Spin current and its heat effect in a multichannel quantum wire with Rashba spin–orbit coupling

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Using the perturbation method, we theoretically study the spin current and its heat effect in a multichannel quantum wire with Rashba spin–orbit coupling. The heat generated by the spin current is calculated. With the increase of the width of the quantum wire, the spin current and the heat generated both exhibit period oscillations with equal amplitudes. When the quantum-channel number is doubled, the oscillation periods of the spin current and of the heat generated both decrease by a factor of 2. For the spin current $j_{s,xy}$, the amplitude increases with the decrease of the quantum channel; while the amplitude of the spin current $j_{s,yx}$ remains the same. Therefore we conclude that the effect of the quantum-channel number on the spin current $j_{s,xy}$ is greater than that on the spin current $j_{s,yx}$. The strength of the Rashba spin–orbit coupling is tunable by the gate voltage, and the gate voltage can be varied experimentally, which implies a new method of detecting the spin current. In addition, we can control the amplitude and the oscillation period of the spin current by controlling the number of the quantum channels. All these characteristics of the spin current will be very important for detecting and controlling the spin current, and especially for designing new spintronic devices in the future.

Keywords: multichannel quantum wire, Rashba spin–orbit coupling, spin current, heat produced by spin current

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1. Introduction

Spintronics, as an important subdiscipline of condensed matter physics, has aroused the broad interest of scientists, and has become a new research hotspot.[1−5] Recently, the spin current, the most important physical quantity in spintronics, has been extensively studied. Many interesting and fundamental phenomena, such as the spin Hall effect[6,7] and the spin precession in systems with spin–orbit coupling, have been discovered and are under further investigation.[8−12] In addition, the effect of spin–orbit coupling creates a way to manipulate spins by an electric field, which may have great potential applications in future spintronics devices.[13,14] The Rashba spin–orbit coupling term arises when the confining potential of the quantum structure lacks inversion symmetry. The Rashba spin–orbit coupling effect is found to be more pronounced in some semiconductor layers (e.g. InAs or InGaAs), and its strength can be controlled by applying a gate voltage.[15,16] More recently, the spin current in a quantum structure with Rashba spin–orbit coupling has been extensively studied, and a definition for the spin current has been proposed.[17−19] They found that in order to completely describe the spin transport, besides the spin current (or the linear spin current), one needs to introduce an angular spin current. The two spin currents describe the translational and the rotational motion (precession) of the spin, respectively.[20,21] The definitions of these spin currents were given and their physical properties were discussed. The spin currents in a parabolically confined semiconductor heterojunction quantum wire with Dresselhaus spin–orbit coupling were theoretically studied by Sheng *et al.*[22] They found that a persistent spin current exists along either the transverse or the longitudinal direction of the quantum wire, and the net linear spin current density has a maximum in the middle of the quantum wire, which implies a way to detect the spin current. They did not discuss the effect of increasing the quantum wire width on the spin current. More recently, the heat produced by the spin current has become a research focus. Considering a simple case without the spin flip process, Sun *et al.* gave a formula to calculate the heat produced by the spin currents, which laid the foundation for the further

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study of the heat produced by the spin currents.[17,21] However, we notice that systematic research on the heat produced by spin currents is still lacking so far.

In this paper, we study the spin current and its heat effect in a confined semiconductor multichannel quantum wire with Rashba spin–orbit coupling. We discuss the variations of the spin current and of its heat effect with the quantum wire width and the quantum-channel number. We can control the amplitude and the oscillation period of the spin current by controlling the number of the quantum channels. These characteristics of the spin current and of its heat effect may offer a way to detect and control the spin current.

The rest of the paper is organized as follows. In Section 2, based on the perturbation theory, we first introduce the Hamiltonian and the wave function of a single electron in a quantum wire with Rashba spin–orbit coupling. The formulas and the calculation method are then described. In Section 3, the numerical results and our discussion are presented. Finally, a brief summary is given in Section 4.

