Sudden death of entanglement of two atoms interacting with thermal fields*

Luo Cheng-Li(罗成立)†, Miao Long(缪 龙), Zheng Xiao-Lan(郑小兰), Chen Zi-Hong(陈子翃), and Liao Chang-Geng(廖长庚)

Department of Physics, Fuzhou University, Fuzhou 350002, *China*

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We investigate the entanglement dynamics of two initially entangled atoms each interacting with a thermal field. We show that the two entangled atoms become completely disentangled in a finite time and that the lost information cannot return to the atomic system when the mean photon number of the thermal field exceeds a critical value (3.3584), even though the whole system is lossless. Then we study how the detuning between the atomic transition frequency and the field frequency and the disparity between two coupling rates would affect the evolution of the entanglement of the atomic system.

Keywords: entanglement sudden death, thermal field, asymmetric, detuning

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1. Introduction

Quantum entanglement is a special type of nonlocal correlation of quantum systems. A pair of particles can be entangled even if they are remotely located. Quantum entanglement can be applied in both quantum computations and communications.[1−7] To make full use of entanglement we need a thorough understanding of its dynamics. Yu and Eberly found some striking new features of entanglement.[8] They considered two initially entangled qubits which individually interact with vacuum noise and demonstrated that the entanglement can vanish completely in a finite time. This counter-intuitive phenomenon is termed entanglement sudden death (ESD). They showed that the time when ESD occurs is shorter than local decoherent time for arbitrary entangled states (pure and mixed). A deeper understanding of ESD under classical noise has been presented in Ref. [9], which shows that the entanglement of initially mixed two-qubit states may fall abruptly to zero in a finite time due to interaction with classical noise. Yönaç *et al.* investigated the entanglement dynamics of two isolated atoms, each in a perfect one-mode near-resonant cavity, in Ref. [10]. They demonstrated that the evolution of the entanglement of the two non-interacting atoms will show some striking new features, including the ESD effect. They also investigated the evolution of the entanglement between two qubits, each interacting with a single-mode coherent state field, and reported a collapse and revival of entanglement that is similar to the collapse and revival of single-atom properties in cavity quantum electrodynamics (QED).[11] Luo *et al.* found the similar properties of two initially entangled macroscopic fields.[12] Kim *et al.* studied the interaction of the thermal field and a quantum system composed of two qubits and found that two atoms can become entangled through their interaction with a highly chaotic system depending on the initial preparation of the atoms.[13] In Ref. [14], Han *et al.* studied the entanglement dynamics of atoms locally coupled to a cavity field and found the finite-time disentanglement induced by a single-mode thermal field. ESD has been studied theoretically in a variety of systems[15−21] and demonstrated experimentally in photonic systems[22] and atomic ensembles.[23]

The finite-time disentanglement induced by thermal fields is quite different from that induced by initial vacuum cavity fields[10] or coherent state fields.[12] In this paper, we try to make an overall and in-depth analysis of the finite-time disentanglement induced by thermal fields. Unlike Ref. [14], we consider four different type Bell-like states as the initial state of atoms and compare the evolution of entanglement of our sys-

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†Corresponding author. E-mail: chengli-luo@hotmail.com

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tem with that of three different systems. We find that the lost information cannot return to the atomic system when the mean photon number of the thermal field exceeds a critical value (3.3584), even though the whole system is lossless. We also show another interesting phenomenon in that the atomic information leakage rate becomes less sensitive to the initial state of the atoms when the mean photon number of the thermal field increases. Then we study how the detuning between the atomic transition frequency and the field frequency and the disparity between two coupling rates would affect the evolution of two initially entangled atoms. Finally, we think that it may be significant for understanding the decoherence effect in quantum information since the thermal field is the quantum field closest to the classical one.[24]

The paper is organized as follows. In Section 2, we present the model and its theoretical description. In Section 3, we show that the entanglement of the two atoms can be fully destroyed in a finite time via resonant atom–field interaction and then recovered, but the completely lost entanglement will not return to the atomic system when the mean photon number of the thermal field exceeds a critical value (3.3584). We show that the time at which ESD starts to occur for the case of asymmetric atom–cavity coupling is longer than that of the symmetric case. In Section 4, we study how the detuning between the atomic transition frequency and the field frequency would affect the evolution of two initially entangled atoms. A summary appears in Section 5.

2. The model

We consider two spatially separated atoms each interacting with a thermal field. We should emphasize that there is no interaction and communication between the two cavities or between the two atoms. Supposing that the two cavities are treated as lossless, in a frame rotating with the photon frequency, we find that the Hamiltonian for the whole system is easily given by (here we set $\hbar = 1$)

$$H_{\rm tot} = \frac{1}{2}\Delta\sigma_z^A + \frac{1}{2}\Delta\sigma_z^B + g_1(a_1^\dagger\sigma_-^A + a_1\sigma_+^A) + g_2(a_2^\dagger\sigma_-^B + a_2\sigma_+^B), \quad (1)$$

where σ_+^i and σ_-^i $(i = A, B)$ are the rising and lowering operators for the i-th atom, $a_j^\dagger$ and a_j $(j = 1, 2)$ are the creation and annihilation operators for the j-th cavity mode, and g_1, g_2 are the atom–cavity coupling strengths and σ_z denotes the usual diagonal Pauli matrix. Δ is the detuning between the atomic transition frequency ω and the field frequency ω_0.

For a single-field mode with frequency ω_0 in a thermal equilibrium, its density operator can be represented by

$$\rho_f = \left[1 - \exp\left(-\frac{\hbar\omega_0}{k_{\rm B}T}\right)\right]\exp\left(-\frac{\hbar\omega_0 a^\dagger a}{k_{\rm B}T}\right), \quad (2)$$

where T is the temperature and $k_{\rm B}$ represents the Boltzmann constant. Spanned in the Fock state basis, it has the following simple form:

$$\rho_f = \sum_{n=0}^{\infty} P_n|n\rangle\langle n|, \quad (3)$$

where $\bar{n} = [\exp(\hbar\omega/k_{\rm B}T) - 1]^{-1}$ is the mean photon number and $P_n = \bar{n}^n/(\bar{n}+1)^{n+1}$. In this paper, we will study two identical thermal fields, namely, the two thermal fields have the same mean photon number.

