Entanglement dynamics of two qubits coupled by Heisenberg XY interaction under a nonuniform magnetic field in a spin bath*

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This paper investigates the entanglement dynamics of a Heisenberg XY model for a two-spin system in the presence of a nonuniform magnetic field. The master equations and the concurrence evolution equations for the initial α state are derived and analysed. It is shown that for the symmetric initial α state, only the nonuniform field can play a role in entanglement dynamics while the uniform field and the bath will not play such a role. For the asymmetric α state, the nonuniform field leads to the beat pattern oscillation of the concurrence evolution. The inhomogeneity of the field can enhance the entanglement by suppressing the decoherence effects of both the spin–orbit interaction and the spin bath.

Keywords: quantum entanglement, decoherence, qubits, Heisenberg model

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1. Introduction

Entanglement[1] is one of the most interesting features of quantum mechanics, which makes it central to studies of quantum information and quantum computation science, such as quantum teleportation,[2,3] quantum cryptography,[4,5] quantum dense coding,[6] etc. These potential applications stimulate investigations of the quantification and control of quantum entanglement.

Considerable effort has been devoted to the study of the interacting Heisenberg spin chain,[7,8] and especially to two parts of a quantum chain,[9,10] which is a very reliable model for various quantum information proposals.[11−13] The magnetic field[14,15] provides an additional effective parameter (besides the temperature T) for controlling entanglement. Kamta *et al.*[16] investigated the entanglement of two-qubit XY systems in the presence of a constant field $B_z\hat{z}$ and found that the thermal entanglement of anisotropic samples can be manipulated and produced for any finite T through changes both in the anisotropy and in the magnetic field strength B_z. Sun *et al.*[17] investigated thermal entanglement in the two-qubit XY model with a nonuniform field along the $\hat{z}$ axis and showed that the entanglement and the critical temperature T_c for the entanglement to disappear may be enhanced under a nonuniform field. Li *et al.*[18] investigated the thermal entanglement of an anisotropic Heisenberg XYZ model with spin–orbit interaction under a nonuniform field. Within these studies, the central topic is the thermal entanglement,[19] i.e., quantum entanglement arising from the interaction of the multiqubit system with the environment or the reservoir at thermal equilibrium.

Just as for solid-state spin nanodevices, the decoherence effect of a fermionic environment arising from nuclear spins has attracted a great deal of attention recently. Yuan *et al.*[20] studied the entanglement dynamics of two interacting spins in a Heisenberg XY high symmetry spin model by using Holstein–Primakoff transformations. They showed that the analytical results are dependent on some particular initial states. In their model, the interaction between the

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spins is isotropic as well as the detuning controlled by an external magnetic field. Rao[21] systematically investigated the dynamical control of qubits in the presence of the decoherence of a spin bath and showed that the loss of entanglement within the two qubits could be controlled by tuning the frequency of the applied radio frequency (RF) field. Ferraro *et al.*[22] investigated the dynamics of two uncoupled qubits interacting with a common spin environment through a Heisenberg XX mechanism and proved the possibility of realizing entanglement sudden death (ESD) as well as entanglement sudden birth (ESB)[23] by appropriately controlling the initial condition of the environment. It is noted that only a uniform magnetic field is studied in these works, the nonuniform case is rarely considered. Therefore, in this paper, the nonuniform external magnetic field is going to be taken into account for the entanglement dynamics of a coupled two-spin Heisenberg XY system interacting with a nuclear spin bath in thermal equilibrium. In addition, since spin–orbit coupling is one of the most important relaxation mechanisms for confined electron spins, it is also included in the qubit–qubit interactions. Entanglement dynamics are sensitive to the initial state of the system and we have investigated the maximally entangled state in a previous work[24] and shown that the ESD phenomenon happens due to the noise from the common bath. We adopt the α state as the initial state for the two qubits in this paper. As the big difference from the other kinds of initial states, no ESD happens and the entanglement can evolve in a beat pattern due to the nonuniform external field.

The paper is organized as follows. In Section 2, we introduce the Hamiltonian of the model and derive the master equation governing the evolution of the system. In Section 3, the analytical and numerical results of the entanglement dynamics are presented. Finally, a brief discussion concludes the paper in Section 4.

