# Structure formation of entanglement entropy in a system of two superconducting qubits coupled with an $LC$-resonator*

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This paper investigates theoretically the evolutions of the entanglement entropy of a system of two coupled-charge-qubits interacting with an $LC$-resonator. It is found that when the initial states of the two qubits are prepared in a given superposition excited state, the evolution of the von Neumann entropy of the system depends significantly on the coupling strength between the two Josephson charge qubits. With the variation of the coupling strength, the evolution of the entanglement entropy of the system forms some structures, especially the periodically bistable properties, which are the first discovered for such a system to our knowledge. It is found that the relative entropy entanglement of the system is also sensitive to the variation of the coupling strength between the two charge qubits, some novel 'collective oscillations' of the relative entropy are found for the system.

**Keywords:** superconducting quantum circuits, evolutions of entanglement entropy, structure formation

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## 1. Introduction

Solid-state superconducting circuits[1,2] consisting of Josephson-junction charge qubit(s), known as artificial atom(s), are versatile systems in which macroscopic quantum states can be engineered and controlled. Due to their low loss, low noise, high coherence, etc., much attention has been drawn their study. Artificial atoms show macroscopic quantum properties.[3−7] It is also possible to study artificial-atom-cavity or circuit-quantum electrodynamics (circuit QED), when the artificial atoms are coupled with superconducting resonator(s). Such a system can be used as an element of quantum information and quantum computation.[8] Recent experiments have shown that many important optical phenomena can be replicated in such artificial-atom-cavity systems, such as generating single microwave photons in a circuit,[9] artificial atom lasing,[10] artificial-atom resonance fluorescence,[11] coherent population trapping,[12] optical nonlinearity,[13,14] electromagnetically induced transparency,[15,16] and so on.

One of its most important advantages is that the quantum circuits are easily manipulated and engineered experimentally. In the case of the cavity QED of an atom-cavity system, a strong coupling for the system is normally required.[17−19] Generally speaking, the coupling between atoms and the cavity depends on the position of the atoms, however, the position of an atom is hard to locate precisely, even in a high-$Q$ regime. To investigate the quantum evolution properties of atoms in the cavity QED, one possible way is to take an effective mean value of the atom-cavity coupling constant of the system.[20] However, for an artificial-atom-cavity system, the situation is totally different. It is possible to technically fabricate the Cooper-pair box at the position of the maximum voltage standing wave, which yields a strong electric dipole interaction between the qubit and a single photon in the cavity.[21]

Macroscopic quantum states in a Josephson junction circuit can be controlled by an external field and the parameters of the circuit. In this context, entanglement plays a fundamental role in quantum information and quantum computing. The entangled states participate as the basic origins of quantum informa-

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tion. In this paper, we will discuss two Cooper-pair boxes coupled with an $LC$ resonator and from the two choices (the constructal formations of von Neumann entropy and the relative entropy entanglements of the system) show which of them is implementable experimentally. The Cooper-pair box in the circuit is treated as a quantum bit (the so-called Josephson charge qubit). The model is the analogue of two two-level atoms coupled with a cavity field. A deep understanding of the entanglement properties of the system is very important. To do this, we investigate carefully the different entanglement entropies of the system and We find some interesting properties of the entanglement entropy of the system. In particular, it is possible to form some entanglement structures in the system.

## 2. Hamiltonian and solution of the Schrödinger equation

We consider a model of two coupled-charge-qubits (for example, as in Ref. [22]) interacting with an $LC$-resonator. The model is supposed to be an ideal one without regarding loss and decoherence. We assume the charging energy to be smaller than the superconducting gap, such that no quasiparticles are present in the Cooper-pair box. In this case, two Cooper-pair boxes have the same properties. At the degeneracy point, a single Cooper-pair box will be referred to a Josephson charge qubit. The $LC$ resonator, which is the analogue of a high-$Q$ cavity, can be described by a quantum harmonic oscillator with a characteristic frequency, being $\omega_r$. There are three coupling capacitances that play a crucial role in the circuit: the capacitance $C_m$ couples two charge qubits to each other, and the capacitances $C_{C1}$, $C_{C2}$ couple two charge qubits and the resonator to each other, respectively.