Firstly, we will use Wootter's concurrence[25] as our entanglement measure, which is given by

$$C(\rho) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (4)$$

where the quantities λ_i are the eigenvalues in decreasing order of the matrix

$$\zeta = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y). \quad (5)$$

Here ρ is our two-qubit reduced-state density matrix, ρ^* denotes the complex conjugation of ρ in the standard bases $\{|\uparrow\rangle_1|\uparrow\rangle_2, |\uparrow\rangle_1|\downarrow\rangle_2, |\downarrow\rangle_1|\uparrow\rangle_2, |\downarrow\rangle_1|\downarrow\rangle_2\}$, and σ_y is the Pauli matrix expressed in the same bases.

For an x-type ρ with the form

$$\rho = \begin{pmatrix} a & 0 & 0 & b \\ 0 & c & d & 0 \\ 0 & d^* & e & 0 \\ b^* & 0 & 0 & f \end{pmatrix}, \quad (6)$$

equation (4) turns into

$$C(\rho) = 2\max\{0, |d| - \sqrt{af}\ , |b| - \sqrt{ec}\}. \quad (7)$$

Equation (7) will be widely used in this paper.

We consider two initially entangled atoms interacting with two thermal fields, respectively. For a partially entangled atomic pure state being a combination of the Bell states usually denoted by $|\Psi^\pm\rangle$, we have

$$|\Psi_{\rm atom}^\pm\rangle = \cos\alpha\,|\uparrow\rangle_A|\downarrow\rangle_B \pm \sin\alpha\,|\downarrow\rangle_A|\uparrow\rangle_B, \quad (8)$$

where the subscripts A and B label two different atoms, $\uparrow$ denotes the excited state and $\downarrow$ denotes the ground state.

The evolution of the system is given by

$$\rho_{\rm tot}(t) = U(t)|\Psi^{\pm}_{\rm atom}\rangle\langle\Psi^{\pm}_{\rm atom}| \otimes \rho_{f1} \otimes \rho_{f2} U^{\dagger}(t), \quad (9)$$

where

$$U(t) = {\rm e}^{-{\rm i}H_{\rm tot}t}. \quad (10)$$

The reduced density matrix $\rho^{AB}(t)$ of the atomic subsystem can be obtained from $\rho_{\rm tot}(t)$ by tracing out the cavity fields, we can calculate the concurrence of $\rho^{AB}(t)$ by virtue of Eq. (7) and denote it by $C^1(\rho_{AB})$ (see Appendix A).

For another initial entangled state ($|\Phi^{\pm}_{\rm atom}\rangle = \cos\alpha\,|\uparrow\rangle_A\,|\uparrow\rangle_B \pm \sin\alpha\,|\downarrow\rangle_A\,|\downarrow\rangle_B$), the concurrence of $\rho^{AB}(t)$ can be obtained in a similar way and denoted by $C^2(\rho_{AB})$ (see Appendix A).

3. The resonant atom–field interaction

In this section we focus on the resonant condition ($\Delta = 0$) and investigate two cases: symmetric ($g_1 = g_2 = g$) and asymmetric ($g_1 \neq g_2$) atom–cavity couplings.

First, we consider the case of symmetric atom–cavity coupling ($g_1 = g_2 = g$) for studying the evolution of the two initially entangled atoms. Set $\alpha = \pi/4$, when the two atoms are initially in the state $(1/\sqrt{2})(|\uparrow\rangle_A|\downarrow\rangle_B + |\downarrow\rangle_A\,|\uparrow\rangle_B)$, the atom–atom concurrence is plotted in Fig. 1(a) as a function of gt with different mean photon numbers ($\bar{n}$) of the thermal field; and the case of the two atoms initially in the state $(1/\sqrt{2})(|\uparrow\rangle_A\,|\uparrow\rangle_B + |\downarrow\rangle_A\,|\downarrow\rangle_B)$ is plotted in Fig. 1(b). Figures 1(a) and 1(b) show that entanglement can fall abruptly to zero if the initial state of the two atoms is a combination of $|\Psi^+\rangle$ or $|\Phi^+\rangle$. The phenomenon is a specific type of decoherence and the two individual thermal fields can be regarded as two specific "reservoirs" to the atoms. The decoherence rate increases with the mean photon number of the field. Due to the atom–field interaction, the atomic entanglement continuously leaks into the fields, leading to atom–field entanglement. When the states of the thermal fields correlated to the specific atomic states become orthogonal, the two atoms are in mixed states (say, after tracing over the field modes) and, moreover, the entanglement of the two atoms disappears.

It can be seen from Figs. 1(a) and 1(b) that the completely lost two-atom entanglement cannot be recovered when the mean photon number of the thermal field exceeds a critical value. (The analytical solution for this plate is quite difficult to obtain, but numerical simulation is an effective approach of investigating this plate. We then find that the critical value is close to 3.3584. All the critical values in this paper are obtained by numerical simulation). This result is different from Refs. [10]–[12], even though those systems and our system are all lossless. If two atoms are initially in a maximally entangled state, the atoms do not exhibit finite-time ESD in initial vacuum cavity fields in the double Jaynes–Cummings model (JCM) (the results of two different Bell states are $\cos^2 gt$ and $\cos^4 gt$, respectively).[10] In Refs. [11] and [12], the completely lost two-atom and two-field entanglements can be recovered, even though the mean photon number of the field is close to 400. That is because coherent states are not strictly classical states since they are pure states, but the thermal field is the quantum field closest to the classical one. In addition, our result is also different from that of Ref. [8] with the atomic spontaneous emission being included. For the initial state $\cos\alpha\,|\uparrow\rangle_A\,|\downarrow\rangle_B \pm \sin\alpha\,|\downarrow\rangle_A\,|\uparrow\rangle_B$, according to Ref. [8], the evolution of the atom–atom entanglement can be obtained by $2\max\{0, \frac{\sin 2\alpha}{2}\,{\rm e}^{-2\Gamma t}\}$, with Γ being the Einstein coefficients for the two-level atoms in the cavities. It is obvious that this initial state will never exhibit ESD. For another initial state $\cos\alpha\,|\uparrow\rangle_A\,|\uparrow\rangle_B \pm \sin\alpha\,|\downarrow\rangle_A\,|\downarrow\rangle_B$, the concurrence for the reduced density matrix ρ_{AB} for the atomic subsystem can be easily obtained in a similar way. It is $2\max\{0, \frac{\sin 2\alpha}{2}\,{\rm e}^{-2\Gamma t} - {\rm e}^{-2\Gamma t}(1-{\rm e}^{-2\Gamma t})\cos^2\alpha\}$. Obviously this initial state will also never exhibit ESD when $\alpha \geq \pi/4$. The time when ESD starts to occur is $t_{\rm ESD} = \ln[\cos\alpha/(\cos\alpha - \sqrt{1-\cos^2\alpha})]/2\Gamma$. The main reason for the difference may be due to the different mean photon number of the cavity fields (their cavities are vacuum, but ours are not).

