Tripartite states Bell-nonlocality sudden death in a spin environment with multisite interaction*

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(Received 14 December 2010; revised manuscript received 16 March 2011)

This paper demonstrates that multipartite Bell-inequality violations can be fully destroyed in a finite time in three-qubit states coupled to a general XY spin-chain with a three-site interaction environment. The Mermin–Ardehali–Belinksii–Klyshko inequality is used to detect the degree of nonlocality, as measured by the extent of their violations. The effects of system-environment couplings, the size of degrees of freedom of the environment and the strength of the three-site interaction on the Bell-inequality violations are given. The results indicate that the Bell-inequality violations of the tripartite states will be completely destroyed by decoherence under certain conditions for the GHZ state. The decoherence-free subspaces of our model are identified and the entanglement of quantum states is also discussed.

Keywords: Bell-type inequality, Bell-nonlocality sudden death, three-site interaction

PACS: 03.65.Ud, 03.67.Mn **DOI:** 10.1088/1674-1056/20/8/080301

1. Introduction

The dynamical behaviour of entanglement under the effects of the environment is a problem of fundamental interest in quantum computation (QC) and quantum information processing (QIP).[1−6] Recently, the dynamics of entanglement received a great deal of attention following the work of Yu and Eberly,[7,8] in which the entanglement between two particles coupled with two independent environments completely vanished within a finite time, called entanglement sudden death (ESD). However, the study of ESD in bipartite systems has been limited because properly quantifying multipartite entanglement, especially for mixed states, is not clear for systems of more than two qubits.[9,10] Very recently, Jaeger and Ann demonstrated the Bell-nonlocality sudden death (BNSD) of a tripartite system by discussing Bell-inequality violations.[11] It showed that multipartite Bell-inequality violations can be fully destroyed in a finite time in three-qubit systems subjected to a multi-local noise environment. The extension of the consideration of the sudden death of nonlocal properties is significant because tripartite systems can exhibit fundamental characteristics which do not exist in bipartite systems. Due to its intrinsic and practical interest, BNSD has attracted much attention.[12−17]

In the real world, QIP will always inevitably be affected by the decoherence induced by the external environment. The evaluation of the extent to which the decoherence affects quantum entanglement is an interesting problem and attracts much attention. Many researchers have investigated the entanglement dynamics of quantum systems under decoherence induced by system–environment interaction and almost all of them considered the environment as an uncorrelated environment.[18−20] In a practical situation, the particles of an environment always interact with each other. What is the effect of the decoherence induced by a correlated environment on quantum states? In order to answer this question, researchers have undertaken many studies. Quan *et al.* studied the decoherence induced by a correlated environment.[21] It was shown that at a critical point of quantum phase transition

*Project supported by the National Natural Science Foundation of China (Grant No. 11004050), the Scientific Research Fund of Hunan Provincial Education Department of China (Grant Nos. 09A013 and 10B013), the Science and Technology Research Foundation of Hunan Province of China (Grant No. 2010FJ4120), the Excellent Talents Program of Hengyang Normal University of China (Grant No. 2010YCJH01), and the Science Foundation of Hengyang Normal University of China (Grant Nos. 10A46 and 10B69).

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(QPT) the decoherence is enhanced. For a two-qubit case under a transverse Ising model, Sun *et al.* discovered that the concurrence decays exponentially with a fourth power of time in the vicinity of the critical point of the environmental system.[22] The multiqubit case was investigated by Ma *et al.*[23] For two-spin coupling to an XY chain with Dzyaloshinsky–Moriya (DM) interaction, Cheng *et al.* found that DM interaction can slightly enhance the decay of the decoherence factor in the weak coupling region and, in the strong coupling region, the decoherence factor is super-sensitive to DM interaction.[24]

In addition, many theoretical studies have paid attention not only to nearest-neighbour spin–spin interactions but also to next-nearest-neighbour ones, as well as the multiple spin-exchange model. In comparison with those models with only nearest neighbour interactions, these more complicated models seem more realistic.[25] For example, the additional terms involving the multisite interaction operators were recognized to be important for the description of many physical systems. In Ref. [26], Cheng *et al.* analysed the disentanglement of a bipartite system in which the two parties coupled to the spin environment with multisite interaction. However, for a three-qubit system in a correlated environment with multisite interaction, the nonlocal behaviour that can be measured by the extent of the violation of the Bell-inequality is still an open issue.