2. Model and method

In our system, a quantum wire with Rasha spin–orbit coupling is connected to two electrode leads. The width of the quantum wire is d. The single electron Hamiltonian for the quantum wire subjected to Rashba spin–orbit coupling can be written as

$$H = \frac{-\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2}\right) + V(x) + H_{\rm R}, \tag{1}$$

where m^* is the effective electron mass. The transverse confining potential is

$$V(x) = \begin{cases} 0, & -d/2 < x < d/2, \\ \infty, & x < -d/2,\ x > d/2. \end{cases}$$

The Rashba spin–orbit coupling term in Hamiltonian $H_{\rm R}$ can be described as

$$H_{\rm R} = {\rm i}\,\alpha_{\rm R}\left(\sigma_y\frac{\partial}{\partial x} - \sigma_x\frac{\partial}{\partial y}\right), \tag{2}$$

where σ_x and σ_y are the x- and the y-components of the Pauli matrices, respectively, and $\alpha_{\rm R}$ represents the Rashba spin–orbit coupling strength.[23,24]

The electron wave functions for the quantum wire with the Rashba spin–orbit coupling can be decomposed into two parts,[21]

$$\psi_{\uparrow\downarrow}(x,y) = \frac{{\rm e}^{{\rm i}\,k_y y}\varphi_{\uparrow\downarrow}(x)}{\sqrt{L}}, \tag{3}$$

where L is the length of the quantum wire. We can solve the transverse eigenfunctions by using the perturbation method as

$$\begin{aligned}\varphi_{\uparrow\downarrow}(x) =& \sqrt{\frac{d}{2}}\sin\left[\frac{n\pi}{d}\left(x+\frac{d}{2}\right)\right] \\ &\pm\sum_{m\neq n}F_m\sin\left[\frac{m\pi}{d}\left(x+\frac{d}{2}\right)\right],\end{aligned} \tag{4}$$

where

$$\begin{aligned}F_m =& \frac{4m^*n\alpha_{\rm R}}{\hbar^2\pi(n^2-m^2)}\int_{-d/2}^{d/2}\sin\left[\frac{m\pi}{d}\left(x+\frac{d}{2}\right)\right] \\ &\times\cos\left[\frac{n\pi}{d}\left(x+\frac{d}{2}\right)\right]{\rm d}x, \\ &(m,n=0,1,2,\ldots).\end{aligned} \tag{5}$$

The associated energy eigenvalues of the n-th mode for the spin-up and the spin-down electrons in the quantum wire are[25,26]

$$E_{\uparrow\downarrow} = \frac{\hbar^2k_{y\uparrow\downarrow}^2}{2m^*} + \frac{\hbar^2n^2\pi^2}{2m^*d^2} \pm \alpha_{\rm R}k_{y\uparrow\downarrow}. \tag{6}$$

Therefore, we can deduce the following relations:

$$k_{y\uparrow}^{\pm}(E_)) = \frac{1}{\hbar^2}\left[-m^*\alpha_{\rm R} \pm m^*\sqrt{\alpha_{\rm R}^2 + \left(\frac{2\hbar^2}{m^*}\right)\left(E_- \frac{\hbar^2n^2\pi^2}{2m^*d^2}\right)}\right], \tag{7}$$

$$k_{y\downarrow}^{\pm}(E_)) = \frac{1}{\hbar^2}\left[m^*\alpha_{\rm R} \pm m^*\sqrt{\alpha_{\rm R}^2 + \left(\frac{2\hbar^2}{m^*}\right)\left(E_- \frac{\hbar^2n^2\pi^2}{2m^*d^2}\right)}\right]. \tag{8}$$

In order to calculate the spin current density, we use the definition proposed by Sun *et al.*[21]

$$\boldsymbol{j}_s = {\rm Re}[\varphi^\dagger\boldsymbol{v}\boldsymbol{s}\varphi], \tag{9}$$

$$\boldsymbol{j}_\omega = {\rm Re}[\varphi^\dagger\boldsymbol{\omega}\times\boldsymbol{s}\varphi], \tag{10}$$

where $\boldsymbol{v} = \boldsymbol{p}/m + (\alpha_{\rm R}/\hbar)(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{\sigma}})$, and $\boldsymbol{\omega} = (2/\hbar^2)(\boldsymbol{p}\times\hat{\boldsymbol{z}})$ are the linear and the angular velocity operators of