The finite-time disentanglement induced by thermal fields can help us analyse the decoherence of a quantum system. As described above, the two thermal fields can be regarded as two "reservoirs" to the atoms and the size of the "reservoir" is dependent on the mean photon number of the thermal field. When $\bar{n} \ll 1$, the atoms can exchange energy with the "reservoirs" leading to the collapses and revivals of the atom–atom entanglement. When $\bar{n} \approx 0$,, a reversible decoherence process could be observed. The time scales for the collapses and revivals of entanglement become shorter when the size of the "reservoir" is increased. When the size of the "reservoir" exceeds a critical value, the completely lost entanglement cannot be recovered (it is closely analogous to the results of the standard irreversible decoherence model, which

considered atomic spontaneous emission and photon leakage).

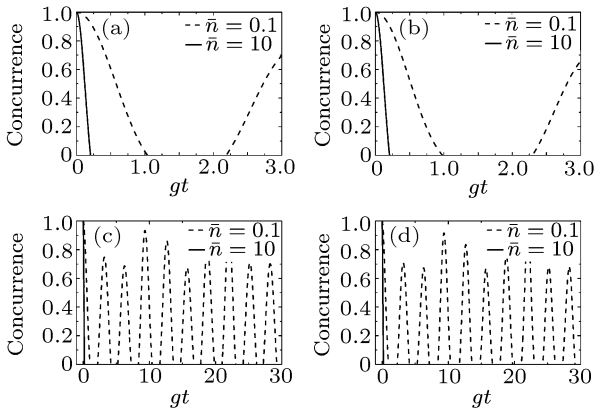


Fig. 1. The evolution of the atom–atom entanglement versus time, dashed and solid lines denote $\bar{n} = 0.1$ and 10, respectively. Panels (a) and (b) denote the initial state of the two atoms $(1/\sqrt{2})(|\uparrow\rangle_A\ |\downarrow\rangle_B + |\downarrow\rangle_A\ |\uparrow\rangle_B)$ and $(1/\sqrt{2})(|\uparrow\rangle_A\ |\uparrow\rangle_B + |\downarrow\rangle_A\ |\downarrow\rangle_B)$ in a short time ($0 \leq gt \leq 3$), respectively. Panels (c) and (d) denote the initial state of the two atoms $(1/\sqrt{2})(|\uparrow\rangle_A\ |\downarrow\rangle_B + |\downarrow\rangle_A\ |\uparrow\rangle_B)$ and $(1/\sqrt{2})(|\uparrow\rangle_A\ |\uparrow\rangle_B + |\downarrow\rangle_A\ |\downarrow\rangle_B)$ in longer time ($0 \leq gt \leq 30$), respectively.

Figures 1(a) and 1(b) also show another interesting phenomenon for which the concurrences of the two different initial entangled states have the same evolution curve when the mean photon number exceeds a critical value (it can be seen in Fig. 2 that the critical value is close to 2.9). The evolutions of the two cases are determined by $C^1(\rho_{AB})$ and $C^2(\rho_{AB})$, respectively. So the difference between the two evolutions is determined by

$$\Phi(t) = |C^1(\rho_{AB}) - C^2(\rho_{AB})|, \tag{11}$$

it is plotted against gt and $\bar{n}$ in Fig. 2. The value of $\Phi(t)$ can represent the disparity of two evolutions. It can be shown that the atomic information leakage rate becomes less sensitive to the initial state of the atoms when the size of the "reservoir" increases.

Then we turn to the case of the asymmetric atom–cavity couplings ($g_1 \neq g_2$). Set $\alpha = \pi/4$, $\bar{n} = 10$ and $g_2 = A \times g_1$ ($0 \leq A \leq 1$). The concurrence of the two atoms initially in state $(1/\sqrt{2})(|\uparrow\rangle_A\ |\downarrow\rangle_B + |\downarrow\rangle_A\ |\uparrow\rangle_B)$ is plotted in Fig. 3 as a function of $g_1 t$ and A, respectively. It can be shown in Fig. 3 that the time at which ESD starts to occur for the case of asymmetric atom–cavity coupling is later than that for the symmetric case ($A = 1$). The time at which ESD starts to occur becomes later when A decreases. However, the time at which ESD starts to occur changes very little when A is less than 0.1.

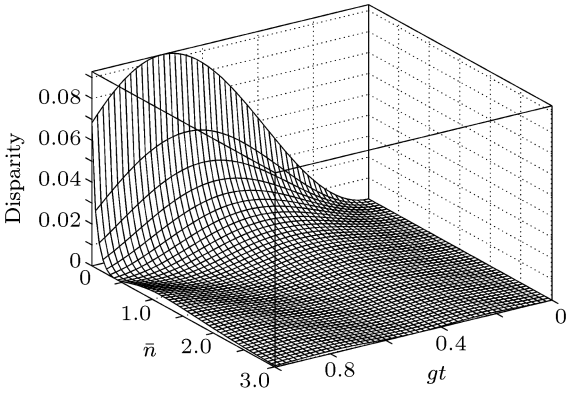


Fig. 2. The disparity between $C(\rho^1_{AB})$ and $C(\rho^2_{AB})$ versus gt and $\bar{n}$.