2. Theoretical model

We choose the environment to be a general XY spin chain with XZY–YZX type of three-site interaction and the three spins are transversely coupled to the chain with homogeneous coupling parameters. The corresponding Hamiltonian reads

$$\begin{aligned} H = & -\sum_{l}^{L} J\Big(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z\Big) \\ & -\sum_{l}^{L}\alpha J(\sigma_{l-1}^x\sigma_l^z\sigma_{l+1}^y - \sigma_{l-1}^y\sigma_l^z\sigma_{l+1}^x) \\ & -\sum_{l}^{L}\frac{gJ}{2}(\sigma_A^z+\sigma_B^z+\sigma_C^z)\sigma_l^z = H_E + H_I. \end{aligned} \quad (1)$$

Here, we set the spin-coupling interaction $J = 1.0$. The parameters α, λ and g are dimensionless. The g denotes the coupling strength between the chain and the three spin-qubits. Noticing that $[\sigma_A^z + \sigma_B^z + \sigma_C^z,\ \sigma_l^\beta] = 0$, we can rewrite Eq. (1) as $H = \sum_{\mu=1}^{\zeta} |\phi_\mu\rangle\langle\phi_\mu| \otimes H_E^{\lambda_\mu}$, where $|\ \phi_\mu\rangle$ $(\mu = 1, \ldots, 8)$ is the μ-th eigenstate of operator $\xi = g(\sigma_A^z + \sigma_B^z + \sigma_C^z)$ corresponding to the μ-th eigenvalues g_μ, and Λ_μ is given by $\lambda_\mu = \lambda + g_\mu$, $H_E^{\lambda_\mu}$ is given from H_E by replacing λ with λ_μ.

In order to obtain the time evolution of the density matrix of the system, we should follow standard procedure by defining the conventional Jordan–Wigner transformation, which maps spin to one-dimensional skinless fermions with creation (annihilation) operators $a_l^\dagger$ (a_l). Then the Fourier transforms of the fermionic operators described by $d_k = \frac{1}{\sqrt{L}}\sum_l a_l\, \mathrm{e}^{-\mathrm{i}2\pi lk/L}$ are introduced. We can then diagonalize Hamiltonian (1) by transforming the fermion operators to momentum space and using the Bogoliubov transformation. The result is

$$H_E^{\lambda_\mu} = \sum\nolimits_k \Lambda_k^{\lambda_\mu}\left(\gamma_{k,\lambda_\mu}^\dagger\gamma_{k,\lambda_\mu} - \frac{1}{2}\right), \quad (2)$$

where the energy spectrum $\Lambda_k^{\lambda_\mu}$ is expressed by

$$\Lambda_k^{\lambda_\mu} = 2\left[\varepsilon_k + \alpha\sin\left(\frac{4\pi k}{L}\right)\right], \quad (3)$$

where $\varepsilon_k = \{[\cos(2\pi k/L) - \lambda_\mu]^2 + \gamma^2\sin^2(2\pi k/L)\}^{1/2}$. The corresponding Bogoliubov transformed fermion operators are defined by

$$\gamma_{k,\lambda_\mu} = \cos\frac{\theta_k^{\lambda_\mu}}{2}d_k - \mathrm{i}\sin\frac{\theta_k^{\lambda_\mu}}{2}d_{-k}^\dagger, \quad (4)$$

with angles

$$\theta_k^{\lambda_\mu} = \arctan[\gamma\sin(2\pi k/L)/(\lambda_\mu - \cos(2\pi k/L))].$$

In terms of these notations, we can obtain the time evolution of quantum states induced by the interaction between the system and the environment. Let the initial state of the composite system be $|\psi(0)\rangle = |\phi(0)\rangle_{ABC} \otimes |\phi(0)\rangle_E$, where $|\phi(0)\rangle_{ABC}$ and $|\phi(0)\rangle_E$ are the initial states of the three central spins and the environment spin chain, respectively. We have $|\psi(t)\rangle = U(t)|\psi(0)\rangle$, where $U(t) = \exp(-\mathrm{i}Ht)$. The reduced density matrix of system can be obtained as[24]

$$\begin{aligned} \rho_s(t) &= \mathrm{Tr}_E|\psi(t)\rangle\langle\psi(t)| \\ &= \sum_{\mu,\nu=1}^{\zeta} a_\mu a_\nu^*\langle\phi_E|U_E^{\dagger(\lambda_\mu)}(t)U_E^{(\lambda_\nu)}(t)|\phi_E\rangle|\mu\rangle\langle\nu|, \end{aligned} \quad (5)$$