2. Model and method

We consider two electronic spins in quantum dots (QD) as the qubits system coupled to a bath composing of N nuclear spins via the Fermi contact hyperfine (HF) interaction.[14,25] A nonuniform external magnetic field is applied over the qubits, which is along the $\hat{z}$ axis. The coupling between the two qubits contains both the spin–spin interaction and the spin–orbit interaction. The spin–spin interaction is modeled by anisotropic Heisenberg interaction, which can be electrically controlled by using a calibrated voltage pulse.[26] The spin–orbit interaction is described by the Dzyaloshinskii–Moriya (DM) interaction.[27−29] The total Hamiltonian for such a system can be expressed as

$$H_{\rm tot} = H_{\rm S} + H_{\rm SB}, \tag{1}$$

where

$$H_{\rm S} = b_1 S_1^z + b_2 S_2^z + \boldsymbol{J}\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + \boldsymbol{D} \cdot (\boldsymbol{S}_1 \times \boldsymbol{S}_2), \tag{2}$$

$$H_{\rm SB} = \sum_{k=1}^{N} g_k (S_1^z + S_2^z) I_k^z. \tag{3}$$

The $H_{\rm S}$ represents the self-Hamiltonian of the qubit system. The $b_i = g_e^* \mu_{\rm B} B_i$ $(i = 1, 2)$ is the Zeeman splitting with the applied magnetic field B_i of the i-th electron spin, the field is assumed to be along the $\hat{z}$ axis. The g_e^* is the effective Landé factor of the electron, $\mu_{\rm B}$ is the Bohr magneton, $\boldsymbol{S}_i = (S_i^x, S_i^y, S_i^z)$ is the spin-1/2 operator of the i-th electron spin, $\boldsymbol{J} = (J_x, J_y, J_z)$ is the real coupling coefficient of the exchange interaction. $\boldsymbol{D}$ is the DM vector arising from the spin–orbit coupling and we choose $\boldsymbol{D} = D\hat{z}$. $H_{\rm SB}$ describes the HF interaction between the central spins and the nuclear bath, I_k^z is the k-th nuclear spin operator of the bath. Due to the renormalization of the appropriate parameters, it is not important whether the spins of the nuclei are 1/2 or not, so it is set to be 1/2 in the following discussion.[30] The qubits-bath coupling strength $g_k = (8\pi/3) g_e \mu_{\rm B} g_n \mu_n |\Psi(\boldsymbol{r}_k)|^2$ is determined by the electron density $|\Psi(\boldsymbol{r}_k)|^2$ at the site $\boldsymbol{r}_k$ of the k-th bath spin (g_n and μ_n are the Landé factor and the magnetons of the nuclei, respectively). For simplicity, it is assumed that the electronic wave function is spread uniformly over the dot volume, that is, the interaction between the qubits and the bath is homogeneous $g_k = g$, since it has been shown that[21,31] the only effect of inhomogeneous interaction is that of destroying certain revival effects. Since the magnetic moment of the electron is much larger $O(10^3)$ in comparison with that of the nucleon, the interaction among the bath spins and their coupling to the external field has a comparatively small effect on the dynamics[21] and so we neglect them here.

We consider the Hamiltonian $H_{\rm S}$ for the anisotropic Heisenberg XY model under a nonuniform field in terms of the raising and lowering operators $S_i^{\pm} = S_i^x \pm S_i^y$ as

$$\begin{aligned} H_{\rm S} =\ & (B+b) S_1^z + (B-b) S_2^z \\ & + \frac{J\gamma}{2}(S_1^+ S_2^+ + S_1^- S_2^-) + \left(\frac{J}{2} + \frac{{\rm i}D}{2}\right) S_1^+ S_2^- \end{aligned}$$