an electron with spin $\boldsymbol{s}$ and unit vector $\hat{\boldsymbol{z}}$, respectively. Substituting Eqs. (3) and (4) into Eqs. (9) and (10), the spin current density can be obtained. There are only three non-zero elements in the linear spin current density, i.e.,

$$\begin{aligned} j_{s,xy} &= -\frac{\alpha_{\rm R}}{2L}(\varphi_\uparrow^*\varphi_\uparrow + \varphi_\downarrow^*\varphi_\downarrow) \\ &\quad + \frac{\hbar^2}{2m^*L}(\varphi_\downarrow^*\varphi_\uparrow' - \varphi_\uparrow^*\varphi_\downarrow'), \qquad (11) \\ j_{s,yx} &= \frac{\alpha_{\rm R}}{2L}(\varphi_\uparrow^*\varphi_\uparrow + \varphi_\downarrow^*\varphi_\downarrow) \\ &\quad + \frac{\hbar^2 k_y}{2m^*L}(\varphi_\uparrow^*\varphi_\downarrow - \varphi_\downarrow^*\varphi_\uparrow), \qquad (12) \\ j_{s,yz} &= \frac{\hbar^2 k_y}{2m^*L}(\varphi_\uparrow^*\varphi_\uparrow - \varphi_\downarrow^*\varphi_\downarrow). \qquad (13) \end{aligned}$$

There is only one non-zero element in the angular spin current density,

$$j_{\omega,y} = \frac{\alpha_{\rm R} k_y}{L}(\varphi_\downarrow^*\varphi_\downarrow - \varphi_\uparrow^*\varphi_\uparrow). \qquad (14)$$

The linear spin current density $\boldsymbol{j}_s$ is a tensor, and its non-zero element, e.g. $\boldsymbol{j}_{s,xy}$, represents an electron moving along the x direction with its spin pointing in the y direction. The angular spin current density $\boldsymbol{j}_\omega$ represents the spin current with the electron spin precessing in the y direction. From Eqs. (7) and (8), we can see that there are four different occupied states for the n-th mode, that is, $k_{y\uparrow}^+$, $k_{y\uparrow}^-$, $k_{y\downarrow}^+$, and $k_{y\downarrow}^-$, which are related to the spin-down and the spin-up states with the electron propagating along the $+y$ and the $-y$ directions, respectively. The states obey the relation $k_{y\uparrow}^+ + k_{y\uparrow}^- + k_{y\downarrow}^+ + k_{y\downarrow}^- = 0$. In equilibrium, the contributions of all occupied states are the same. Therefore, the elements of the linear and the angular spin current densities, including k_y, can be obtained as[21]

$$\begin{aligned} j_{s,xy}^n &= \frac{1}{4}(j_{s,xy\downarrow+}^n + j_{s,xy\uparrow+}^n + j_{s,xy\downarrow-}^n + j_{s,xy\uparrow-}^n) \\ &= -\frac{\alpha_{\rm R}}{2L}(\varphi_\uparrow^*\varphi_\uparrow + \varphi_\downarrow^*\varphi_\downarrow) \\ &\quad + \frac{\hbar^2}{2m^*L}(\varphi_\downarrow^*\varphi_\uparrow' - \varphi_\uparrow^*\varphi_\downarrow'), \qquad (15) \\ j_{s,yx}^n &= \frac{1}{4}(j_{s,yx\downarrow+}^n + j_{s,yx\uparrow+}^n + j_{s,yx\downarrow-}^n + j_{s,yx\uparrow-}^n) \\ &= \frac{\alpha_{\rm R}}{2L}(\varphi_\uparrow^*\varphi_\uparrow + \varphi_\downarrow^*\varphi_\downarrow), \qquad (16) \\ j_{s,yz}^n &= \frac{1}{4}(j_{s,yz\downarrow+}^n + j_{s,yz\uparrow+}^n + j_{s,yz\downarrow-}^n + j_{s,yz\uparrow-}^n) \\ &= 0, \qquad (17) \\ j_{\omega,y}^n &= \frac{1}{4}(j_{\omega,y\downarrow+}^n + j_{\omega,y\uparrow+}^n + j_{\omega,y\downarrow-}^n + j_{\omega,y\uparrow-}^n) \\ &= 0. \qquad (18) \end{aligned}$$

The two nonzero components in the spin current density for the transverse n-th mode are $j_{s,xy}^n$ and $j_{s,yx}^n$. By the summing spin current density $j_{s,ij}^n$ $(i,j = x,y,z)$ from the 1-st to the n-th modes, we can obtain the total spin current density.