where $a_\mu = \langle\phi_\mu|\phi_{ABC}\rangle$, $U_E^{(\lambda_\mu)}(t) = \exp(-\mathrm{i}H_E^{(\lambda_\mu)}t)$ is the projected time evolution operator for the spin chain dressed by the system–environment interaction

parameter λ_μ. We define the decoherence factor $|F_{\mu\nu}(t)|$ as follows:

$$F_{\mu\nu}(t) = \langle \Phi_E | U_E^{\dagger(\lambda_\mu)}(t) U_E^{(\lambda_\nu)}(t) | \Phi_E \rangle. \tag{6}$$

Here the initial state of the system is separable from the initial state of the environment. The initial state of the environment $|\phi(0)\rangle$ is assumed to be

$$|G\rangle_\lambda = \prod_{k=1}^{M} (\cos(\theta_k^\lambda/2)|0\rangle_k|0\rangle_{-k} + \mathrm{i}\sin(\theta_k^\lambda/2)|1\rangle_k|1\rangle_{-k}, \tag{7}$$

where $|0\rangle_k$ and $|1\rangle_k$ denote the vacuum and single excitation of the k-th mode d_k, respectively. Noticing that

$$\gamma_{k,\lambda_\mu} = (\cos\Theta_k^{(\lambda_\mu)})\gamma_{k,\lambda} - \mathrm{i}(\sin\Theta_k^{(\lambda_\mu)})\gamma_{-k,\lambda}^\dagger, \tag{8}$$

where $\Theta_k^{(\lambda_\mu)} = (\theta_k^{(\lambda_\mu)} - \theta_k^{(\lambda)})/2$ and γ_{k,λ_μ}, $\gamma_{k,\lambda}$ are the normal mode dressed by the system–environment interaction and the pure environment, respectively, we can obtain the ground state $|G\rangle_\lambda$ of the Hamiltonian H_E from the ground state $|G\rangle_{\lambda_\mu}$ of the qubit-dressed Hamiltonian $H_E^{\lambda_\mu}$ by the following relation:

$$|G\rangle_\lambda = \prod_{k=1}^{M} \left(\cos\Theta_k^{(\lambda_\mu)} + \mathrm{i}\sin\Theta_k^{(\lambda_\mu)}\gamma_{k,\lambda_\mu}^\dagger\gamma_{-k,\lambda_\mu}^\dagger\right)|G\rangle_{\lambda_\mu}. \tag{9}$$

In this study, we assume that the system is initially in a general three-qubit pure state

$$\begin{aligned}|\phi(0)\rangle_{ABC} = {} & c_1|000\rangle + c_2|001\rangle + c_3|010\rangle + c_4|011\rangle \\ & + c_5|100\rangle + c_6|101\rangle + c_7|110\rangle + c_8|111\rangle,\end{aligned} \tag{10}$$

where the coefficients satisfy the relation for normalization $|c_1|^2 + |c_2|^2 + \cdots + |c_8|^2 = 1$. After careful calculation, we obtain the elements $M_{\mu\nu}$ of the reduced density matrix of the system, $\mu, \nu = 1, 2, \ldots, 8$. Here, we write the $M_{\mu\nu}$ in the following equations:

$$M_{\mu\nu} = c_\mu c_\nu^* F_{\mu\nu}(t), \quad \mu, \nu = 1, 2, \ldots, 8, \tag{11}$$

It should be noted that once all the moduli of $F_{\mu\nu}(t)$ are completely destroyed by decoherence, any state will become a separable one. The moduli of $F_{\mu\nu}(t)$ can be obtained in the following equation:[27]