The Hamiltonian of the system can be written as ($\hbar = 1$):

$$H = H_0 + H_{\rm I}, \tag{1}$$

where $H_0$ represents the non-interaction Hamiltonian of two charge qubits[23] and the quantized $LC$ resonator;[24] $H_{\rm I}$ is the interaction Hamiltonian. The non-interaction Hamiltonian can be written as

$$H_0 = \omega_r a^\dagger a + \sum_{i=1}^{2} (4E_{ci}(n_i - n_g)^2 - E_{ji}\cos\theta_j), \tag{2}$$

where $E_{ci} = e^2/2C_{\Sigma i}$, and $C_{\Sigma i} = \{C_{ji} + [1/(C_{ci} + C_{gi}) + 1/C_{ri}]^{-1}\}$ (for $i = 1, 2$) is the charging energy, $n_i$ is the operator associated to the number of excess Cooper-pairs on the box, $n_g = n_{gi} = C_{gi}V_{gi}/2e$ is the reduced gate charge and it can be controlled by a voltage source $V_{gi}$ in series with a gate capacitance $C_{gi}$, $E_{ji}$ is the Josephson energy, $\theta_i$ denotes the phase of the superconducting current, $n_i$ and $\theta_i$ are treated as operators obeying $[\theta_i, n_i] = {\rm i}$, $\omega_r = 1/\sqrt{L_r C_r}$ is the frequency of the $LC$ resonator, $a^\dagger$ and $a$ are the creation-annihilation operators for photons in the resonator.

Near the charge degeneracy point, $n_g = 1/2$, the electrostatic energy of the two charge states $|n = 0\rangle$ and $|n = 1\rangle$ are equal, $H_0$ can be rewritten in spin-boson model[23] notation as

$$H_0 = \sum_{i=1}^{2} \left( -\frac{1}{2}B_{zi}\sigma_{zi} - \frac{1}{2}B_{xi}\sigma_{xi} + \omega_r a^\dagger a \right), \tag{3}$$

where $B_{zi} = 4E_{ci}(1 - 2n_g)$, $B_{xi} = E_{ji}$, $\sigma_{zi}$ and $\sigma_{xi}$ are referred to the Pauli spin operators in the basis $\{|0\rangle, |1\rangle\}$. While the interaction Hamiltonian including two parts, one is the interaction between two Josephson charge qubits and the other is interaction between the $LC$ resonator and the charge qubit, which is given by

$$H_{\rm I} = -{\rm i}\sum_{i=1}^{2}[\, g_i(n_i - n_g)(a^\dagger - a)] + J\sigma_{z1}\sigma_{z2}, \tag{4}$$

where

$$g_i = \frac{e}{C_{ji} + C_{ri} + \frac{C_{ji}C_{ri}}{C_{ci}+C_{gi}}} \times \sqrt{2\omega_r \left[ C_{ri} + \frac{(C_{ci} + C_{gi})C_{ji}}{C_{ci} + C_{gi} + C_{ji}} \right]} \tag{5}$$

denotes the coupling strength between the charge qubit and the cavity field; while $J$ denotes the coupling strength between two Josephson charge qubits.[22] In fact, $J$ may have a different form according to different designs of the superconducting circuit. For the present theoretical discussion, we do not consider the detail of the form of $J$. Under the rotating wave approximation, the interaction Hamiltonian can be written as

$$H_{\rm I} = -{\rm i}\sum_{i=1}^{2}[\, g_i(a\sigma_i^\dagger - a^\dagger\sigma_i)] + J\sigma_{z1}\sigma_{z2}, \tag{6}$$

where $\sigma_i^\dagger$ and $\sigma_i$ are the raising and the lowering operators, respectively. In this manner, the Hamiltonian (6) is similar to the Jaynes–Cummings model.