In order to further understand the physical properties of the spin current, we consider another physical effect of the spin current: the heat produced by it. We consider a uniform isotropic conductor with a linear spin current $\boldsymbol{j}_s$ and a charge current $\boldsymbol{j}_{\rm e}$, and consider a simple case without the spin flip process, so that $\boldsymbol{j}_\omega = 0$, then the heat Q produced in unit volume and in unit time by the currents will be[21]

$$Q = \rho\left(\sum_{ij} j_{s,ij}^2 + \sum_i j_{{\rm e},i}^2\right) = \rho(\boldsymbol{j}_s^2 + \boldsymbol{j}_{\rm e}^2), \qquad (19)$$

where ρ is the semiconductor resistivity, the term $\rho\boldsymbol{j}_{\rm e}^2$ gives the heat produced by the charge current and the term $\rho\boldsymbol{j}_s^2$ gives the heat produced by the linear spin current. So the heat produced can be expressed by $\boldsymbol{j}_s^2$ (for the case of $\boldsymbol{j}_\omega = 0$). Note that the heat Q produced depends on $\boldsymbol{j}_s^2$, and the calculating formula will be

$$Q = \rho j_{s,ij}^2. \qquad (20)$$

3. Calculation results and analysis

Now we calculate and analyse the spin current and its heat effect in a semiconductor multichannel quantum wire with Rashba spin–orbit coupling. The electron effective mass takes the value of the InGaAs quantum wire $m^* = 0.042m_{\rm e}$, and the strength of the Rashba spin–orbit coupling takes the value of $\alpha_{\rm R} = 1.0 \times 10^{-11}$ eV/m. For the hard-wall confining potential, we consider two different cases with quantum-channel numbers three and six. The width of each quantum wire is 300 nm.[27–29]

Figure 1 shows the spin current density $j_{s,xy}$ as a function of the quantum wire width. From the figure, we can see that with the width of the quantum wire increasing, the spin current exhibits periodic oscillations with equal amplitudes. When the quantum-channel number is doubled, the oscillation period decreases by a factor of 2 and the amplitude of the spin current decreases with the increase of the quantum-channel number. This is because as the number of the mode increases, more quantum channels are occupied, so the oscillation becomes more intense and the amplitude becomes smaller.

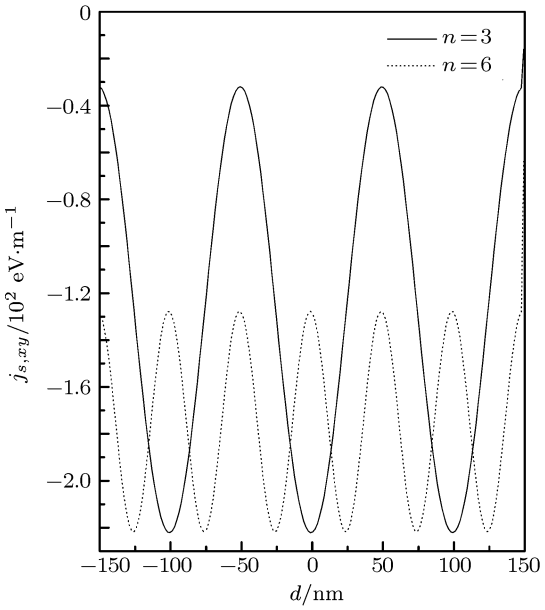


Fig. 1. The spin current $j_{s,xy}$ as a function of the quantum wire width. The numbers of the quantum channel are $n = 3$ and 6, with the energy eigenvalue for the electron $E = 5.9 \times 10^{-3}$ eV, the strength of the Rashba spin–orbit coupling $\alpha_{\rm R} = 1.0 \times 10^{-11}$ eV/m, and the length of the quantum wire $L = 300$ nm.