$$\begin{aligned}|F_{\mu\nu}(t)| = \prod_{k>0} [& 1 + 2\sin(2\Theta_k^{\lambda_\mu})\sin(2\Theta_k^{\lambda_\nu})\sin(\Lambda_k^{\lambda_\mu}t)\sin(\Lambda_k^{\lambda_\nu}t)\cos(\Lambda_k^{\lambda_\mu}t - \Lambda_k^{\lambda_\nu}t) \\ & - 4\sin^2(\Lambda_k^{\lambda_\mu} - \Lambda_k^{\lambda_\nu})\sin^2(\Lambda_k^{\lambda_\mu}t)\sin^2(\Lambda_k^{\lambda_\nu}t)\sin(2\Theta_k^{\lambda_\mu})\sin(2\Theta_k^{\lambda_\nu}) \\ & - \sin^2(2\Theta_k^{\lambda_\nu})\sin^2(\Lambda_k^{\lambda_\nu}t) - \sin^2(2\Theta_k^{\lambda_\mu})\sin^2(\Lambda_k^{\lambda_\mu}t)]^{1/2},\end{aligned} \tag{12}$$

where $\lambda_{\mu,\nu}$, $\mu, \nu = 1, \ldots, 8$, take the following expressions:

$$\begin{aligned}\lambda_1 = \lambda + \frac{3g}{2}, \quad \lambda_{2,3,5} = \lambda + \frac{g}{2}, \\ \lambda_{4,6,7} = \lambda - \frac{g}{2}, \quad \lambda_8 = \lambda - \frac{3g}{2}.\end{aligned} \tag{13}$$

3. The violation of Mermin–Ardehali–Belinksii–Klyshko (MABK) inequality

For N-qubit systems, there are two kinds of multipartite Bell inequalities used to detect the degree of nonlocality, as measured by the extent of their violations. The first is the inequality put forward by Zukowski and Brukner.[28] The second is the MABK inequality.[29–31] Although it is not clear whether these two kinds of calculations are valid for $N > 3$, the nonlocal correlation for $N = 3$ is equivalent for these two inequalities. Here we use the MABK inequality as a criterion for Bell locality, because it is more easily computed than the first inequality. A quantum state ρ violates the MABK inequality if

$$|\langle \mathrm{B}_N \rangle_\rho| > 1. \tag{14}$$

In the simplest case of two qubits, $N = 2$, the operator B_N on the left-hand side of the MABK inequality is just the Clauser–Home–Shimony–Holt (CHSH) operator,[32] namely

$$B_2 = \frac{1}{2}\left[M_A M_B + M_A M_B' + M_A' M_B - M_A' M_B'\right]. \tag{15}$$

In the case of three qubits, the specific form of operator B_N is expressed as

$$\begin{aligned}B_3 = \frac{1}{2}[& M_A M_B M_C' + M_A M_B' M_C \\ & + M_A' M_B M_C - M_A' M_B' M_C'],\end{aligned} \tag{16}$$

the primed and unprimed terms denote the two different directions in which we measure the corresponding

party; the measurement operators M_K and M'_K, denoting the operators corresponding to measurements on the qubit K (here labeled $K = A$, B, or C), while the second corresponds to a measurement performed along a direction differing by θ relative to that performed on the first qubit, are

$$\begin{pmatrix} M_K \\ M'_K \end{pmatrix} = R(\theta_K) \begin{pmatrix} M_A \\ M'_A \end{pmatrix}, \tag{17}$$

where

$$R(\theta_K) = \begin{pmatrix} \cos\theta_K & -\sin\theta_K \\ \sin\theta_K & \cos\theta_K \end{pmatrix}. \tag{18}$$

In the case of three qubits, there are two such rotation angles θ_B and θ_C; the corresponding measurement operators for qubits A, B and C can be written in terms of Pauli operators as

$$M_A = \sigma_y \otimes I \otimes I, \tag{19a}$$
$$M'_A = \sigma_x \otimes I \otimes I, \tag{19b}$$
$$M_B = I \otimes [\cos(\theta_B)\sigma_y - \sin(\theta_B)\sigma_x] \otimes I, \tag{19c}$$
$$M'_B = I \otimes [\cos(\theta_B)\sigma_y + \sin(\theta_B)\sigma_x] \otimes I, \tag{19d}$$
$$M_C = I \otimes I \otimes [\cos(\theta_C)\sigma_y - \sin(\theta_C)\sigma_x], \tag{19e}$$
$$M'_C = I \otimes I \otimes [\cos(\theta_C)\sigma_y + \sin(\theta_C)\sigma_x]. \tag{19f}$$