We will consider that the charge qubits resonate with the $LC$ oscillator $\hbar\omega_r = E_j$. We define a set of new quantum superposition states: $\{|e\rangle = \frac{\sqrt{2}}{2}(|1\rangle - |0\rangle)$, $|g\rangle = \frac{\sqrt{2}}{2}(|1\rangle + |0\rangle)\}$. For two qubits, the states $|e_j\rangle$ and $|g_j\rangle$ ( $j = 1, 2$) express qubits 1 and 2, respectively. We assume that it is possible to prepare the charge qubits in the excited state $|e\rangle$ initially and that there are no photons inside the $LC$ resonator at the beginning. In this case, the initial state vector of the system can be written as $|\psi(0)\rangle = |e_1, e_2, 0\rangle$.

The Schrödinger equation of the system in the interaction picture can be written as,

$$\mathrm{i}\frac{\partial|\psi(t)\rangle}{\partial t} = H_{\mathrm{I}}|\psi(t)\rangle. \tag{7}$$

For simplicity and without loss of generality, we choose that the coupling coefficients between the two charge qubits and the cavity are same, and noted as $g$. The time evolution of the wave function can be given by

$$\begin{aligned}|\psi(t)\rangle = {} & C_1(t)|e_1, e_2, 0\rangle + C_2(t)|e_1, g_2, 1\rangle \\ & + C_3|g_1, e_2, 1\rangle + C_4(t)|g_1, g_2, 2\rangle,\end{aligned} \tag{8}$$

where $C_i$'s are obtained to be

$$\begin{aligned}C_1 = {} & \frac{2}{3}\,\mathrm{e}^{-\mathrm{i}Jt} + \frac{\sqrt{6g^2+J^2}-J}{6\sqrt{6g^2+J^2}}\,\mathrm{e}^{\mathrm{i}\sqrt{6g^2+J^2}\,t} \\ & + \frac{J+\sqrt{6g^2+J^2}}{6\sqrt{6g^2+J^2}}\,\mathrm{e}^{-\mathrm{i}\sqrt{6g^2+J^2}\,t},\end{aligned} \tag{9}$$

$$\begin{aligned}C_2 = C_3 = {} & \frac{g}{2\mathrm{i}\sqrt{6g^2+J^2}}\,\mathrm{e}^{\mathrm{i}\sqrt{6g^2+J^2}\,t} \\ & - \frac{g}{2\mathrm{i}\sqrt{6g^2+J^2}}\,\mathrm{e}^{-\mathrm{i}\sqrt{6g^2+J^2}\,t},\end{aligned} \tag{10}$$

$$\begin{aligned}C_4 = {} & \frac{\sqrt{2}}{3}\,\mathrm{e}^{-\mathrm{i}Jt} + \frac{\sqrt{2}(\sqrt{6g^2+J^2}-J)}{6\sqrt{6g^2+J^2}}\,\mathrm{e}^{\mathrm{i}\sqrt{6g^2+J^2}\,t} \\ & + \frac{\sqrt{2}(J+\sqrt{6g^2+J^2})}{6\sqrt{6g^2+J^2}}\,\mathrm{e}^{-\mathrm{i}\sqrt{6g^2+J^2}\,t}.\end{aligned} \tag{11}$$

The density matrix of the system can be described as

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)|, \tag{12}$$

where $|\psi(t)\rangle$ is given by Eqs. (8) and (9)–(11). Correspondingly, the reduced density matrix for two charge qubits is given by

$$\rho_{a_1a_2}(t) = \mathrm{Tr}_f|\psi(t)\rangle\langle\psi(t)|, \tag{13}$$

where the subindex $f$ expresses the trace over photon numbers. Similarly, one can define the reduced matrix density for the cavity field by conventional means.

## 3. The evolution of the entropy of two coupled charge qubits with the cavity field

The evolution of entropy of a quantum system is very important for understanding the quantum information. To meet our goal, we employ quantum reduced von Neumann entropy to measure the entropy evolution of two charge qubits coupling with the cavity field. The von Neumann entropy of subsystem is described by a reduced density matrix and defined by

$$S_i(t) = -\mathrm{Tr}[\,\rho_i(t)\ln\rho_i(t)], \quad (i = a_1a_2, f), \tag{14}$$