$$+\left(\frac{J}{2}-\frac{\mathrm{i}D}{2}\right)S_1^- S_2^+, \tag{4}$$

where $B=(b_1+b_2)/2$ is the mean external magnetic field and $b=(b_1-b_2)/2$ measures the degree of the inhomogeneity of the field, $J=(J_x+J_y)/2$ is the mean coupling coefficient in the xy-plane, $\gamma=(J_x-J_y)/(J_x+J_y)$ $(-1\leq\gamma\leq 1)$ measures the anisotropy of the system, $\gamma=0$ for the isotropic XY model and $\gamma=\pm 1$ for the Ising model.[16] The initial state of the qubit-bath system is assumed to be uncorrelated, i.e., $\rho_{\rm tot}(0)=\rho_{\rm S}(0)\otimes\rho_{\rm B}(0)$, where $\rho_{\rm S}$, $\rho_{\rm B}$ represent the density matrices of the system and the bath, respectively. The evolution of the total system is governed by the dynamical equation

$$\rho_{\rm tot}(t)=U(t)\rho_{\rm S}(0)\otimes\rho_{\rm B}(0)U^{\dagger}(t), \tag{5}$$

where, $U(t)=\exp(-\mathrm{i}H_{\rm tot}t)$ is the time-evolution operator. The reduced matrix of the system is obtained by tracing over the bath degrees of freedom $\rho_{\rm S}(t)={\rm Tr}_{\rm B}\rho_{\rm tot}(t)$.

The initial density matrix of the spin bath satisfies the Boltzmann distribution, that is, $\rho_{\rm B}(0)=(1/Z_\beta)\exp(-\beta M)$,[32,33] where $M=\sum_k I_k^z$ is the magnetization of the bath, β is the inverse spin temperature, $Z_\beta={\rm Tr}(\,{\rm e}^{\beta M})=[2\cosh(\beta/2)]^N$ is the partition function and the Boltzmann constant has been set to 1. Such a choice corresponds to a thermally polarized bath with the polarization $P_{\rm B}=\langle M\rangle/(N/2)=\tanh(\beta/2)$. When $\beta=0$, $\rho_{\rm B}(0)=2^{-N}\mathbf{1}_1\otimes\cdots\otimes\mathbf{1}_N$ corresponds to an unpolarized bath, where $\mathbf{1}_k$ is the unity matrix for the k-th bath spin.

In the standard bases[34] $\{|++\rangle,|+-\rangle,|-+\rangle,|--\rangle\}$, the initial state of the qubits is considered to be the α states[35,36] defined as

$$\rho_{\rm S}(0)=\frac{1}{3}\begin{pmatrix}\alpha&0&0&0\\0&1&1&0\\0&1&1&0\\0&0&0&1-\alpha\end{pmatrix} \tag{6}$$

with the parameter α satisfying $0\leq\alpha\leq 1$, so concurrence $C(0)=\frac{2}{3}[1-\sqrt{\alpha(1-\alpha)}]$. The α states belong to the "X" states,[37]

$$\rho_{\rm S}=\begin{pmatrix}\rho_{11}&0&0&\rho_{14}\\0&\rho_{22}&\rho_{23}&0\\0&\rho_{23}^*&\rho_{33}&0\\\rho_{14}^*&0&0&\rho_{44}\end{pmatrix}. \tag{7}$$

The X states have a robust character in that the initial X structure is maintained during the evolution under the influences of the environment.

Then, for $t\geq 0$, the equations of the matrix elements are given by

$$\rho_{11}(t)=\frac{1}{3Z_\beta}\sum_{i=1}^{N}M_i\Big\{\alpha\cos^2\Omega t+\Big[\alpha(B+\Delta_i)^2+(1-\alpha)\frac{J^2\gamma^2}{4}\Big]\frac{\sin^2\Omega t}{\Omega^2}\Big\}, \tag{8}$$

$$\rho_{22}(t)=\frac{1}{3}\Big\{\cos^2\omega t+\Big[\Big(b+\frac{J}{2}\Big)^2+\frac{D^2}{4}\Big]\times\frac{\sin^2\omega t}{\omega^2}-\frac{D\sin 2\omega t}{2\omega}\Big\}, \tag{9}$$

$$\rho_{33}(t)=\frac{1}{3}\Big\{\cos^2\omega t+\Big[(b-\frac{J}{2})^2+\frac{D^2}{4}\Big]\frac{\sin^2\omega t}{\omega^2}+\frac{D\sin 2\omega t}{2\omega}\Big\}, \tag{10}$$