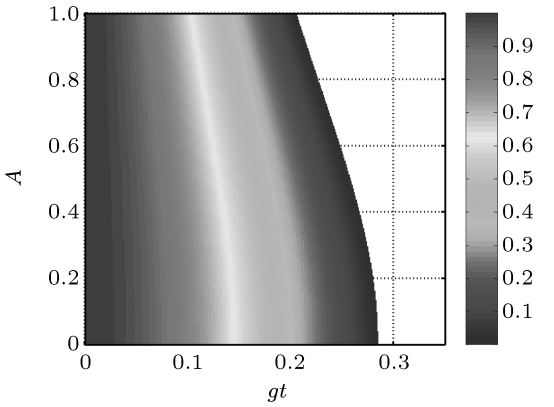


Fig. 3. The evolution of the atom–atom entanglement versus $g_1 t$ and A for $(1/\sqrt{2})(|\uparrow\rangle_A\ |\downarrow\rangle_B + |\downarrow\rangle_A\ |\uparrow\rangle_B)$ with $\bar{n} = 10$, where $A = g_1/g_2$.

4. The effect of detuning

We study how the atom–field detuning affects the evolution of the two-atom entanglement, the detuning Δ between the atomic transition frequency and the field frequency will not be equal to zero in this section. We here set $g_1 = g_2 = g$, $\alpha = \pi/4$ and $\bar{n} = 10$ (for the case of resonant interaction, the completely lost two-atom entanglement cannot be recovered). The concurrence of the two atoms initially in state $(1/\sqrt{2})(|\uparrow\rangle_A\ |\downarrow\rangle_B + |\downarrow\rangle_A\ |\uparrow\rangle_B)$ is plotted in Fig. 4(a) as a function of gt with different atom–field detunings. It shows that the completely lost two-atom entanglement can be recovered when Δ exceeds a critical value (we find that the critical value is close to $2.646g$). This result is different from that of Ref. [10], the completely lost two-atom entanglement cannot be recovered in their system even if the detuning is large enough. Perhaps that is because their result is based on the Markov–Born approximation,

which means that the atomic information leaking into the reservoirs will never return to the atomic system, but our system does not consider atomic spontaneous emission.

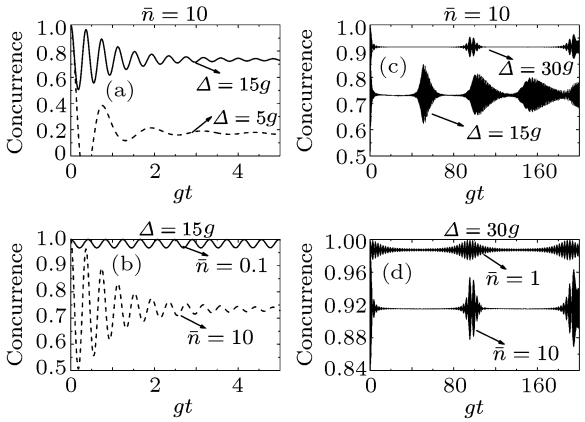


Fig. 4. (a) The evolution of the atom–atom entanglement versus gt for $(1/\sqrt{2})(|\uparrow\rangle_A\,|\downarrow\rangle_B + |\downarrow\rangle_A\,|\uparrow\rangle_B)$ with $\Delta = 5g$ (solid), $15g$ (dashed) when $\bar{n} = 10$; (b) the evolution of the atom–atom entanglement versus gt for $(1/\sqrt{2})(|\uparrow\rangle_A\,|\downarrow\rangle_B + |\downarrow\rangle_A\,|\uparrow\rangle_B)$ with $\bar{n} = 0.1$ (dashed), 10 (solid) when $\Delta = 15g$; (c) the evolution of the atom–atom entanglement versus gt for $(1/\sqrt{2})(|\uparrow\rangle_A\,|\downarrow\rangle_B + |\downarrow\rangle_A\,|\uparrow\rangle_B)$ with $\Delta = 15g$, $30g$ when $\bar{n} = 10$; (d) the evolution of the atom–atom entanglement versus gt for $(1/\sqrt{2})(|\uparrow\rangle_A\,|\downarrow\rangle_B + |\downarrow\rangle_A\,|\uparrow\rangle_B)$ with $\bar{n} = 1$, 10 when $\Delta = 30g$.

It also can be seen that the entanglement cannot completely disappear when Δ exceeds another critical value (we find that the critical value is close to $7.175g$). This phenomenon should be useful for retaining the entanglement, which is a major resource in quantum information processing and quantum computing.

The main reason for the appearance of the two phenomena is that the increase of Δ means the decrease of the energy exchange between the atomic system and the cavity.[26] So the atomic entanglement leaking into the cavity decreases and then the correlation between the atomic system and the cavities decreases after one or several full periods of the energy exchange.

Figure 4(b) shows that the variation range of $C(\rho_{AB}^1)$ increases with the increase of the mean photon number of the thermal field. This is because the influence of the "reservoirs" on the atomic system increases with the increase of the size of the "reservoirs". The atomic entanglement leaking into the cavities increases with the increase of the size of the "reservoirs". In addition, the correlation between the atomic system and the cavities also increases with the increase of the size of the "reservoirs" after one or several full periods of energy exchange.

It can be seen from Figs. 4(a) and 4(b) that the function of the concurrence of the two atoms oscillates rapidly. The oscillation period is $T = 2\pi/\sqrt{\Delta^2 + (4\bar{n}+1)g^2}$, which is similar to the corresponding period of the atomic population inversion. This formula can well explain Figs. 4(a) and 4(b). The reason is that in Eq. (14) we have as many summations as the original inversion calculations.[27]

It can be shown in Figs. 4(c) and 4(d) that the envelope surrounding the oscillations is a periodic function. However, it is very difficult to find the period from Eqs. (14), (15), (21) and (22). Now we try to find the period under the condition of large detuning (i.e. for $\Delta \gg g_{1,2}$).