where $S_{a_1a_2}(t)$ denotes the (von Neumann) entropy of two charge qubits, $S_f(t)$ denotes the entropy of the cavity field. According to the theory of Araki and Leib,[25] the entropy of the subsystem and the entropy of the whole system satisfy the triangle inequality

$$\begin{aligned}\|S_{a_1a_2}(t) - S_f(t)\| \le {} & S_{a_1a_2f}(t) \\ \le {} & \|S_{a_1a_2}(t) + S_f(t)\|,\end{aligned} \tag{15}$$

where $S_{a_1a_2f}(t) = -\,\mathrm{tr}[\,\rho(t)\ln\rho(t)]$ is the entropy of the whole system. The $\rho(t)$ is the positive density matrix of the system. Suppose that, at the beginning, two charge qubits and a cavity field are prepared in a disentangled pure state, so the entropy of the whole system is zero. The eigenvalue of $\rho_f(t)$ is equal to the eigenvalue of $\rho_{a_1a_2}(t)$, so we can obtain the equality $S_{a_1a_2}(0) = S_f(0)$. Based on the above analysis, in order to obtain the entropy of the whole system, we just need to calculate the quantum reduced entropy of the two charge qubits $S_{a_1a_2}(t)$, which can be expressed as follows:[26]

$$S_{a_1a_2}(t) = -\sum_{j=1}^{4}\lambda_j(t)\ln\lambda_j(t), \tag{16}$$

where $\lambda_j$'s are eignevalues of the reduced density matrix of the two charge qubits, and $S_{a_1a_2}(t)$ reflects the degree of entanglement between the two charge qubits and the cavity field. The $S_{a_1a_2}(t) = 0$ means that the charge qubits disentangle with the cavity field, which $S_{a_1a_2}(t) \neq 0$ means that the charge qubits entangle with the cavity field. We analyse numerically the reduced entropy of entanglement of two charge qubits coupling with a cavity field under different coupling parameters.

In Fig. 1, we discuss the case of weak coupling strength between two Josephson charge qubits, that

is the situation of the coupling intensity $J \leq 1g$. In Figs. 1(a)–1(d), we choose different coupling strengths between two charge qubits and see how the von Neumann entropy varies with time. The figures show that the evolution of the von Neumann entropy of charge qubits coupling with a cavity field strongly depends on the coupling strength between the charge qubits. At $J = 0g$, that is no coupling between two charge qubits, the entropy of the system varies periodically. However, for other values of the coupling constant, although the entropy varies with time, the periodicity is not so good, it forms an irregular evolution even when $J$ tends to $1g$. At most, one may say that in the week coupling case, the evolution of the system forms a quasi-periodicity.