(i) Assume that the initial state of the system is a three-qubit GHZ state,

$$|\phi(0)\rangle_{ABC} = c_1|000\rangle + c_8|111\rangle, \tag{20}$$

then the expectation value of the B_3 operator for the GHZ state can be obtained as

$$|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)| = \mathrm{tr}[B_{3\rho_{\rm GHZ}}] = 4|c_1||c_8||F_{18}||\cos\theta_{BC\delta}|, \tag{21}$$

where $\theta_{BC\delta} = \theta_B + \theta_C + \delta$. Particularly, we consider the case in which the initial state is a standard GHZ one, namely,

$$|\phi(0)\rangle_{ABC} = \frac{\sqrt{2}}{2}|000\rangle + \frac{\sqrt{2}}{2}|111\rangle, \tag{22}$$

then we have

$$|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)| = 2|F_{18}||\cos\theta_{BC\delta}|, \tag{23}$$

One can calculate the expectation value of the MABK operator by substituting Eq. (12) into the above equation. When the time is short enough, we can have $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)| > 1$, that is, there exists Bell-nonlocality.

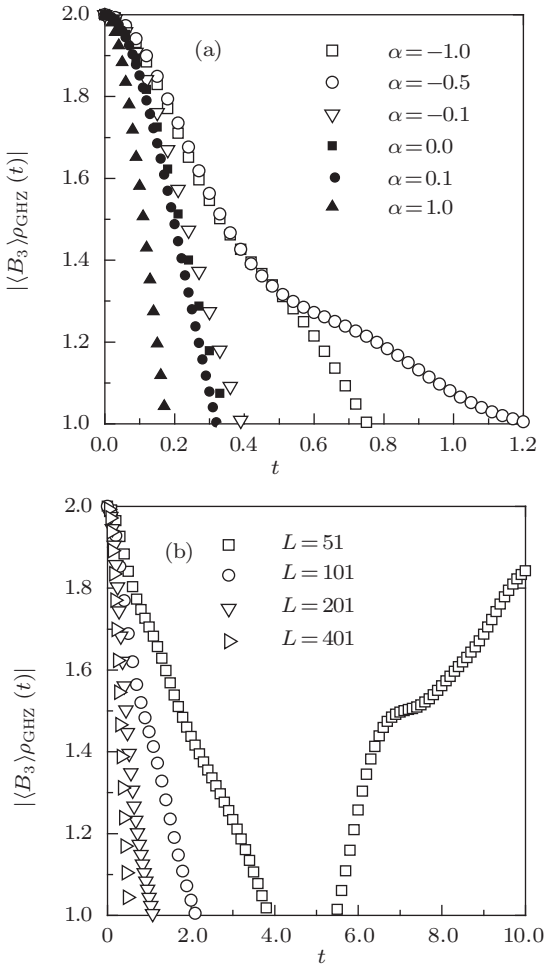


Fig. 1. The $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|$ versus time t when the initial state is a standard GHZ state, where $3g = 0.05$, $\gamma = 1.0$, $\lambda = 1.0$, and $\theta_{BC\delta} = \pi$. Panel (a) for $L = 801$ and (b) for $\alpha = 0.1$.

Firstly, we consider the effect of the three-site interaction α and the total site of the spin chain L on the time evolution of Bell-nonlocality violations of the three qubits interacting with the environment in the weak-coupling region ($g \ll 1$). In Fig. 1(a), $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|$ is plotted as a function of time t for different values of α, where the parameters $L = 801$, $3g = 0.05$, $\gamma = 1.0$, $\theta_{BC\delta} = \pi$. It is clear that BNSD occurs for definite time. It can be observed that the Bell-nonlocality $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|$ decays more sharply with the increasing intensity in the interval $\alpha > 0$. However, the plot also indicates that the effects of the three-site interaction on the decay of Bell-nonlocality is not remarkable in the region $\alpha > 0$. In the region $\alpha < 0$, it is found that the decay of the Bell-nonlocality can be delayed remarkably with the increasing intensity along the negative direction. To check this phenomenon, we carry out an analysis similar to that in Ref. [26]. In a short time, when $\lambda \to 1$, one has $|F_{\mu\nu}(t)| = \mathrm{e}^{-(\tau_1+\tau_2+\tau_3)t^2}$, where τ_1, τ_2, and τ_3 are