Figure 2 shows the spin current density $j_{s,yx}$ as a function of the quantum wire width. By comparison with Fig. 1, we can see that the spin current densities present periodic oscillations with equal amplitudes. The oscillation period decreases with the increase of the quantum-channel number. The variations of the spin current density $j_{s,yx}$ have the same tendency as that of the spin current density $j_{s,xy}$. The value of the spin current density $j_{s,xy}$ is negative, while the value of the spin current density $j_{s,yx}$ is positive. When the number of the quantum channel is 6, the amplitude of the spin current density $j_{s,yx}$ is larger than that of the spin current density $j_{s,xy}$. Although both spin current densities have two opposite directions, their variation trends are not completely symmetrical. With the increase of the strength of the Rashba spin–orbit coupling, the spin current density greatly increases, which has been theoretically proved by Sun *et al.*[21] They also found that the linear or the angular spin current can generate a linear or an angular electric field, which is measurable by existing technology, and the electric field is proportional to the magnitudes of the two spin currents. Therefore, in our case, for a Rashba spin–orbit coupling quantum wire, the net linear spin current should also induce a linear electric field. Furthermore, it will exhibit periodic oscillations in the quantum wire. These characteristics of the induced electric field, which is measurable, may offer a way to detect the spin current.[21]

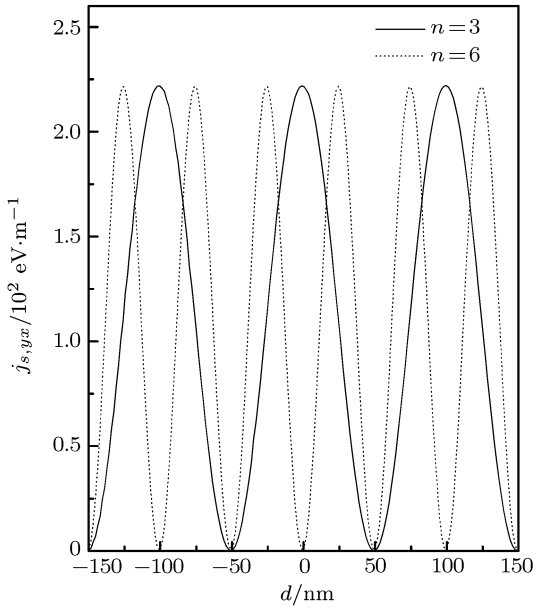


Fig. 2. The spin current $j_{s,yx}$ as a function of the quantum wire width. The numbers of the quantum channel are $n = 3$ and 6, with the energy eigenvalue for the electron $E = 5.9 \times 10^{-3}$ eV, the strength of the Rashba spin–orbit coupling $\alpha_{\rm R} = 1.0 \times 10^{-11}$ eV/m, and the length of the quantum wire $L = 300$ nm.

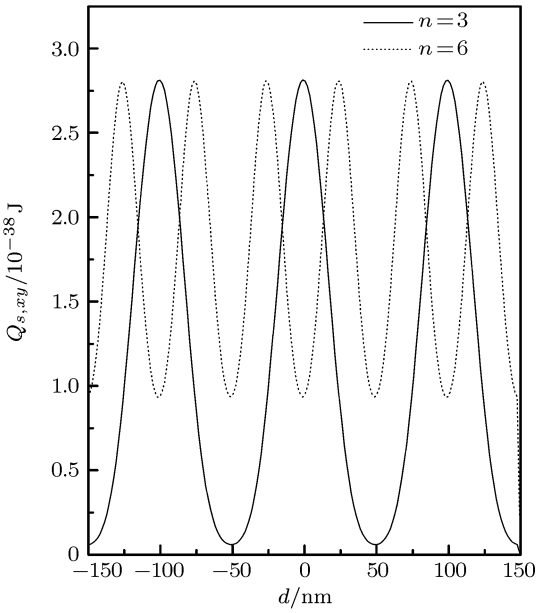


Fig. 3. The heat produced by the spin current $j_{s,xy}$ as a function of quantum wire width. The numbers of the quantum channel are $n = 3$ and 6, with the energy eigenvalue for the electron $E = 5.9 \times 10^{-3}$ eV, the strength of the Rashba spin–orbit coupling $\alpha_{\rm R} = 1.0 \times 10^{-11}$ eV/m, and the length of the quantum wire $L = 300$ nm.

