$\hat{H}_{\rm int} = \hbar g\hat{\sigma}_n\hat{a}^\dagger\hat{a}, \tag{23}$$

where $E = \sqrt{\epsilon^2 + \Delta^2}/2$ is energy splitting of the qubit and n is a vector representing the direction of the qubit basis before diagonalization relative to the energy basis

$$\hat{\sigma}_n = \cos\eta\hat{\sigma}_z + \sin\eta\hat{\sigma}_x, \tag{24}$$

where $\eta = \arctan(\Delta/\epsilon)$. The eigenstates are denoted by the superposition of $|0\rangle$ and $|1\rangle$

$$|\uparrow\rangle = \cos\frac{\eta}{2}|0\rangle + \sin\frac{\eta}{2}|1\rangle,$$

$$|\downarrow\rangle = -\sin\frac{\eta}{2}|0\rangle + \cos\frac{\eta}{2}|1\rangle. \tag{25}$$

From Eq. (23) we find that the fluctuation induced by external noise in particle number $\hat{n} = \hat{a}^\dagger\hat{a}$ can cause transition between qubit states. Therefore, the qubit acts as a spectrum analyser. We separate the interaction Hamiltonian into two parts, one is $\hbar g\cos\eta\hat{\sigma}_z\hat{a}^\dagger\hat{a}$ and the other is $\hbar g\sin\eta\hat{\sigma}_x\hat{a}^\dagger\hat{a}$. The first part does not affect the repeatability of QND measurement, as discussed in previous sections. The second part will cause "spin flip" transition due to the coupling between the qubit off-diagonal term and an external variable. According to the equation of motion, the time evolution of the density matrix is obtained in the interaction picture as

$$\dot{\rho}^{\rm I}(t) = \frac{1}{{\rm i}\hbar}[\hat{H}^{\rm I}_{\rm int}(t), \rho^{\rm I}(t)]. \tag{26}$$

We choose the initial state as a product of the qubit and the detector, i.e., $\rho(0) = \rho_0(0)\otimes\rho_{\rm D}(0)$. Then, tracing Eq. (26) over the detector degrees and expanding its right-hand side to the second order, we obtain the time evolution of reduced density matrix ρ_0. Equation (26) becomes

$$\begin{aligned}\dot{\rho}^{\rm I}_0(t) = &-\frac{1}{\hbar^2}\int_0^t {\rm d}\tau\{[\hat{\sigma}^{\rm I}_n(t), \hat{\sigma}^{\rm I}_n(t-\tau)\\ &\times\rho^{\rm I}(t-\tau)]\langle\hat{n}^{\rm I}(\tau)\hat{n}\rangle_{\rm D}\\ &-[\hat{\sigma}^{\rm I}_n(t), \rho^{\rm I}(t-\tau)\hat{\sigma}^{\rm I}_n(t-\tau)]\langle\hat{n}\hat{n}^{\rm I}(\tau)\rangle_{\rm D}\}.\end{aligned} \tag{27}$$

After taking matrix elements between eigenstates of $\hat{H}_{\rm s}$, we finally obtain the equation of motion that can

describe the reduced density matrix evolution

$$\dot{\rho}^{\rm I}_{kk'}(t) = -\int_0^t {\rm d}\tau \sum_{l,l'} M_{kk'll'}\rho^{\rm I}_{ll'}(t), \tag{28}$$

in which we introduced the matrix

$$\begin{aligned} M_{kk'll'} \;=\; & g^2\{\langle\hat{n}^{\rm I}(\tau)\hat{n}\rangle[(\hat{\sigma}^{\rm I}_n(t)\hat{\sigma}^{\rm I}_n(t-\tau))_{kl}\delta_{l'k'} \\ & -(\hat{\sigma}^{\rm I}_n(t-\tau))_{kl}(\hat{\sigma}^{\rm I}_n(t))_{l'k'}] \\ & +\langle\hat{n}\hat{n}^{\rm I}(\tau)\rangle[(\hat{\sigma}^{\rm I}_n(t-\tau)\hat{\sigma}^{\rm I}_n(t))_{l'k'}\delta_{kl} \\ & -(\hat{\sigma}^{\rm I}_n(t))_{kl}(\hat{\sigma}^{\rm I}_n(t-\tau))_{l'k'}]\}. \end{aligned} \tag{29}$$