Under the condition of large detuning, i.e. for $\Delta \gg g_{1,2}$, there is no energy exchange between the atomic system and the cavities. We here set $g_1 = g_2 = g$, the effective Hamiltonian is given by[26]

$$\begin{aligned} H \;=\; & \lambda(|\uparrow\rangle_A\langle\uparrow|\; a_1 a_1^\dagger - |\downarrow\rangle_A\langle\downarrow|\; a_1^\dagger a_1 + |\uparrow\rangle_B \\ & \times \langle\uparrow|\; a_2 a_2^\dagger - |\downarrow\rangle_B\langle\downarrow|\; a_2^\dagger a_2), \end{aligned} \tag{12}$$

where $\lambda = g^2/\Delta$, $|\uparrow\rangle_j$ and $|\downarrow\rangle_j$ ($j = A, B$) are the excited and ground states of the j-th atom, respectively.

If the initial states of atoms A and B are

$$|\Psi_{\rm atom}^{\pm}\rangle = \frac{\sqrt{2}}{2}(|\uparrow\rangle_A\,|\downarrow\rangle_B \pm |\downarrow\rangle_A\,|\uparrow\rangle_B), \tag{13}$$

then evolution of the system is given by

$$\begin{aligned} \rho_{\rm tot}(t) \;=\; & \frac{1}{2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_n P_m \\ & \times (\,{\rm e}^{-{\rm i}(n+1)\lambda t}\,{\rm e}^{{\rm i}m\lambda t}\;|\uparrow\rangle_A|n\rangle_1\;|\downarrow\rangle_B|m\rangle_2 \\ & + {\rm e}^{{\rm i}n\lambda t}\,{\rm e}^{-{\rm i}(m+1)\lambda t}\;|\downarrow\rangle_A|n\rangle_1\;|\uparrow\rangle_B|m\rangle_2) \\ & \times (\,{\rm e}^{{\rm i}(n+1)\lambda t}\,{\rm e}^{-{\rm i}m\lambda t}\;{}_A\langle\uparrow|_1\;\langle n|\;{}_B\langle\downarrow|_2\;\langle m| \\ & + {\rm e}^{-{\rm i}n\lambda t}\,{\rm e}^{{\rm i}(m+1)\lambda t}\;{}_A\langle\downarrow|_1\;\langle n|\;{}_B\langle\uparrow|_2\;\langle m|). \end{aligned} \tag{14}$$

The information about the entanglement of two atoms is contained in the reduced density matrix ρ_{AB} for the atomic subsystem, which can be obtained from Eq. (14) by tracing out the cavity fields. It can be shown in Eqs. (6) and (7) that the concurrence for this density matrix is

$$C^3(\rho_{AB}) = 2\max\{0, \sqrt{a_1^2 + a_2^2}\}, \tag{15}$$

where

$$a_1 = \frac{1}{2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_n P_m \cos 2(n-m)\lambda t,$$

and

$$a_2 = \frac{1}{2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_n P_m \sin 2(n-m)\lambda t.$$

It is obvious that $C^3(\rho_{AB})$ is a periodic function and its period is $T = \pi/\lambda$.

If the initial states of atoms A and B are

$$|\Phi^{\pm}_{\rm atom}\rangle = \frac{\sqrt{2}}{2}(|\uparrow\rangle_A\,|\uparrow\rangle_B \pm |\downarrow\rangle_A\,|\downarrow\rangle_B), \qquad (16)$$

the concurrence for the reduced density matrix ρ_{AB} for the atomic subsystem can be easily obtained in a similar way. It is

$$C^4(\rho_{AB}) = 2\max\left\{0, \sqrt{b_1^2 + b_2^2}\right\}, \qquad (17)$$

where

$$b_1 = \frac{1}{2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_n P_m \cos 2(n+m+2)\lambda t,$$

and

$$b_2 = \frac{1}{2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_n P_m \sin 2(n+m+2)\lambda t.$$

It is obvious that $C^4(\rho_{AB})$ is also a periodic function and its period is $T = \pi/\lambda$.

We have shown above that the period is $T = \pi\Delta/g^2$, which is in good agreement with that of Fig. 4(c). It needs to be noted that the period is independent of the mean photon number of the thermal field, which can be proved by Fig. 4(d). However, it needs to be emphasized that $C^3(\rho_{AB})$ and $C^4(\rho_{AB})$ are equal to 1 and will have infinite period when the cavity field is vacuum (namely, the mean number of the field is equal to 0). That is to say, the envelope function is independent of the detuning and equal to 1 when the cavity field is vacuum.

5. Conclusion

In summary, we have analysed the evolution of the entanglement of two atoms initially in an entangled state when each atom interacts with a thermal field. We should emphasize that the whole system is lossless. We have shown that the non-interacting atoms A and B can abruptly lose their entanglement for the case of symmetric atom–cavity coupling and resonant interaction. Given the lossless nature of the evolution, we can expect the recovery of the original entanglement to occur in a periodic way following each sudden death event, this is evident in Refs. [10]–[12]. However, this conclusion does not apply to this system, even though the whole system is also lossless. In this system, whether the completely lost two-atom entanglement can be recovered or not, this problem is dependent on the mean photon number of the thermal field. The completely lost two-atom entanglement cannot be recovered when the mean photon number of the thermal field exceeds a critical value (3.3584). Overall, with the increase of the mean photon number of the thermal field, the degree of disorder (entropy) of the thermal field increases. The increase of the degree of disorder means that the atom–atom entanglement will disappear more rapidly and be more difficult to recover via atom–field interaction.

This phenomenon can help us to analyse the decoherence of a quantum system, two thermal fields can be regarded as two "reservoirs" to the atoms, the size of the "reservoir" is dependent on the mean photon number of the thermal field. With the increase of the size of the "reservoir", the atomic entanglement leaking into the cavity increases and cannot gradually return to the atomic system leading to the disappearance of the entanglement of the atomic system. In addition, we have also proved that the atomic information leakage rate becomes less sensitive to the initial state of the atoms when the size of the "reservoirs" increases.