equal to $8E(K)\gamma^2 g^2/(\lambda-1)^2$, $32F(K)\gamma^2 g\alpha/(\lambda-1)^2$ and $32G(K)\gamma^2\alpha^2/(\lambda-1)^2$, respectively; $E(k)$, $F(k)$ and $G(k)$ are equal to $4\pi^2\sum(k/L)^2$, $8\pi^3\sum(k/L)^3$ and $16\pi^4\sum(k/L)^4$, respectively. From the above approximate analysis we may expect that when the parameter λ is adjusted to the point$\lambda=1$, the Bell inequality $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|$ will exponentially decay in the second power of time. It is easy to find that the Bell inequality $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|$ is determined by τ_1, τ_2, and τ_3, while we can also find that the expression $\tau_1+\tau_2+\tau_3$ is a second-order polynomial against variable α. In Fig. 1(b), we display the effect of the degrees of freedom of an environment on Bell nonlocality. From the figure, we can see that the expectation of the operator decays exponentially to zero with the increase of time. Thus, BNSD can always take place at some definite timescale $\tau_{\rm BNSD}$; namely, there exists a finite time after which we have $|\langle B_3\rangle_{\rho_{\rm GHZ}}(\tau_{\rm BNSD})|\leq 1$. With the increase of the size L, one can find the decrease of the timescale. Furthermore, one can see that there are some revivals of Bell nonlocality in Fig. 1(b). With regard to the revivals, the numerical results imply that the larger the size is, the smaller the revival of Bell nonlocality is. That is, the oscillation of Bell nonlocality is suppressed by enlarging the size of the environmental spin chain in the weak-coupling region.

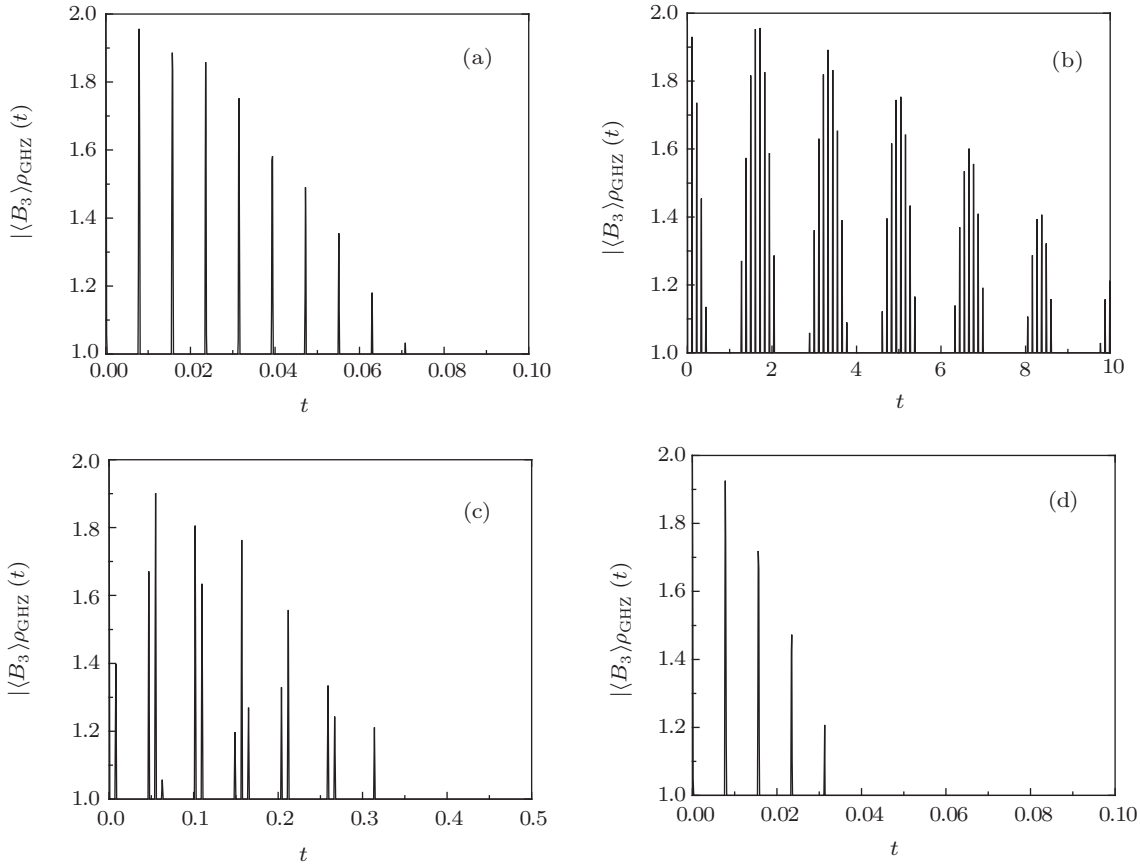


Fig. 2. The $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|$ versus time t with $\alpha=-0.5$ (a), $\alpha=0.0$ (b), $\alpha=0.1$ (c), and $\alpha=1.0$ (d), respectively, when the initial state is a standard GHZ state, where $3g=100$, $\gamma=1.0$, $\lambda=1.0$, and $\theta_{BC\delta}=\pi$.

