Quantum logic networks for cloning a quantum state near a given state

Zhou Yan-Hui(周彦辉)†

Department of Basic Courses, Zhengzhou College of Science & Technology, Zhengzhou 450064, *China*

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Two quantum logic networks are proposed to simulate a cloning machine that copies the states near a given one. Probabilistic cloning based on the first network is realized and the cloning probability of success based on the second network is 100%. Therefore, the second network is more motivative than the first one.

Keywords: given state, network, quantum cloning

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1. Introduction

The no-cloning theorem is one of the most fundamental differences between classical and quantum information.[1] It states that an arbitrary quantum state cannot be cloned exactly. Fortunately, probabilistic cloning is allowed by quantum dynamics, i.e., one can obtain an approximate copy in a deterministic cloning process or perfect copies in a probabilistic cloning process.[2] Quantum cloning has not only fundamental significance in quantum mechanics, but also many promising applications to quantum information processing, such as quantum computation[3] and quantum cryptography.[4]

The rapid development of quantum information theory over the last decade has stimulated the investigation of optimal approximate quantum copying transformations, which produce two or more copies of a state with maximum fidelity based on quantum cloning theory. The universal quantum cloning machine (UQCM) was proposed by Bužek and Hilley,[5] which cloned an arbitrary quantum state. The quality of the copies was independent of the input state and the fidelity was 5/6. The phase-covariant quantum cloning machine (PCCM) was proposed by Bruß *et al.*[6] with the fidelity $1/2 + 1/\sqrt{8}$, of which the fidelity was higher than that of UQCM because partial information about the input state was known. In addition, some other schemes have also been proposed for the implementation of different optimal quantum cloning machines.[7–9] However, in some special cases where the states to be cloned are near a given state, we can clone states with a higher fidelity than that of UQCM and PCCM. This type of quantum cloning machine was established by Bužek and Hilley in Ref. [5] and copies states in the neighbourhood of a given state very well. In their protocol, the input and copy qubits become entangled with the ancilla via unitary transformation. After the selective measurement, a good copy of the input state is given probabilistically. In 2003, Zheng[10] proposed a scheme to realize this cloning machine by applying a unitary transformation on two qubits without select measurement. In 2008, Zhang *et al.*[11] proposed a network to realize this cloning machine based on cavity quantum electrodynamics.

In this paper, we construct two quantum networks for cloning a quantum state near a given state. The first one extends the idea of Zhang *et al.*,[11] and is better than that of Zhang *et al.* due to its simple and fewer operations, which is important for reducing operation time and simplifying the experiment. The clone based on the first network is probabilistic, while making use of the second network, and a good copy of the input state is given no matter what the measurement result of the ancilla is, so the probability of success is 100%. Therefore the second network is more motivative than the first one.

The remainder of the paper is organized as follows. In Section 2, we propose the first network for cloning a quantum state near a given state. In Section 3, we present the second network. Finally, a summary is given in Section 4.

†Corresponding author. E-mail: yanhuizhou@126.com

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2. The first network

The process for cloning a state near a given one is determined by the unitary transformation relation[5]

$$|1\rangle_A|0\rangle_B|Q_0\rangle_x \to \frac{1}{\sqrt{2}}(|11\rangle_{AB}|Q_1\rangle_x + |00\rangle_{AB}|Q_0\rangle_x),$$
$$|0\rangle_A|0\rangle_B|Q_0\rangle_x \to \frac{1}{\sqrt{2}}(|10\rangle_{AB} + |01\rangle_{AB})|Q_1\rangle_x, \quad (1)$$

where A is the initial qubit to be cloned, B is the copy qubit and x denotes the ancilla. The states of the ancilla $|Q_0\rangle_x$ and $|Q_1\rangle_x$ satisfy the relations $\langle Q_0|Q_0\rangle = \langle Q_1|Q_1\rangle = 1$ and $\langle Q_0|Q_1\rangle = 0$.