The time at which ESD starts to occur becomes longer with the increase of the detuning between the atomic transition frequency and the field frequency and the disparity between two coupling rates. Unlike Ref. [8], we have found that the completely lost two-atom entanglement can be recovered when Δ exceeds a critical value ($2.646g$) for the same $\bar{n}$ with the resonant case that the completely lost two-atom entanglement cannot be recovered and the entanglement cannot completely disappear when Δ exceeds another critical value ($7.175g$). Finally, we have shown that the concurrence of the two atoms causes rapid oscillation with period $T = 2\pi/\sqrt{\Delta^2 + (4\bar{n}+1)g^2}$ and the envelope surrounding the oscillations is a periodic function (under the condition of large detuning, the period is $\pi\Delta/g^2$).

Appendix A: The evolution of two initial entangled atoms for a general situation

We will calculate the evolution of two initial entangled atoms for a general situation. If the initial state of atoms A and B is

$$|\Psi^{\pm}_{\rm atom}\rangle = \cos\alpha\,|\uparrow\rangle_A\,|\downarrow\rangle_B \pm \sin\alpha\,|\downarrow\rangle_A\,|\uparrow\rangle_B, \qquad (A1)$$

then the initial density matrix of the total system is given by

$$\begin{aligned}\rho_{\rm tot}(0) =& (\cos\alpha\,|\uparrow\rangle_A\,|\downarrow\rangle_B \pm \sin\alpha\,|\downarrow\rangle_A\,|\uparrow\rangle_B)\\ &\times (\cos\alpha\ {}_A\langle\uparrow|_B\langle\downarrow| \pm \sin\alpha_A\langle\downarrow|_B\langle\uparrow|)\\ &\otimes \sum_{n=0}^{\infty} P_n \mid n\rangle_1\langle n \mid \otimes \sum_{m=0}^{\infty} P_m \mid m\rangle_2\langle m \mid .\end{aligned} \tag{A2}$$

The evolution of the system is given by

$$\begin{aligned}\rho_{\rm tot}(t) =& \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_nP_m\Bigg\{\cos\alpha\Bigg[\left(\cos\frac{\Omega_nt}{2} - \frac{{\rm i}\Delta}{\Omega_n}\sin\frac{\Omega_nt}{2}\right)|\uparrow\rangle_A \mid n\rangle_1 - \frac{2{\rm i}g_1\sqrt{n+1}}{\Omega_n}\sin\frac{\Omega_nt}{2}\,|\downarrow\rangle_A \mid n+1\rangle_1\Bigg]\\ &\times\Bigg[\left(\cos\frac{\Omega_{m-1}t}{2} + \frac{{\rm i}\Delta}{\Omega_{m-1}}\sin\frac{\Omega_{m-1}t}{2}\right)|\downarrow\rangle_B \mid m\rangle_2 - \frac{2{\rm i}g_2\sqrt{m}}{\Omega_{m-1}}\sin\frac{\Omega_{m-1}t}{2}\,|\uparrow\rangle_B \mid m-1\rangle_2\Bigg]\\ &\pm\sin\alpha\Bigg[\left(\cos\frac{\Omega_{n-1}t}{2} + \frac{{\rm i}\Delta}{\Omega_{n-1}}\sin\frac{\Omega_{n-1}t}{2}\right)|\downarrow\rangle_A \mid n\rangle_1 - \frac{2{\rm i}g_1\sqrt{n}}{\Omega_{n-1}}\sin\frac{\Omega_{n-1}t}{2}\,|\uparrow\rangle_A \mid n-1\rangle_1\Bigg]\\ &\times\Bigg[\left(\cos\frac{\Omega_mt}{2} - \frac{{\rm i}\Delta}{\Omega_m}\sin\frac{\Omega_mt}{2}\right)|\uparrow\rangle_B \mid m\rangle_2 - \frac{2{\rm i}g_2\sqrt{m+1}}{\Omega_m}\sin\frac{\Omega_mt}{2}\,|\downarrow\rangle_B \mid m+1\rangle_2\Bigg]\Bigg\}\\ &\times\Bigg\{\cos\alpha\Bigg[\left(\cos\frac{\Omega_nt}{2} + \frac{{\rm i}\Delta}{\Omega_n}\sin\frac{\Omega_nt}{2}\right)_A\langle\uparrow|_1\,\langle n \mid + \frac{2{\rm i}g_1\sqrt{n+1}}{\Omega_n}\sin\frac{\Omega_nt}{2}{}_A\langle\downarrow|_1\,\langle n+1 \mid\Bigg]\\ &\times\Bigg[\left(\cos\frac{\Omega_{m-1}t}{2} - \frac{{\rm i}\Delta}{\Omega_{m-1}}\sin\frac{\Omega_{m-1}t}{2}\right)_B\langle\downarrow|_2\,\langle m \mid + \frac{2{\rm i}g_2\sqrt{m}}{\Omega_{m-1}}\sin\frac{\Omega_{m-1}t}{2}{}_B\langle\uparrow|_2\,\langle m-1 \mid\Bigg]\\ &\pm\sin\alpha\Bigg[\left(\cos\frac{\Omega_{n-1}t}{2} - \frac{{\rm i}\Delta}{\Omega_{n-1}}\sin\frac{\Omega_{n-1}t}{2}\right)_A\langle\downarrow|_1\,\langle n \mid + \frac{2{\rm i}g_1\sqrt{n}}{\Omega_{n-1}}\sin\frac{\Omega_{n-1}t}{2}{}_A\langle\uparrow|_1\,\langle n-1 \mid\Bigg]\\ &\times\Bigg[\left(\cos\frac{\Omega_mt}{2} + \frac{{\rm i}\Delta}{\Omega_m}\sin\frac{\Omega_mt}{2}\right)_B\langle\uparrow|_2\,\langle m \mid + \frac{2{\rm i}g_2\sqrt{m+1}}{\Omega_m}\sin\frac{\Omega_mt}{2}{}_B\langle\downarrow|_2\,\langle m+1 \mid\Bigg]\Bigg\},\end{aligned} \tag{A3}$$

where $\Omega_n = \sqrt{\Delta^2 + 4g_1^2(n+1)}$ and $\Omega_m = \sqrt{\Delta^2 + 4g_2^2(m+1)}$.