To calculate the energy relaxation rate and dephasing rate, we compute the transition rate between qubit eigenstates by computing the evolution of the diagonal terms of reduced density matrix ρ_{11} and ρ_{00}, and the decay of the off-diagonal term ρ_{01} with the initial state being a $\hat{\sigma}_x$ eigenstate. The transition rates between the two qubit eigenstates can be derived from Eq. (28),

$$\begin{aligned} \Gamma_\downarrow &= g^2\sin^2\eta S_{nn}(\omega_{01}), \\ \Gamma_\uparrow &= g^2\sin^2\eta S_{nn}(-\omega_{01}), \end{aligned} \tag{30}$$

where $\Gamma_\uparrow$ and $\Gamma_\downarrow$ are rates at which the qubit is excited from the ground state to an excited state and decay from an excited state to the ground state, respectively, $\omega_{01} = E/\hbar$, $S_{nn}(\omega) = \int_0^t {\rm d}\tau\, {\rm e}^{{\rm i}\omega\tau}\langle\hat{n}(\tau)\hat{n}(0)\rangle$. If the noise source is in the thermal equilibrium at temperature T, the transition rate must satisfy the balance condition $\Gamma_\uparrow/\Gamma_\downarrow = \exp(-\beta\hbar\omega_{01})$ with effective temperature $\beta = 1/k_{\rm B}T_{\rm eff}$. It means that the qubit energy can be absorbed or emitted by detector, and the ratio between positive and negative noise spectral density depends on the effective temperature. We can also obtain the expression of dephasing rate Γ_ϕ,

$$\begin{aligned} \Gamma_\phi \;=\; & \frac{1}{2}g^2\sin^2\eta[S_{nn}(\omega_{01}) + S_{nn}(-\omega_{01})] \\ & + g^2\cos^2\eta S_{nn}(\omega=0) \\ =\; & \frac{\Gamma_\uparrow + \Gamma_\downarrow}{2} + \gamma_\phi, \end{aligned} \tag{31}$$

where γ_ϕ is the qubit pure dephasing rate. For the case $\Delta = 0$, the measurement is a perfect QND measurement, which cannot cause transition between qubit states, in which the dephasing rate is

$$\Gamma_\phi = \gamma_\phi = g^2\int_{-\infty}^{\infty}{\rm d}\tau\langle\hat{n}^{\rm I}(\tau)\hat{n}\rangle_{\rm D}. \tag{32}$$

To calculate the dephasing rate caused by fluctuation of the particle number of the oscillator during measurement, we should calculate correlator $\langle\delta\hat{n}(\tau)\delta\hat{n}\rangle$, where $\delta\hat{n}$ is quantum fluctuations around the mean of photons in the resonator. For a driving resonator with frequency $\omega_{\rm d}$, the following equation is valid[22]

$$\hat{a}(t) = {\rm e}^{-{\rm i}\omega_{\rm d}t}[\alpha + \hat{d}(t)], \tag{33}$$

where α is a classic part corresponding to the coherence state $|\alpha\rangle$, $\hat{d}(t)$ is a quantum part which can annihilate the coherence state to the vacuum state. Then the correlator becomes

$$\langle\delta\hat{n}(\tau)\delta\hat{n}(0)\rangle = \langle\hat{d}(\tau)\hat{d}(0)\rangle = \bar{n}\,{\rm e}^{{\rm i}\delta\omega t - {\rm i}\kappa t/2}, \tag{34}$$

where $\overline{n}$ is the average photo number in the resonator. Under the condition that $\delta\omega = 0$ and weak coupling $g \ll \kappa$, the measurement-induced dephasing rate is

$$\Gamma_{\rm m} = \kappa\bar{n}\theta_0^2, \tag{35}$$

where $\theta_0 = \arctan(2g/\kappa) \approx 2g/\kappa$ is the phase shift during measurement, which is identical with that in Refs. [20] and [21] with a coefficient of expression difference caused by the difference of the lower limit of the integral. It is quite a contrast to Eq. (17) having double peaks for matching the frequencies of the qubit. With the increase of amplitude of the off-diagonal term of the density matrix, the pure dephasing rate becomes weak and relaxation will be dominant.