$$\rho_{44}(t)=\frac{1}{3Z_\beta}\sum_{i=1}^{N}M_i\Big\{(1-\alpha)\cos^2\Omega t+\Big[(1-\alpha)(B+\Delta_i)^2+\alpha\frac{J^2\gamma^2}{4}\Big]\times\frac{\sin^2\Omega t}{\Omega^2}\Big\}, \tag{11}$$

$$\rho_{23}(t)=\frac{1}{3}\Big\{\cos^2\omega t-\Big[b^2+\frac{(D+\mathrm{i}J)^2}{4}\Big]\times\frac{\sin^2\omega t}{\omega^2}-\frac{\mathrm{i}b\sin 2\omega t}{\omega}\Big\}, \tag{12}$$

$$\rho_{14}(t)=\frac{1}{6Z_\beta}\sum_{i=1}^{N}M_i(1-2\alpha)J\gamma\Big[(B+\Delta_i)\frac{\sin^2\Omega t}{\Omega^2}+\mathrm{i}\frac{\sin 2\Omega t}{2\Omega}\Big], \tag{13}$$

where $M_i=\langle i|\exp(-\beta M)|i\rangle$, $|i\rangle$ is the i-th basis states of the bath, $\Omega=\sqrt{(B+\Delta_i)^2+\frac{1}{4}J^2\gamma^2}$, $\Delta_i=\langle i|g\sum_k I_k^z|i\rangle$, $\omega=\sqrt{b^2+\frac{1}{4}(J^2+D^2)}$.

3. Entanglement evolution

We adopt Wootters' concurrence[38,39] to quantify the degree of entanglement between the two qubits in this paper. The concurrence is conveniently defined for both pure and mixed states and it can be calculated explicitly from the density matrix of the bipartite system,

$$C(\rho)=\max\{0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\}, \tag{14}$$

where λ_i are the eigenvalues in decreasing order of the matrix $\widetilde{\rho}$,

$$\widetilde{\rho} = \rho_{\rm S}(\sigma_y \otimes \sigma_y)\rho_{\rm S}^*(\sigma_y \otimes \sigma_y), \tag{15}$$

where $\rho_{\rm S}^*$ denotes the complex conjugation of $\rho_{\rm S}$, σ_y is the pure imaginary Pauli matrix. The C varies from 0 for a separable state to 1 for a maximally entangled state. Since the X structure is preserved during the evolution determined by the Hamiltonian of Eq. (4), the concurrence at time t for an X-state can be simplified to

$$C(t) = 2\max\{0, K_1(t), K_2(t)\}, \tag{16}$$

where,

$$K_1(t) = |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}, \tag{17}$$

$$K_2(t) = |\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)}. \tag{18}$$

By substituting Eqs. (8)–(13), we can obtain the concurrence evolution of the system. Evidently, it is not easy for us to obtain simple analytical expressions for the concurrence of α states. We will give some numerical simulations in the following discussion, as well as analytical results for some fixed parameters. Note that all parameters are dimensionless in this paper.

Apparently, the concurrence evolution behaves in an oscillating way, which is related to the magnetic field, the interaction between the two qubits and the interaction between the qubits and the bath. First of all, by examining Eq. (13), we notice that if $\alpha = \frac{1}{2}$ (or $J = 0$, or $\gamma = 0$), then $\rho_{14}(t) = 0$, i.e., $K_1 < 0$, the concurrence would be determined by $K_2(t)$,

$$K_2(t) = |\rho_{23}(t)| - \frac{1}{6}. \tag{19}$$

In this case, it can be seen from Eq. (12) that the mean field B and the bath will not have any influence on the entanglement dynamics of the system while only the inhomogeneity of the field b will play a role, which is caused by the symmetry of the initial states of the system. Moreover, the more symmetry the initial state has, the much less influence the mean field and the bath coupling have over the entanglement dynamics of the system.