The information about the entanglement of two atoms is contained in the reduced density matrix ρ_{AB} for the atomic subsystem, which can be obtained from Eq. (14) by tracing out the cavity fields. It can be shown in Eqs. (6) and (7) that the concurrence for this density matrix is

$$C^1(\rho_{AB}) = 2\max\{0, \mid d \mid -\sqrt{af}\}, \tag{A4}$$

the matrix elements are given by

$$\begin{aligned}a =& \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Bigg[\cos^2\alpha P_nP_{m+1}\left(\cos^2\frac{\Omega_nt}{2} + \frac{\Delta^2}{\Omega_n^2}\sin^2\frac{\Omega_nt}{2}\right)\frac{4g_2^2(m+1)}{\Omega_m^2}\sin^2\frac{\Omega_mt}{2}\\ &+ \sin^2\alpha P_{n+1}P_m\frac{4g_1^2(n+1)}{\Omega_n^2}\sin^2\frac{\Omega_nt}{2}\left(\cos^2\frac{\Omega_mt}{2} + \frac{\Delta^2}{\Omega_m^2}\sin^2\frac{\Omega_mt}{2}\right)\Bigg],\end{aligned} \tag{A5}$$

$$\begin{aligned}|d| =& \frac{\sin 2\alpha}{2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}P_nP_m\sqrt{\left(\cos^2\frac{\Omega_nt}{2} + \frac{\Delta^2}{\Omega_n^2}\sin^2\frac{\Omega_nt}{2}\right)\left(\cos^2\frac{\Omega_mt}{2} + \frac{\Delta^2}{\Omega_m^2}\sin^2\frac{\Omega_mt}{2}\right)}\\ &\times\sqrt{\left(\cos^2\frac{\Omega_{n-1}t}{2} + \frac{\Delta^2}{\Omega_{n-1}^2}\sin^2\frac{\Omega_{n-1}t}{2}\right)\left(\cos^2\frac{\Omega_{m-1}t}{2} + \frac{\Delta^2}{\Omega_{m-1}^2}\sin^2\frac{\Omega_{m-1}t}{2}\right)},\end{aligned} \tag{A6}$$

$$\begin{aligned}f =& \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Bigg[\cos^2\alpha P_{n-1}P_m\frac{4g_1^2n}{\Omega_{n-1}^2}\sin^2\frac{\Omega_{n-1}t}{2}\left(\cos^2\frac{\Omega_{m-1}t}{2} + \frac{\Delta^2}{\Omega_{m-1}^2}\sin^2\frac{\Omega_{m-1}t}{2}\right)\\ &+ \sin^2\alpha P_nP_{m-1}\left(\cos^2\frac{\Omega_{n-1}t}{2} + \frac{\Delta^2}{\Omega_{n-1}^2}\sin^2\frac{\Omega_{n-1}t}{2}\right)\frac{4g_2^2m}{\Omega_{m-1}^2}\sin^2\frac{\Omega_{m-1}t}{2}\Bigg].\end{aligned} \tag{A7}$$

We find that the concurrences of the two different initial states $\cos\alpha\,|\uparrow\rangle_A\,|\downarrow\rangle_B + \sin\alpha\,|\downarrow\rangle_A\,|\uparrow\rangle_B$ and $\cos\alpha\,|\uparrow\rangle_A\,|\downarrow\rangle_B - \sin\alpha\,|\downarrow\rangle_A\,|\uparrow\rangle_B$ have the same evolution curve.

If the initial states of atoms A and B are

$$\mid \Phi_{\rm atom}^{\pm}\rangle = \cos\alpha\,|\uparrow\rangle_A\,|\uparrow\rangle_B \pm \sin\alpha\,|\downarrow\rangle_A\,|\downarrow\rangle_B, \tag{A8}$$

then the initial density matrixes of the total system are given by

$$\rho_{\rm tot}(0) = (\cos\alpha\,|\uparrow\rangle_A\,|\uparrow\rangle_B \pm \sin\alpha\,|\downarrow\rangle_A\,|\downarrow\rangle_B)(\cos\alpha_A\langle\uparrow|_B\,\langle\uparrow| \pm \sin\alpha_A\langle\downarrow|_B\,\langle\downarrow|) \otimes \sum_{n=0}^{\infty} P_n \mid n\rangle_1\langle n \mid \otimes \sum_{m=0}^{\infty} P_m \mid m\rangle_2\langle m \mid . \tag{A9}$$