The first network that we propose for cloning a state near a given one is shown in Fig. 1. We assume that the qubit A to be cloned is in the superposition state

$$|\phi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A, \quad (2)$$

where $|\alpha|^2+|\beta|^2 = 1$ and $|\alpha|^2 \ll |\beta|^2$. After the circuit shown in Fig. 1, we have

$$\begin{aligned} &\hat{X}_C\hat{X}_B\hat{X}_A\hat{P}_{ABC}\hat{X}_B\hat{P}_{BA}\hat{H}_B|\phi\rangle_A|0\rangle_B|0\rangle_C \\ \to\ &\frac{1}{\sqrt{2}}[\beta|11\rangle_{AB} + \alpha(|10\rangle + |01\rangle)_{AB}]|1\rangle_C \\ &+\frac{1}{\sqrt{2}}\beta|00\rangle_{AB}|0\rangle_C, \end{aligned} \quad (3)$$

where the $\hat{X}_i$ is a NOT gate and the $\hat{H}_i$ is a Hadamard gate, which all act on the qubit i. $\hat{P}_{kl}$ is the controlled NOT (CNOT) gate with k being the controlling qubit and l being the target qubit. The $\hat{P}_{k_1k_2l}$ is also a CNOT gate with k_1 and k_2 being the controlling qubits and l being the target qubit.

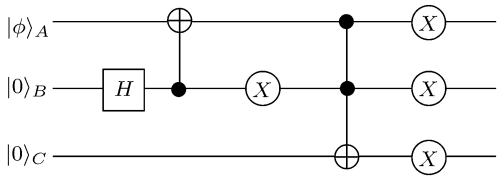


Fig. 1. The first quantum logic network for cloning a state near a given one.

Then a selective measurement is performed on the ancilla. If the outcome is $|1\rangle_C$, the state of system reduces to

$$\frac{1}{\sqrt{|\alpha|^2+1}}[\beta|11\rangle_{AB} + \alpha(|10\rangle + |01\rangle)_{AB}]. \quad (4)$$

The reduced density operator for A and B is written as

$$\rho = \frac{1}{|\alpha|^2+1}[(\alpha|0\rangle + \beta|1\rangle)(\langle 0|\alpha^* + \langle 1|\beta^*) + |\alpha|^2|1\rangle\langle 1|]. \quad (5)$$

The fidelity is given as

$$\begin{aligned} F &= (\langle 0|\alpha^* + \langle 1|\beta^*)\rho(\alpha|0\rangle + \beta|1\rangle) \\ &= \frac{1+|\alpha\beta|^2}{1+|\alpha|^2}. \end{aligned} \quad (6)$$

The probability of success is $P = 1-|\beta|^2/2$ due to the selective measurement. The fidelity F and the probability of success P are relevant to the initial state. The figure for F and P changing with $|\beta|^2$ is shown in Fig. 2. It is shown that the larger $|\beta|^2$ is, the higher the fidelity is. Meanwhile, the probability of success decreases with the increase of $|\beta|^2$, i.e., in order to increase the fidelity, we need to reduce the probability of success for cost. We also give the limit value of the fidelity F and the probability of success P. When $|\beta|^2 = 1$, the fidelity F achieves the maximum value of 1 and the probability of success P attains the minimum value of 0.5, while $|\beta|^2 = 0$, the fidelity F achieves the minimum value of 0.5 and the probability of success P reaches the maximum value of 1.

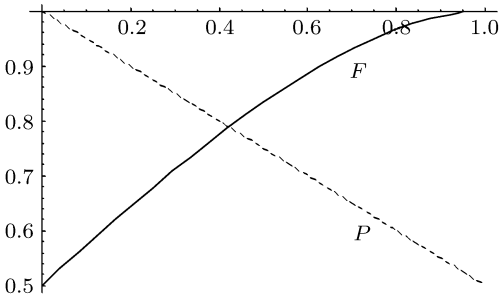


Fig. 2. Curves for F and P changing with $|\beta|^2$.

3. The second network

The quantum cloning based on the first network is a probabilistic one. Next, we propose a second network to simulate the cloning machine, which copies states near a given one with a probability of 100% nad is shown in Fig. 3.

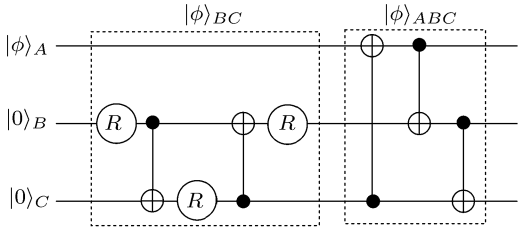


Fig. 3. The second quantum network for cloning a quantum state near a given state.