For asymmetric $\alpha \neq \frac{1}{2}$ states, the exact expression of concurrence is slightly more complicated. Without any loss of generality, we consider some special cases with $\alpha = 0$. When $D = 0$, $g = 0$, and $\beta = 0$, the following expressions can be obtained

$$K_2(t) = -K_1(t) = \frac{1}{3}\sqrt{1 - \frac{J^2b^2\sin^4\omega t}{\omega^4}}$$

$$- \left|\frac{J\gamma\sin\Omega t}{6\Omega}\right|\sqrt{1 - \frac{J^2\gamma^2\sin^2\Omega t}{4\Omega^2}}$$

$$\approx \frac{1}{3}\left[1 - \frac{J^2b^2\sin^4\omega t}{2\omega^4} - \left|\frac{J\gamma\sin\Omega t}{6\Omega}\right|\left(1 - \frac{J^2\gamma^2\sin^2\Omega t}{8\Omega^2}\right)\right]. \tag{20}$$

It can be noted that the concurrence is mainly determined by $K_2(t)$, since $K_1(t)$ is negative for most of the time. The oscillation period is closely related to B and b, that is to say, the entanglement oscillation behaviours can be controlled by the external nonuniform magnetic field.

When the field is uniform (i.e., $b = 0$),

$$C(t) \approx \frac{2}{3}\left[1 - \left|\frac{J\gamma\sin\Omega t}{6\Omega}\right|\left(1 - \frac{J^2\gamma^2\sin^2\Omega t}{8\Omega^2}\right)\right]$$

is always bigger than zero. The oscillation period for the concurrence approaches π/Ω (i.e., $\frac{\pi}{\sqrt{B^2+J^2\gamma^2/4}}$) for $B > 0$, and $\langle C(t)\rangle \to 2/3$ for large B, these results are shown in Fig. 1. The contour plot of concurrence as a function of the mean field B and time t is shown in Fig. 1(a), while figure 1(b) shows curves of $C(t)$ for some given B, thus detailed information can be obtained. It can be found that the entanglement is promoted on the whole by increasing B. It is noticed that if $b \gg J/2$, the second term of Eq. (20) can also be deleted, and then it corresponds to the situation of the uniform field where b does not play a role.

Figure 2(a) shows the contour plot of concurrence as a function of B and t for the nonuniform field with $b = 1$. It is easy to see that the entanglement develops as B is increased. When the magnetic field is inhomogeneous (i.e., $b \neq 0$) and b is not large enough, obviously from Eq. (20), it results in a beat pattern for the concurrence evolution because when $\omega \neq \Omega$ there are two oscillation frequencies, which is shown in Fig. 2(b). If the mean field is relatively large $B \gg 0$, the last term of Eq. (20) can be omitted and

$$C(t) \approx \frac{2}{3}\left(1 - \frac{J^2b^2\sin^4\omega t}{2\omega^4}\right), \tag{21}$$

then the oscillation period approaches to π/ω (i.e., $\pi/\sqrt{b^2 + J^2/4}$). As shown in the figures, the larger the mean magnetic field B is, the more regular the periodical evolution of entanglement oscillation is. When B is large enough, the period of oscillation turns out to be stable and almost regular periodical oscillation appears. From Eq. (21), it can be seen that the entanglement of the system can be manipulated and enhanced by the inhomogeneity of the field, which will be proved in the next figure.

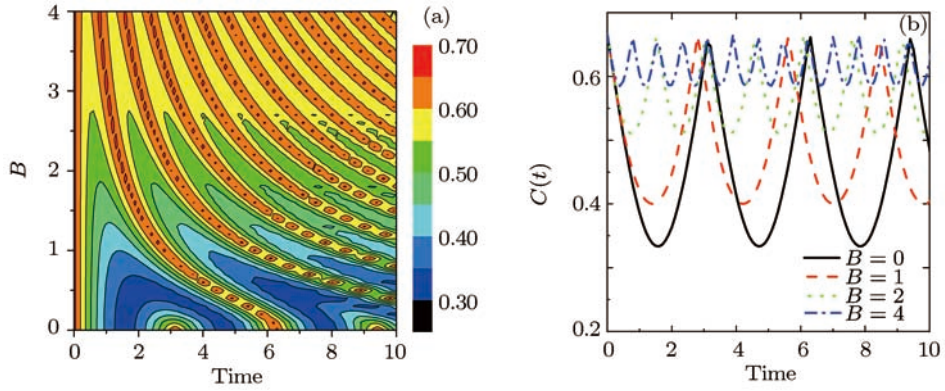


Fig. 1. (colour online) Plots of concurrence versus the mean field B and time t with the inhomogeneity $b = 0$. The given parameters are $\alpha = 0$, $D = 0$, $g = 0$, $\beta = 0$, $J = 1$, $\gamma = 1$, $N = 10$.