The evolution of the system is given by

$$\begin{aligned}
\rho_{\rm tot}(t) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_n P_m \Bigg\{ & \cos\alpha\left[\left(\cos\frac{\Omega_n t}{2} - \frac{{\rm i}\Delta}{\Omega_n}\sin\frac{\Omega_n t}{2}\right)|\uparrow\rangle_A \mid n\rangle_1 - \frac{2{\rm i}g_1\sqrt{n+1}}{\Omega_n}\sin\frac{\Omega_n t}{2}\,|\downarrow\rangle_A \mid n+1\rangle_1\right] \\
& \times \left[\left(\cos\frac{\Omega_m t}{2} - \frac{{\rm i}\Delta}{\Omega_m}\sin\frac{\Omega_m t}{2}\right)|\uparrow\rangle_B \mid m\rangle_2 - \frac{2{\rm i}g_2\sqrt{m+1}}{\Omega_m}\sin\frac{\Omega_m t}{2}\,|\downarrow\rangle_B \mid m+1\rangle_2\right] \\
& \pm \sin\alpha\left[\left(\cos\frac{\Omega_{n-1} t}{2} + \frac{{\rm i}\Delta}{\Omega_{n-1}}\sin\frac{\Omega_{n-1} t}{2}\right)|\downarrow\rangle_A \mid n\rangle_1 - \frac{2{\rm i}g_1\sqrt{n}}{\Omega_{n-1}}\sin\frac{\Omega_{n-1} t}{2}\,|\uparrow\rangle_A \mid n-1\rangle_1\right] \\
& \times \left[\left(\cos\frac{\Omega_{m-1} t}{2} + \frac{{\rm i}\Delta}{\Omega_{m-1}}\sin\frac{\Omega_{m-1} t}{2}\right)|\downarrow\rangle_B \mid m\rangle_2 - \frac{2{\rm i}g_2\sqrt{m}}{\Omega_{m-1}}\sin\frac{\Omega_{m-1} t}{2}\,|\uparrow\rangle_B \mid m-1\rangle_2\right]\Bigg\} \\
& \times \Bigg\{\cos\alpha\left[\left(\cos\frac{\Omega_n t}{2} + \frac{{\rm i}\Delta}{\Omega_n}\sin\frac{\Omega_n t}{2}\right)_A\langle\uparrow|_1\,\langle n \mid + \frac{2{\rm i}g_1\sqrt{n+1}}{\Omega_n}\sin\frac{\Omega_n t}{2}{}_A\langle\downarrow|_1\,\langle n+1 \mid\right] \\
& \times \left[\left(\cos\frac{\Omega_m t}{2} + \frac{{\rm i}\Delta}{\Omega_m}\sin\frac{\Omega_m t}{2}\right)_B\langle\uparrow|_2\,\langle m \mid + \frac{2{\rm i}g_2\sqrt{m+1}}{\Omega_m}\sin\frac{\Omega_m t}{2}{}_B\langle\downarrow|_2\,\langle m+1 \mid\right] \\
& \pm \sin\alpha\left[\left(\cos\frac{\Omega_{n-1} t}{2} - \frac{{\rm i}\Delta}{\Omega_{n-1}}\sin\frac{\Omega_{n-1} t}{2}\right)_A\langle\downarrow|_1\,\langle n \mid + \frac{2{\rm i}g_1\sqrt{n}}{\Omega_{n-1}}\sin\frac{\Omega_{n-1} t}{2}{}_A\langle\uparrow|_1\,\langle n-1 \mid\right] \\
& \times \left[\left(\cos\frac{\Omega_{m-1} t}{2} - \frac{{\rm i}\Delta}{\Omega_{m-1}}\sin\frac{\Omega_{m-1} t}{2}\right)_B\langle\downarrow|_2\,\langle m \mid + \frac{2{\rm i}g_2\sqrt{m}}{\Omega_{m-1}}\sin\frac{\Omega_{m-1} t}{2}{}_B\langle\uparrow|_2\,\langle m-1 \mid\right]\Bigg\},
\end{aligned} \tag{A10}$$

where $\Omega_n = \sqrt{\Delta^2 + 4g_1^2(n+1)}$ and $\Omega_m = \sqrt{\Delta^2 + 4g_2^2(m+1)}$.

The information about the entanglement of two atoms is contained in the reduced density matrix ρ_{AB} for the atomic subsystem, which can be obtained from Eq. (A6) by tracing out the cavity fields. It can be shown in Eqs. (6) and (7) that the concurrence for this density matrix is

$$C^2(\rho_{AB}) = 2\max\{0, \mid b \mid -\sqrt{ce}\}, \tag{A11}$$

the matrix elements are given by

$$\begin{aligned}
|b| = {} & \frac{\sin 2\alpha}{2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} P_n P_m \sqrt{\left(\cos^2\frac{\Omega_n t}{2} + \frac{\Delta^2}{\Omega_n^2}\sin^2\frac{\Omega_n t}{2}\right)\left(\cos^2\frac{\Omega_m t}{2} + \frac{\Delta^2}{\Omega_m^2}\sin^2\frac{\Omega_m t}{2}\right)} \\
& \times \sqrt{\left(\cos^2\frac{\Omega_{n-1} t}{2} + \frac{\Delta^2}{\Omega_{n-1}^2}\sin^2\frac{\Omega_{n-1} t}{2}\right)\left(\cos^2\frac{\Omega_{m-1} t}{2} + \frac{\Delta^2}{\Omega_{m-1}^2}\sin^2\frac{\Omega_{m-1} t}{2}\right)},
\end{aligned} \tag{A12}$$

$$\begin{aligned}
c = {} & \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Bigg[\cos^2\alpha P_n P_{m-1}\left(\cos^2\frac{\Omega_n t}{2} + \frac{\Delta^2}{\Omega_n^2}\sin^2\frac{\Omega_n t}{2}\right)\frac{4g_2^2 m}{\Omega_{m-1}^2}\sin^2\frac{\Omega_{m-1} t}{2} \\
& + \sin^2\alpha P_{n+1} P_m \frac{4g_1^2(n+1)}{\Omega_n^2}\sin^2\frac{\Omega_n t}{2}\left(\cos^2\frac{\Omega_{m-1} t}{2} + \frac{\Delta^2}{\Omega_{m-1}^2}\sin^2\frac{\Omega_{m-1} t}{2}\right)\Bigg],
\end{aligned} \tag{A13}$$

$$\begin{aligned}
e = {} & \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Bigg[\cos^2\alpha P_{n-1} P_m \frac{4g_1^2 n}{\Omega_{n-1}^2}\sin^2\frac{\Omega_{n-1} t}{2}\left(\cos^2\frac{\Omega_m t}{2} + \frac{\Delta^2}{\Omega_m^2}\sin^2\frac{\Omega_m t}{2}\right) \\
& + \sin^2\alpha P_n P_{m+1}\left(\cos^2\frac{\Omega_{n-1} t}{2} + \frac{\Delta^2}{\Omega_{n-1}^2}\sin^2\frac{\Omega_{n-1} t}{2}\right)\frac{4g_2^2(m+1)}{\Omega_m^2}\sin^2\frac{\Omega_m t}{2}\Bigg],
\end{aligned} \tag{A14}$$

We find that the concurrences of the two different initial states $\cos\alpha\,|\uparrow\rangle_A\,|\uparrow\rangle_B + \sin\alpha\,|\downarrow\rangle_A\,|\downarrow\rangle_B$ and $\cos\alpha\,|\uparrow\rangle_A\,|\uparrow\rangle_B - \sin\alpha\,|\downarrow\rangle_A\,|\downarrow\rangle_B$ have the same evolution curve.

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