We decompose the quantum logic network into two parts for clarity. In the first part qubits B and C are prepared in a specific state $|\phi\rangle_{BC}$, and qubit A is entangled with qubits B and C to distribute the information about the initial state among the three qubits in the second part. Assume that the qubit A to be cloned is in the state

$$|\phi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A, \tag{7}$$

where $|\alpha|^2 \ll |\beta|^2$. The qubits B and C are prepared in the state $|0\rangle_B|0\rangle_C$. After the first part of the circuit, the state $|\phi\rangle_{BC}$ is constructed as

$$|\phi\rangle_{BC} = \hat{R}_B(\theta_3)\hat{P}_{CB}\hat{R}_C(\theta_2)\hat{P}_{BC}\hat{R}_B(\theta_1)|0\rangle_B|0\rangle_C. \tag{8}$$

where $\hat{P}_{kl}$ is the controlled NOT gate with k being the controlling qubit and l being the target qubit. The $\hat{R}_j(\theta)$ is the single-qubit rotation, which is defined as

$$\begin{aligned}\hat{R}_j(\theta)|0\rangle_j &= \cos\theta|0\rangle_j + \sin\theta|1\rangle_j,\\ \hat{R}_j(\theta)|1\rangle_j &= -\sin\theta|0\rangle_j + \cos\theta|1\rangle_j.\end{aligned} \tag{9}$$

In order to realize the cloning of the quantum state near a given one, we choose the angles of the single-qubit rotation θ_i as follows:

$$\begin{aligned}\theta_1 = \theta_3 &= \arcsin\sqrt{\frac{1}{2} + \frac{\sqrt{5}}{10}},\\ \theta_2 &= -\arcsin\sqrt{\frac{1}{2} - \frac{\sqrt{5}}{6}}.\end{aligned} \tag{10}$$

After the second part of the network, we obtain the following state:

$$\begin{aligned}|\phi\rangle_{ABC} &= \hat{P}_{BC}\hat{P}_{AB}\hat{P}_{CA}|\phi\rangle_A|\phi\rangle_{BC}\\ &= \frac{1}{\sqrt{3}}[(\beta|00\rangle + \alpha|10\rangle + \alpha|01\rangle)_{AB}|1\rangle_C\\ &\quad + (\alpha|11\rangle + \beta|10\rangle + \beta|01\rangle)_{AB}|0\rangle_C].\end{aligned} \tag{11}$$

Then a measurement is performed on the ancilla. The outcome $|1\rangle_C$ reduces the state of the system to

$$|\phi^1\rangle_{AB} = \frac{1}{\sqrt{1+|\alpha|^2}}(\beta|00\rangle + \alpha|10\rangle + \alpha|01\rangle)_{AB}. \tag{12}$$

Next we need to perform the σ_x on A and B, respectively, to obtain

$$\begin{aligned}&\sigma_x^B\sigma_x^A|\phi^1\rangle_{AB}\\ &= \frac{1}{\sqrt{1+|\alpha|^2}}(\beta|11\rangle + \alpha|10\rangle + \alpha|01\rangle)_{AB},\end{aligned} \tag{13}$$

which is the result we want, as shown in Eq. (4). The fidelity is

$$\begin{aligned}F_1 &= (\langle 0|\alpha^* + \langle 1|\beta^*)\rho(\alpha|0\rangle + \beta|1\rangle)\\ &= \frac{1+|\alpha\beta|^2}{1+|\alpha|^2}.\end{aligned} \tag{14}$$

Otherwise, if the measurement result is $|0\rangle_C$, the state for A and B reduces to

$$|\phi^2\rangle_{AB} = \frac{1}{\sqrt{1+|\beta|^2}}(\alpha|11\rangle + \beta|10\rangle + \beta|01\rangle)_{AB}. \tag{15}$$

In order to obtain a good copy, we need to perform the following operation:

$$\begin{aligned}&\sigma_A^x\hat{P}_{AB}H_A\sigma_A^x\hat{P}_{AB}|\phi^2\rangle_{AB}\\ &= \frac{1}{\sqrt{2+2\beta^2}}[2\beta|11\rangle_{BC} + \alpha(|10\rangle + |01\rangle)_{AB}].\end{aligned} \tag{16}$$

The fidelity is

$$F_2 = \frac{1}{2+2|\beta|^2}[4|\beta|^4 + 4|\alpha\beta|^2 + |\alpha|^2]. \tag{17}$$

The fidelity F_1 is a perfect one, shown in Eq. (6), which is equal to the one in Ref. [5], and F_2 is not as perfect as F_1. The curves for F_1 and F_2 changing with $|\beta|^2$ are shown in Fig. 4, where the dashed line denotes F_1 and the solid line refers to F_2. If we assume $|\alpha|^2 = 0.2$ and $|\beta|^2 = 0.8$, then fidelity $F_1 = 0.967$ and fidelity $F_2 = 0.944$. For $|\alpha|^2 = 0.3$ and $|\beta|^2 = 0.7$, fidelity $F_1 = 0.931$, and fidelity $F_2 = 0.912$. Fidelity F_2 decreases slightly, but fidelities F_1 and F_2 are almost equal when $|\alpha|^2 \ll |\beta|^2$. Thus no matter what the measurement result of ancilla is, our scheme is successful and a good copy of the input state can be attained.