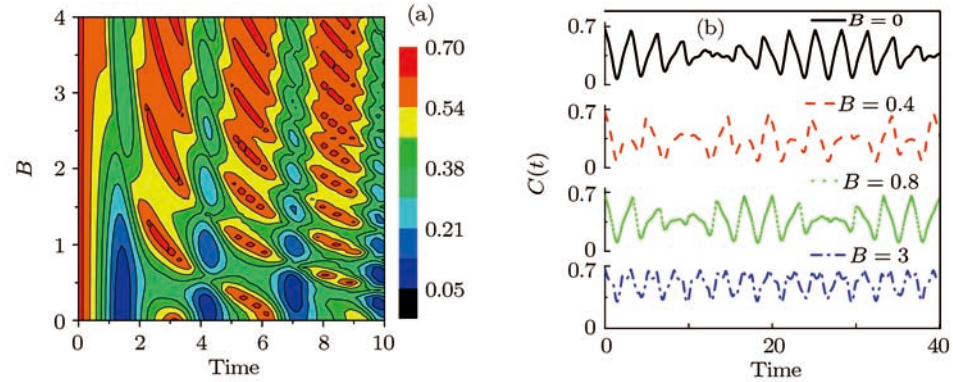


Fig. 2. (colour online) Plots of concurrence versus the mean field B and time t with the inhomogeneity $b = 1$. The other parameters are the same as those in Fig. 1.

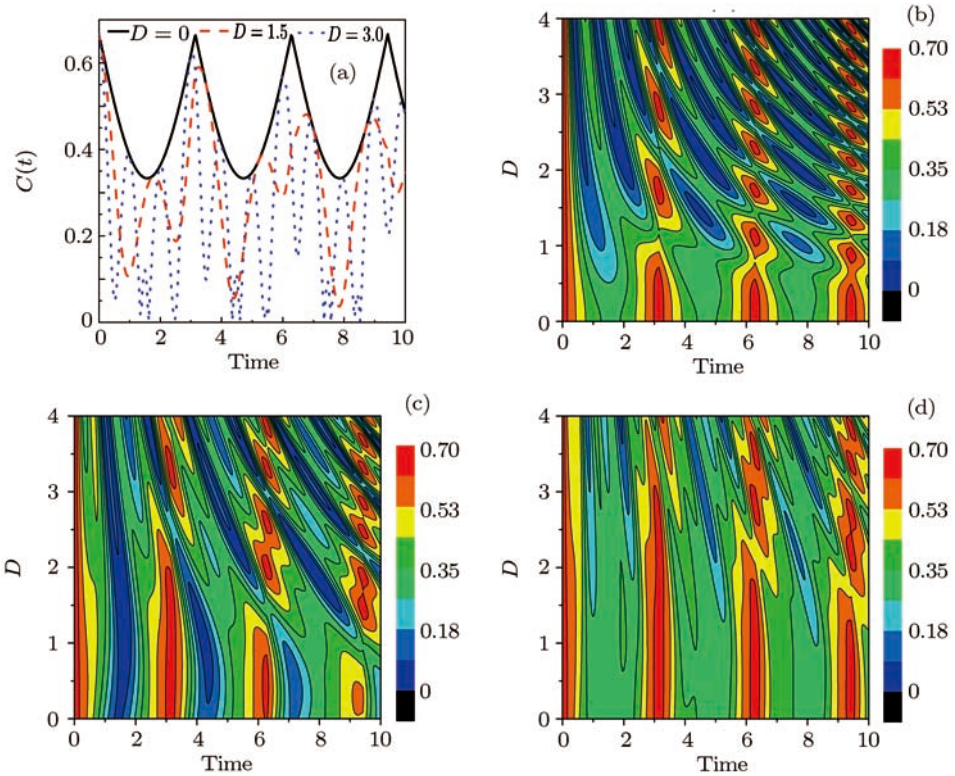


Fig. 3. (colour online) Curves of concurrence evolution with $D = 0$ (solid), $D = 1.5$ (dashed), $D = 3$ (dotted) in (a) with $b = 0$. Contour plots of concurrence versus D and t for (b) $b = 0$, (c) $b = 1$, (d) $b = 2$. The other parameters are the same as those in Fig. 1 except $B = 0$.