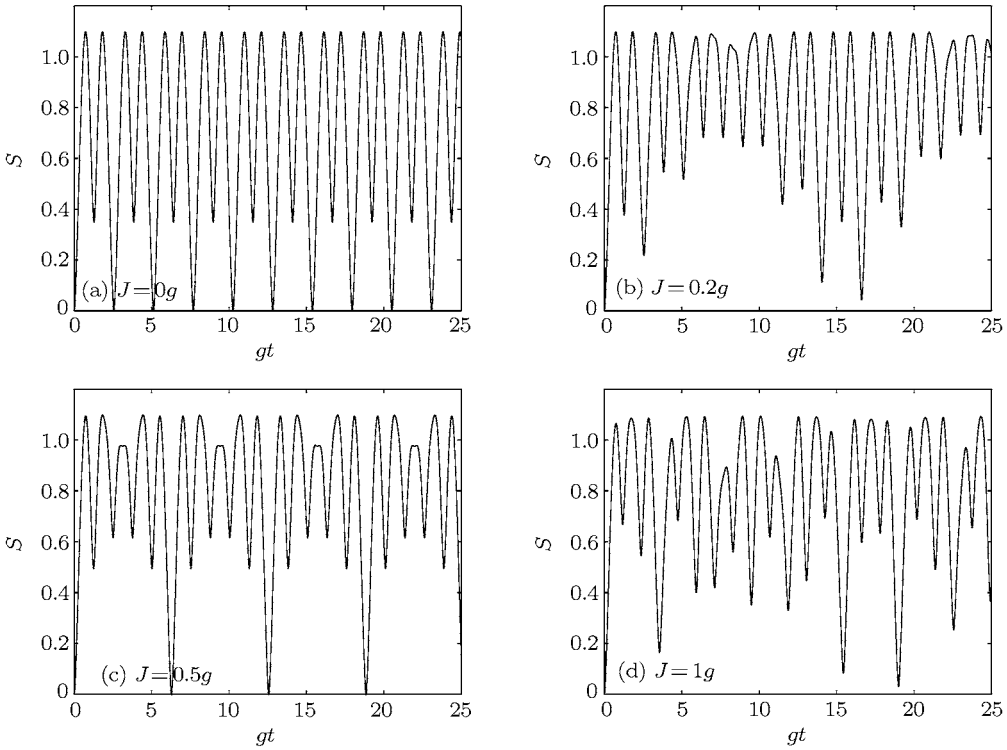



**Fig. 1.** The evolution of the von Neumann entropy of two charge qubits and a cavity field for a weak coupling strength between two Josephson charge qubits, $J/g = 0$, 0.2, 0.5, and 1, for (a), (b), (c), and (d), respectively.

We further consider the case of the strong coupling between two charge qubits, which are of $J > 1g$. In Fig. 2, we take $J/g = 2$, 4, 10, and 20, for Figs. 2(a)–2(d), respectively. Comparing Fig. 2(a) with Fig. 1(d), one can see that the evolution of the von Neumann reduced entropy is less regular also. However, with the increase of the value of $J$, the evolution of the system is very interesting. It shows a tendency to form the doubly periodic structure from Fig. 2(a) to 2(d) with the increase of $J$ gradually. In Fig. 2(d), the periodicity of the system is not a normal one, but a doubly periodic one. This property is first discovered here for an artificial atom–cavity system to our knowledge.

We analyse the evolving regularity of von Neumann reduced entropy. Suppose that the 'normalized period' of the reduced entropy is noted by $x$. At the point $gt = (n-1)x$, $(n = 1, 2, \ldots)$, the reduced entropy is smaller than that at any other time. It implies that the degree of correlation of the two charge qubits with an $LC$ resonator is the weakest at that time and the system can return to this state after a periodic time. At the point $gt = (2n-1)x/4$, $(n = 1, 2, \ldots)$, the reduced entropy has the largest values, for example, at the condition of $J = 5g$. This means that the two charge qubits strongly correlate with the cavity field and that they are in a mixed state. At the point $gt = (2n-1)x/2$, $(n = 1, 2, \ldots)$, the reduced entropy reaches a sub-minimum value. Figure 2(d) shows a good double-peak periodicity structure. This property cannot be seen in the case of only one single charge qubit coupled with an $LC$ resonator. Obvi-

ously, the structural forms basically depend on the artificial atomic structure of two charge qubits. Also, a similar double-peak entanglement structure property, even though not exactly the same, can also be observed for a two-atom-cavity- coupling system.[27] In that case, the doubly periodic structure of entanglement is described by the concurrence and related to the atomic motion and mean cavity photon numbers. Here, we discuss the evolution of the reduced von Neumann entropy. In fact, in the pure state case, the von Neumann entropy can be also used to describe the entanglement of the quantum system.[28] From this viewpoint, we can say that for the artificial atomic system, in the case of two charge qubits coupling with an $LC$-resonator, it is possible to realize a doubly periodic entanglement structure. This entanglement of 'artificial atoms' coupled with a cavity field can be controlled by the coupling strength between two charge qubits.