From Eq. (32), we find that the decay of coherence is caused by intrinsic dephasing and aggravated by measurement-induced dephasing. In addition, the deviation from the initial value of $\langle\sigma_z\rangle$ becomes bigger as time goes by due to transition between the qubit states. Therefore, qubit state is acquired; meanwhile, the repeatability of measurement is destroyed. It means that it should not be considered as a QND measurement. Based on this effect, it can be used for "back action cooling" in many systems, such as a cantilever coupled to an optical cavity,[23] a noisy qubit coupled to a mechanical resonator,[24] a nanomechanical resonator coupled to a driven superconducting resonantor[25] and cooling a micromirror by radiation pressure inside an optical cavity.[26]

4. Conclusion

We have analysed the QND measurement of a flux qubit coupled to a noisy detector. The analytical results reveal that both measurement probability and measurement-induced dephasing have a similar trend in that their values have peaks for matching one of the qubit frequencies and reduce at the resonance point, at which the driving frequency equals the bare harmonic oscillator frequency. With the increase of the

resonator decay rate, both the curves of dephasing rate and measurement probability become flat. Due to the coupling between the non-diagonal term of qubit and the variable of detector, the noise of the detector perturbs the initial qubit state. As a result, the transition between the qubit states destroys the initial information of the qubit. Therefore, there will be energy exchange between the qubit and the detector according to the value of the effective temperature of noise, which can be used for the cooling of "back action". Conversely, the behaviour of a qubit which acts as a spectrum analyser can also be used for analysing the property of noise.

References

[1] Walls D F and Milburn G J 1994 *Quantum Optics* (Berlin: Springer-Verlag)

[2] Caves C M, Thorne K S, Drever R W P, Sandberg V D and Zimmermann M 1980 *Rev. Mod. Phys.* **52** 341

[3] Milburn G J and Walls D F 1980 *Phys. Rev.* A **28** 2065

[4] Imoto N, Haus H A and Yamamoto Y 1985 *Phys. Rev.* A **32** 2287

[5] Bachor H A, Levenson M D, Walls D F, Perlmutter S H and Shelby R M 1988 *Phys. Rev.* A **38** 180

[6] Larson J and Abdel-Aty M 2009 *Phys. Rev.* A **80** 053609

[7] Brune M, Haroche S and Raimond J M 1988 *Phys. Rev.* A **45** 5139

[8] Boissonneault M, Gambetta J M and Blais A 2009 *Phys. Rev.* A **79** 013819

[9] Shi Z G, Chen X W, Zhu X X and Song K H 2009 *Chin. Phys.* B **18** 910

[10] Lupascu A, Saito S, Picot T, De Groot P C, Harmans C J P M and Mooij J E 2007 *Nat. Phys.* **3** 119

[11] Bocko M F and Onofrio R 1996 *Rev. Mod. Phys.* **68** 755

[12] Makhlin Y, Schon G and Shnirman A 2000 *Phys. Rev. Lett.* **85** 4578

[13] Ji Y H, Lai H F, Cai S H and Wang Z S 2010 *Chin. Phys.* B **19** 030310

[14] Clerk A A, Girvin S M and Stone A D 2003 *Phys. Rev.* B **67** 165324

[15] Averin D V *arXiv: cond-mat*/0004364v1

[16] Viola L, Onofrio R and Calarco T 1997 *Phys. Rev.* A **229** 23

[17] Chirolli L and Burkard G 2009 *Phys. Rev.* B **80** 184509

[18] Clerk A A, Devoret M H, Girvin S M, Marquardt F and Schoelkopf R J 2010 *Rev. Mod. Phys.* **82** 1155

[19] Ashhab S, You J Q and Nori F 2009 *New J. Phys.* **11** 083017

[20] Gambetta J, Blais A, Schuster D I, Wallraff A, Frunzio L, Majer J, Devoret M H, Girvin S M and Schoelkopf R J 2006 *Phys. Rev.* A **74** 042318

[21] Gambetta J, Blais A, Boissonneault M, Houck A A, Schuster D I and Girvin S M 2008 *Phys. Rev.* A **77** 012112

[22] Blais A, Huang R S, Wallraff A, Girvin S M and Schoelkopf R J 2004 *Phys. Rev.* A **69** 062320

[23] Marquardt F, Chen J P, Clerk A A and Girvin S M 2007 *Phys. Rev. Lett.* **99** 093902

[24] Wang Y D, Li Y, Xue F, Bruder C and Semba K 2009 *Phys. Rev.* B **80** 144508

[25] Rocheleau T, Ndukum T, Macklin C, Hertzberg J B, Clerk A A and Schwab K C 2010 *Nature* **463** 72

[26] Gigan S, Bohm H R, Paternostro M, Blaser F, Langer G, Hertzberg J B, Schwab K C, Bauerle D, Aspelmeyer M and Zeilinger A 2006 *Nature* **444** 67