In order to further understand the property of the spin current density, we calculate the heat produced by the spin current. The variation trends of the heat are shown in Fig. 3.

Figure 3 shows the heat produced by the spin current density $j_{s,xy}$ as a function of the quantum wire width. From the figure, we can see that with the increase of the width of the quantum wire, the heat exhibits periodic oscillations with equal amplitudes. When the quantum-channel number is doubled, the oscillation period decreases by a factor of 2 and the amplitude of the spin current decreases with the increase of the quantum-channel number, which is basically coincident with the variation trend of the spin current $j_{s,xy}$. The amplitudes of the heat in the mode for $n = 3$ are greater than that for $n = 6$.

Figure 4 shows the heat produced by the spin current density $j_{s,yx}$ as a function of the quantum wire

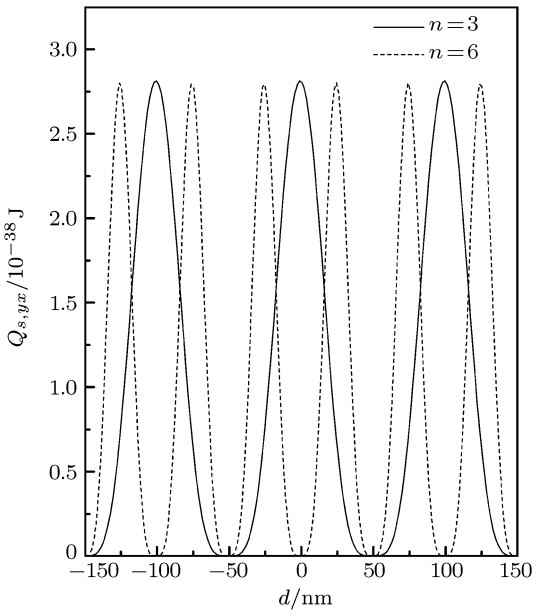


Fig. 4. The heat produced by the spin current $j_{s,yx}$ as a function of quantum wire width. The numbers of the quantum channel are $n = 3$ and 6, with the energy eigenvalue for the electron $E = 5.9 \times 10^{-3}$ eV, the strength of the Rashba spin–orbit coupling $\alpha_{\rm R} = 1.0 \times 10^{-11}$ eV/m, and the length of the quantum wire $L = 300$ nm.

width. Comparing Fig. 4 with Fig. 3, when the quantum-channel number is doubled, the oscillation period decreases by a factor of 2. We can see that the increase in the quantum-channel number leads to a decrease of the amplitude of the heat produced by the spin current density $j_{s,xy}$, and there is no obvious effect on the amplitude of the heat produced by the spin current density $j_{s,yx}$. Therefore we conclude that the effect of the quantum-channel number on the spin current density $j_{s,xy}$ is greater than that on the spin current density $j_{s,yx}$.

4. Conclusion

Using the perturbation method, we study theoretically the spin current density and its heat effect in a multichannel quantum wire with Rashba spin–orbit coupling. The heat generated by the spin current is calculated. By theoretical calculation, we find that with the increase of the width of the quantum wire, the spin current and the heat produced both exhibit periodic oscillations with equal amplitudes. When the quantum-channel number is doubled, the oscillation periods decrease by a factor of 2. We conclude that the effect of the quantum-channel number on the spin current density $j_{s,xy}$ is greater than that on the spin current density $j_{s,yx}$. For the spin current $j_{s,xy}$, we can control the amplitude of the spin current by controlling the number of the quantum channel. The strength of the Rashba spin–orbit coupling is tunable by the gate voltage, and the gate voltage can be varied experimentally, which may offer a new method to detect spin currents. All of these spin current characteristics will be very important for detecting and controlling spin current, and especially for designing new spintronic devices in the future.

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