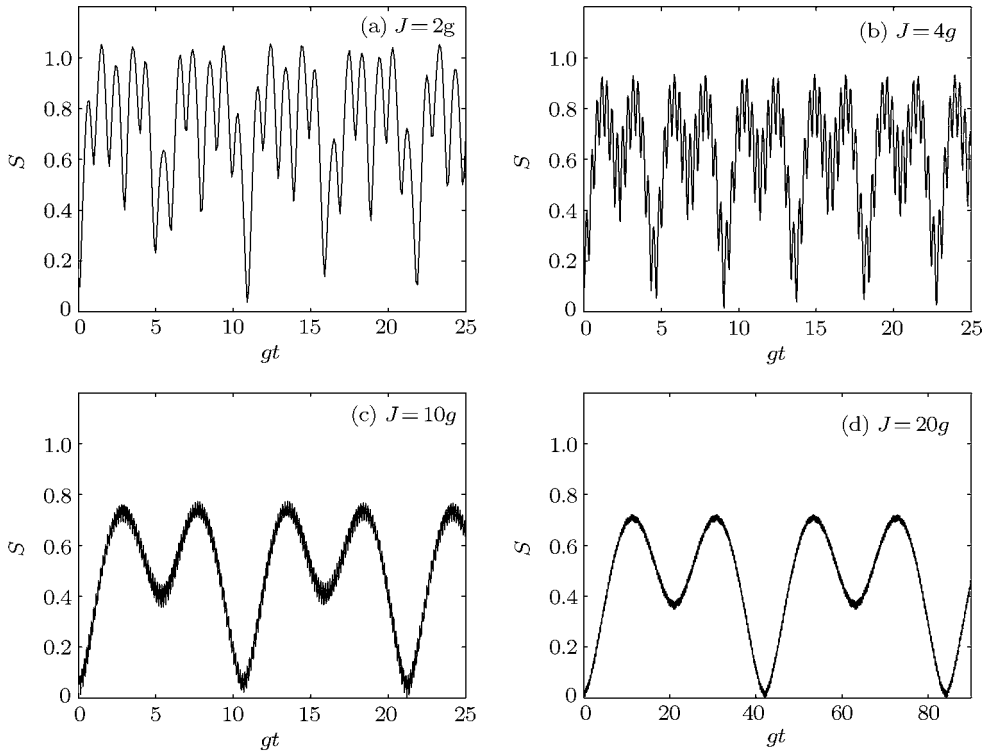



**Fig. 2.** The evolution of the von Neumann reduced entropy of two charge qubits in the cavity field for a strong coupling strength between two Josephson charge qubits, $J/g = 2$, 4, 10 and 20, for (a), (b), (c), and (d), respectively.

Physically, the entropy is normally considered to be a state function. In thermal equilibrium, the entropy of an isolated system increases. However, here we have an open system. When the artificial atomic system is coupled with a cavity, the transition between the energy levels accomplishes photon absorbtion and emission. From Eqs. (9)–(11), it is easy to see that the system oscillates with time depending on the coupling constants of the system $J$ and $g$'s. With the variation of both $J$ and $g$'s, the evolution property of the system varies with time, as well as with parameters $J$ and $g$'s. Photons drive the system transferring between two states of two qubits. Finally, with the variation of the parameters $J$ and $g$, the system reaches a double steady state. That is, the system reaches the bistable equilibrium state. This is a very interesting property that we have found for this system. In fact, this property coincides with the experimentally realized coherent quantum state storage and transfers between a two phase qubit via a resonant cavity,[29] in principle, although the model is somewhat different between two cases. Bistable periodicity may find some special applications in quantum information processing and storage. In addition, from the figures one can also see that with the variation of the coupling strength between two charge qubits, both the period and the degree of the entropy entanglement vary gradually and significantly.

## 4. The relative entropy between two coupled charge qubits

It is well known that relative entropy plays an important role in understanding quantum information.[30,31] According to the above result, it is natural to consider that the states of two charge qubits may be changed from a pure state to a mixed state. In this case, relative entropy is suitable for this double-mixed-quantum-state system. The relative entropy is defined as[30,31]

$$E(\rho) = \min_{\sigma \in D} S(\rho \| \sigma), \tag{17}$$

where $S(\rho\|\sigma) = \mathrm{Tr}[\,\rho(\ln\rho - \ln\sigma)]$ is the quantum relative entropy, $D$ is the set of two-particle separable states, $\sigma$ expresses the separable state. Some relevant references [32] and [33] have studied the calculation method for relative entropy entanglement. If the density matrix of a two-particle quantum state can be expressed as

$$\rho = \sum_{n_1,n_2} a_{n_1,n_2} \mid \psi_{n_1}\phi_{n_2}\rangle\langle\psi_{n_1}\phi_{n_2} \mid, \tag{18}$$

where $|\psi_n\rangle(|\phi_n\rangle)$ are basic vectors of subsystem A(B). The relative entropy entanglement is written as

$$E_i(\rho_j) = -\sum_n a_{n,n} \ln a_{n,n} + \mathrm{Tr}\rho_j \ln \rho_j,$$
$$(i = a_1 a_2, f). \tag{19}$$