Furthermore, the influence of DM interaction is considered and the plots of the concurrence $C(t)$ with D are shown in Fig. 3. It is shown that the oscillation of the concurrence becomes much more intensive with the increase of D since the frequency ω is getting larger. In addition, by comparing the evolution curves of Fig. 3(a) with and without D, it is confirmed that DM interaction, as one of the dominant decoherence mechanisms of the qubit, decays the entanglement of the system greatly. By comparing Figs. 3(b)–3(d), it is important to see that the stable oscillation area of entanglement is extended as b increases, which is easy to understand because when D is relatively weak its effect cannot become evident, and the oscillation period of concurrence is stable. This indicates that the decoherence effect caused by the spin–orbit interaction can be suppressed by properly increasing the inhomogeneity b of the field in a certain period of time.

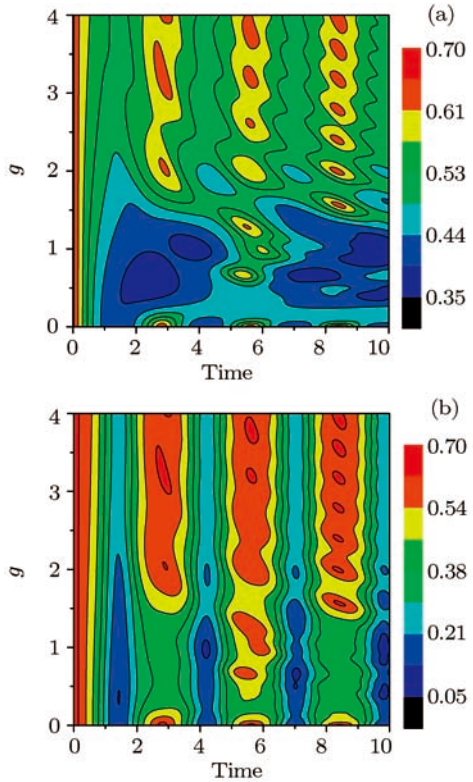


Fig. 4. (colour online) Contour plots of concurrence versus the coupling strength g and time t for (a) $b = 0$, (b) $b = 1$ with $B = 1$. The other parameters are the same as those in Fig. 1.

It is easy to verify the effects of the spin bath on the entanglement of the system. The bath will decay the initial entanglement of the system by comparing the entanglement with $0 < g < 2$ and $g = 0$ in Fig. 4. However, if the bath coupling strength g is strong enough, it can induce entanglement revival by comparing the area of plots with $0 < g < 2$ and $g > 2$ in Fig. 4. It is important to find out, by comparing Fig. 4(a) with Fig. 4(b), if the entanglement of the system can be enhanced by increasing the inhomogeneity of the field to a certain extent, that is to say, if the nonuniform field can inhibit the dechoherence effects of the bath.

It is necessary to point out that the analysis for the finite-temperature case has also been considered and it will not change the main results above. Therefore, it is restricted to the $\beta = 0$ case for simplicity in the context.

4. Conclusions

In conclusion, we have investigated a two-coupled-spin system in a Heisenberg XY model interacting with a spin bath in thermal equilibrium. A nonuniform external magnetic field is applied along the z direction. Since it is necessary to consider the nonuniform external magnetic field at each qubit for performing quantum computing, we focus on the influence of a nonuniform field on the dynamics.

The nonuniform field can have a dramatic influence on the entanglement of the system. Especially for the symmetric initial α state, only the nonuniform field can have an effect on the entanglement dynamics while the uniform field and the bath will not play a part. For the asymmetric α state, the nonuniform field leads to the beat pattern for the concurrence evolution.

Moreover, it is important to see that the uniform field can not only promote the entanglement of the system, but also the inhomogeneity of the field can enhance entanglement by depressing the decoherence effect of the spin–orbit interaction. Last but not least, the nonuniform field can also suppress the decoherence effect caused by the spin bath and increase the entanglement.

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