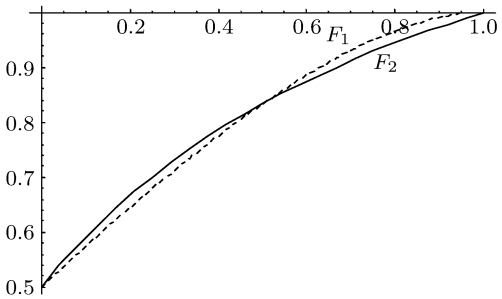


Fig. 4. Curves for F_1 and F_2 changing with $|\beta|^2$.

There are usually three types of quantum cloning machines used to clone different states. They are: the UQCM, which clones a completely unknown arbitrary quantum state;[5] the PCCM, which clones a state where the modules of the complex coefficients of the input state are known but their phases are unknown;[6] and the real state cloning machine, which clones a state where the phases of the complex coefficients are known but their modules are unknown.[12] A cloning machine that can copy states near a given one is a beneficial complement to the system of cloning machines. When information about a quantum state is

partially known and $|\alpha|^2 \ll |\beta|^2$, this cloning machine appears to be better than the others due to its higher fidelity. It is known that the fidelity of a UQCM is 5/6.[5] If $|\beta|^2 = 1/2$, the fidelities of our schemes are $F_1 = F_2 = 5/6$, which is just equal to the fidelity of a QUCM. So, if $|\beta|^2 > 1/2$, the fidelities of our schemes are higher than a UQCM regardless of what the measurement result of the ancilla is.

We now give a brief discussion about the experimental feasibility of the proposed schemes. The main component of our networks is the CNOT gate. It is well known that the CNOT gate can be realized by a controlled phase gate before and after a Hadamard gate, and many schemes for implementing a controlled phase gate and a CNOT gate have been proposed using different physical systems, such as the cavity quantum electrodynamics system,[13,14] the linear optical system[15] and the ion trap system.[16] A UQCM based on the network in Ref. [17], which is similar to our second network, has been realized with linear optical devices.[18] So our scheme is achievable with the present technology and can provide a theoretical guideline for experiments.

Next, we briefly describe the implementation of the networks in cavity quantum electrodynamics (QED). The controlled-Z gates proposed in Refs. [13] and [19] can be used directly to realize our network. Consider a system of a three-level atom interacting with a single-mode cavity resonantly. The atom states are denoted by $|g\rangle$, $|e\rangle$, and $|i\rangle$. The level $|i\rangle$ is not involved in the interactions. We can adjust the frequency of the cavity mode to resonate with the energy level transition $|g\rangle \leftrightarrow |e\rangle$. The interaction Hamiltonian is

$$H_i = \sum_{j=1}^{3} g_j (a^+ s_j^- + a^- s_j^+), \quad \hbar = 1, \tag{18}$$

where g_j is the coupling constant between the j-th atom and the cavity mode. The quantum information is encoded in the states $\{|g\rangle_A, |i\rangle_A, |g\rangle_B, |e\rangle_B, |g\rangle_C, |i\rangle_c\}$. When the coupling constant $g_B \ll g_A = g_C$ and interaction time $\tau = \pi/g_B$, the three-qubit controlled-Z gate can be obtained.[19] Similarly, in the system of two atoms interacting simultaneously with the cavity, when the coupling constant $g_B \ll g_A$ and interaction time $\tau = \pi/g_B$, the two-qubit controlled-Z gate can be obtained.[13] The single-qubit gate can be realized by applying laser pulses to atoms to adjust the amplitude, frequency and strength of the single mode laser field.[20]

The three-qubit controlled-Z gate, the two-qubit controlled-Z gate and the single-qubit gate all can be realized. The technique of cavity QED is commonly used at present and the time spent in implementing the controlled-Z gates is the same in our scheme, so the above networks can be easily realized in cavity QED.

4. Summary

We proposed two quantum logic networks for the process of copying states that are in the neighbourbood of the given ones. The second network is better than the first one due to its higher probability of success. The second network is composed of single- and two-qubit logic gates only, so the second one is more easily achieved in experiment.

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