By using the expression of the density matrix of the system, the relative entropy entanglement for the charge qubits can be written as

$$\begin{aligned} E_{a_1a_2}(\rho_{a_1a_2}) &= -\sum_n a_{n,n} \ln a_{n,n} - S(\rho_{a_1a_2}) \\ &= -\sum_n a_{n,n} \ln a_{n,n} \\ &\quad + \sum_{i=1}^{4} \lambda_i(t) \ln \lambda_i(t). \end{aligned} \tag{20}$$

According to numerical computation, we obtain the relative entropy entanglement of two charge qubits in different conditions as shown in Fig. 4.

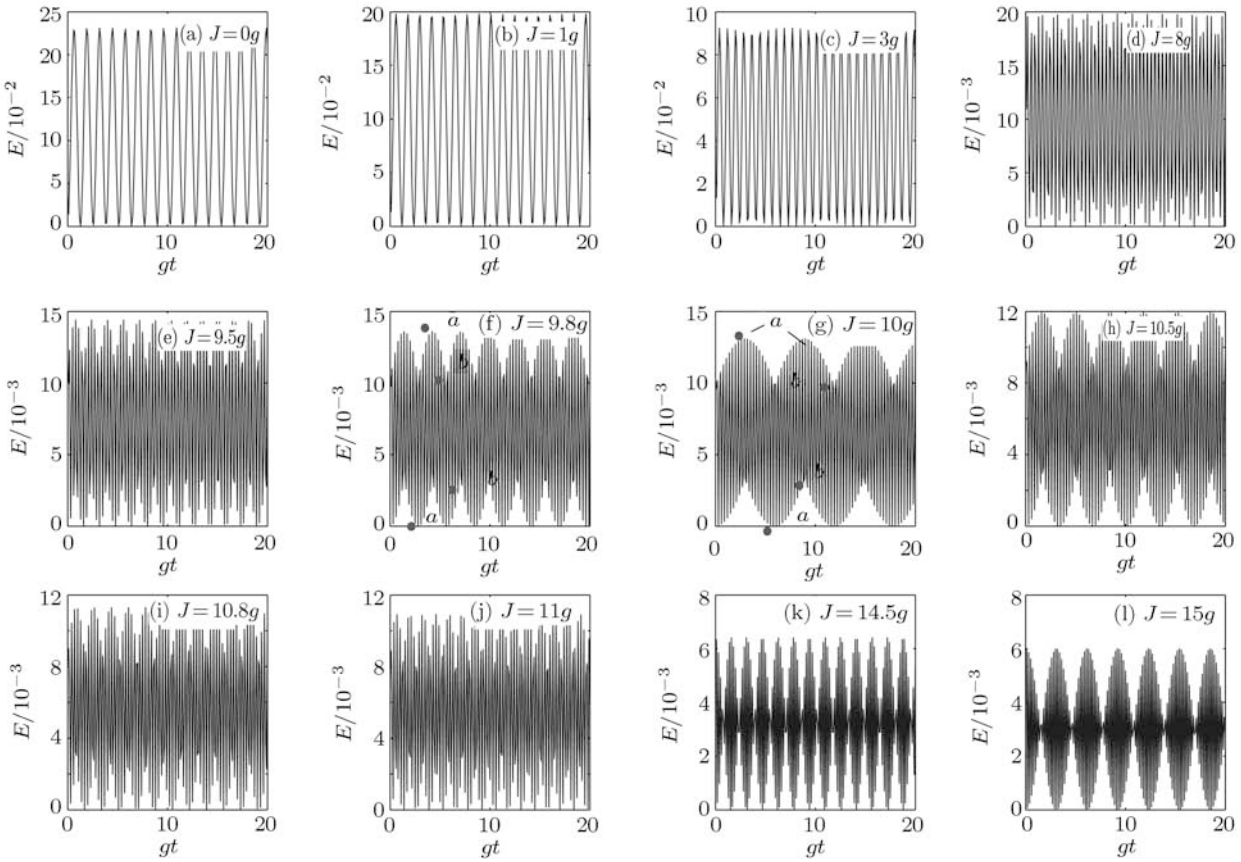



**Fig. 3.** Evolutions of the relative entropy of two charge qubits in different coupling strengths $J$.

Figures 3(a)–3(h) show the properties of the relative entropy of entanglement of two charge qubits. The evolution of the relative entropy of entanglement of the system is very interesting. From those figures one can see that for relatively small values of charge qubit strength, the evolution of the relative entropy entanglement exhibits a single-peak phenomenon. However, with the increase of the values of the coupling strength between the two charge qubits, the evolution of the relative entropy of entanglement exhibits interesting collective beat-like oscillation characteristics and also the degrees of relative entropy entanglement decrease gradually. If one carefully investigates the figures, one can find another interesting property, being that the collective beat-like oscillations have different phase characteristics. From Figs. 3(b) to 3(e), with the variations of the coupling strength $J$ of two charge qubits, the evolution of the relative entropy seems to become less regular. However, this is a process of variation to the structure of the evolution. Up to Fig. 3(f) for $J = 9.8$, the collective beat-like oscillation appears evidently. However, it exhibits a 'counter-phase beat-like oscillation', that is each collective maximum (a peak, labeled by $a$) corresponds to a collective deep anti-valley (labeled by $b$ state, we call them 'trapping states'); and the minimum (anti-peak) corresponds to a collective deep valley. We refer to this phenomenon as 'counter-phase beat-like oscillation'. In Fig. 3(g) ($J = 10g$), this phenomenon is evident. However, with the variation of the coupling strength (from Fig. 3(h) to Fig. 3(l)), the 'collective counter-phase oscillations' collapse and to the peak-to-peak states (alternatively called anti-notes states), and 'trapping state'-to-'trapping state' (or called notes states), as shown in Fig. 3(l). The evolution of the