We now turn to investigate the effect of this three-site interaction on the time evolution of the Bell-nonlocality violations of the three qubits interacting with the environment in the strong-coupling region ($g\gg 1$). Figure 2 is a plot of $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|$ as a function of time t when $3g=100$, where parameters $L=801$, $\gamma=1.0$, $\lambda=1.0$, $\theta_{BC\delta}=\pi$. It is seen that the decay is characterized by an oscillatory Gaussian envelope and the width of the Gaussian envelope is very sensitive to the three-site interaction α. To explain these features, we use a similar tactic to that employed by Ref. [21]. When ($g\gg 1$), we obtain the following from the angle of the Bogoliubov transformation: $\theta_k^{\lambda_1}\approx 0$ and $\theta_k^{\lambda_8}\approx\pi$. Consequently, we can obtain $|F_{18}(t)|\approx\prod|\cos^2\Theta_k^{\lambda_1}{\rm e}^{{\rm i}\Lambda_k t}+\sin^2\Theta_k^{\lambda_8}{\rm e}^{-{\rm i}\Lambda_k t}|$, where $\Lambda_k=\Lambda_k^{\lambda_1}+\Lambda_k^{\lambda_8}$. This expression is similar to that given in Ref. [26]. We can follow the mathematical procedure and derive the approximate expression for $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|\approx 2\exp(-s_L^2 t/2)|\cos(\Lambda t)|^{(L-1)/2}$, where Λ is the mean value of Λ_k and approximately equals

$4g+\gamma^2/g$; $s_L^2=\sum\sin^2(2\Theta_k^{\lambda_1})\delta_k^2$, where the quantity δ_k describes the deviation of Λ_k from its mean values and is approximately equal to $-(\gamma^2/g)\cos(4\pi k/L)+4\alpha\sin(4\pi k/L)$. This is the reason why the three-site interaction can enhance the decay of $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|$ dramatically. From the above approximate analysis, one can also find that the evolvement of $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|$ shows an oscillating and decaying exponential function, as shown in Fig. 2. It is clear that BNSD occurs for definite times and then the nonlocality is revived. However, the nonlocality will eventually not revive when the time comes to a certain value.

(ii) Assume that the initial state of the system is a three-qubit W state,

$$|\phi(0)\rangle_{ABC}=c_2|001\rangle+c_3|010\rangle+c_5|100\rangle. \tag{24}$$

From Eq. (11), we can obtain: $M_{\mu\nu}(t)=M_{\mu\nu}(0)$. Thus, the evolution of the W state can be expressed as

$$\rho_W(t)=\sum\nolimits_{\mu\nu}M_{\mu\nu}(t)=\sum\nolimits_{\mu\nu}M_{\mu\nu}(0)=\rho_W(0). \tag{25}$$

So the generalized W class states do not evolve under the spin environment with three-site interaction. In this sense, the W state is called a decoherence-free quantum state and can be used to design noiseless quantum code.[33−35]

4. Dynamical entanglement of three qubits

Now we consider the time evolution of the entanglement of three-qubit states. Due to its good operational and calculation properties, negativity has been used to measure the multipartite entanglement of quantum states with high dimensions. For a given quantum state with density matrix ρ, the negativity of ρ is defined as[36]