relative entropy forms another kind of structure. From the above discussions, one can see that the relative entropy of the two charge qubit system forms different structures with the variation of the coupling strength between the two charge qubits. From the numeric calculation, we find that the structural formation is sensitive with respect to the value of the parameter $J$. It is well known that the quantum information is related to the quantum measurements and variation of quantum states. It could be possible to use these properties to construct quantum logic gates and other applications of quantum computations.

# 5. Conclusion

In conclusion, we have discussed an artificial atomic system, which is two superconducting charge qubits coupling with an $LC$-resonator. We focused on the evolutionary properties of the von Neumann entropy and the relative entropy of the system. It is found that the evolutions of both von Neumann entropy and relative entropy can form some interesting structures. The evolution of the von Neumann entropy of the system exhibits a double periodic structure. This is a bistable entanglement structure. The formation of the structure depends on the variation of the coupling strength of the two charge qubits. A similar property for the entanglement of atoms in an atom-cavity system has also been discussed in Ref. [27]. Therefore, this shows that the double periodic structure may be a general property of two-qubit-cavity-coupling systems. Since von Neumann entropy can be used to describe the entanglement of quantum pure states, this property is considered to be very interesting. The relative entropy evolves also with the variation of the coupling strength between two charge qubits. We showed that it is possible to observe different evolutionary structures of relative entropy, such as an 'anti-phase structure' (see Fig. 3(g)) and a 'beat-like structure' (see Fig. 3(l)), which are first discovered theoretically for the artificial-atomic-cavity-coupling system.

It is well known that superconducting charge qubit devices can be used to prepare entanglement states by adjusting some parameters. Due to their technical advantage and highly integrative characteristics, superconducting circuit quantum electrodynamic systems are considered to be very important in quantum information processing. The above-discussed properties may have some special applications in implementing quantum logical gates and the transfer of quantum information.

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