$$N(\rho)=\frac{||\rho^{{\rm T}_i}||-1}{2}, \tag{26}$$

where $||\rho^{{\rm T}_i}||$ is the sum of the absolute values of the eigenvalues of $\rho^{{\rm T}_i}$, and $\rho^{{\rm T}_i}$ denotes the partial transposition of density matrix ρ with respect to party i. If $N(\rho)>0$, then the two-subsystem state is entangled. We know that negativity can be applied to a multipartite quantum state. Here, we employ negativity to measure the tripartite quantum state. For example, a tripartite quantum state ρ_{ABC} can be split into three bipartitions $AB-C$, $BC-A$, and $AC-B$. Every bipartition gives a negativity, so there are three negativities N_{AB-C}, N_{BC-A}, N_{AC-B} to measure the quantum correlation between the one group with two parties and the other group with one party. The residual entanglement of a reduced density matrix will be considered too, which can be obtained by tracing one party of the density matrix ρ_{ABC} to ρ_{AB}^r, ρ_{BC}^r or ρ_{CA}^r, and we can analyse the residual entanglement with negativities N_{A-B}, N_{B-C}, and N_{C-A}, respectively. Here, we consider the dynamical entanglement of the GHZ state under an XY environment with three-site interaction as a case where the W state does not perceive the presence of the environment. Here, we employ negativity to analyse the dynamical entanglement of the standard GHZ state. With calculation, we obtain the time evolution of the entanglement of the GHZ state,

$$N_{AB-C}(t)=\frac{1}{2}|F_{18}(t)| \tag{27}$$

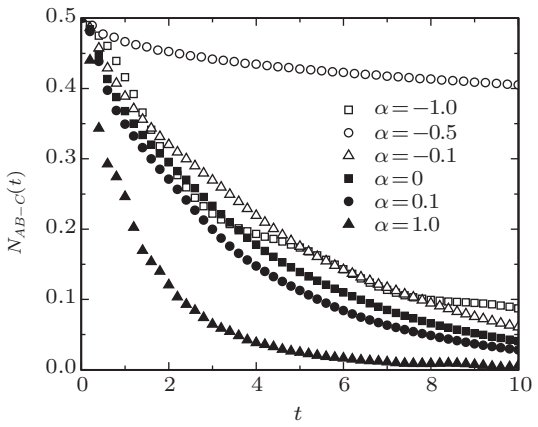


Fig. 3. Negativity of the GHZ state as a function of time t and α in the weak-coupling region, where $3g=0.05$, $\lambda=1.0$, $\gamma=1.0$, and $L=801$.

For the symmetry of the GHZ state, the other two negativities of N_{BC-A} and N_{CA-B} take the same expression as N_{AB-C}. Tracing one party of the GHZ state, we find that the reduced density matrix is separable, so the residual entanglements of N_{A-B}, N_{B-C}, and N_{C-A} are zero. In Fig. 3, we plot $N_{AB-C}(t)$ as a function of time t for different values of α. In a short time, it is seen that the entanglement decays more sharply with increasing the intensity of three-site interaction in the interval $\alpha\geq 0$. However, the decay of the entanglement can be delayed remarkably by increasing the intensity of three-site interaction along the negative direction. With its expression, the negativity of the GHZ state takes a similar expression to

that of the Bell violation, so the effect of the other parameters on the negativity is similar to the case of Bell violations. The only difference is that the Bell violation can be vanished in a finite time.

We notice that the dynamics of multipartite entanglement in other environments has also been investigated. In Ref. [37], the authors studied the entanglement dynamics of three qubits in contact with independent Markovian or non-Markovian reservoirs. It was found that the GHZ state can exhibit ESD under certain conditions. In addition, it was shown that the non-Markovian reservoirs can recover the entanglement at instantaneous points or after a finite interval of entanglement disappearance, which is called entanglement sudden birth. The difference is that the Bell violation can be vanished in a finite time. Finally, the difference between the dynamics of entanglement in this paper and the Markovian or non-Markovian environment is that the entanglement does not exhibit sudden death.

5. Conclusion

In summary, we have studied the nonlocal behaviour of a three-qubit system homogeneously coupled to a general XY spin-chain with a three-site interaction environment by calculating the extent of violation of Bell inequality. It has been demonstrated that there exists the destruction of Bell-nonlocality behaviour due to the decoherence induced by the three-site interaction environment. In the weak-coupling region, the effects of the three-site interaction and the degrees of freedom of an environment on the Bell-nonlocality $|\langle B_3\rangle_{\rho_{\rm GHZ}}(t)|$ were also considered. In the strong-coupling region, we found that the decay of Bell-nonlocality is characterized by an oscillatory Gaussian envelope, and the width of the Gaussian envelope is very sensitive to the three-site interaction. The decoherence-free subspaces of our model have been identified. We calculated the entanglement evolution in our model and compare it with the results of the violation of Bell inequality. Finally, the difference between the dynamics of entanglement in our model and the Markovian or non-Markovian environment is also discussed along with the nonexhibition of ESD.

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