Interface of quasicrystal and crystal*

Fan Tian-You(范天佑)[a)†], Xie Ling-Yun(解凌云)[a)],
Fan Lei(范 蕾)[b)], and Wang Qing-Zhao(王清昭)[c)]

[a)] *School of Science, Beijing Institute of Technology, Beijing* 100081, *China*
[b)] *School of Life Science, Beijing Institute of Technology, Beijing* 100081, *China*
[c)] *Department of Mathematics and Statistics, University of Minnesota Duluth, Duluth, MN* 55812, *USA*

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A model on the coexisting phase of quasicrystal-crystal is proposed, with which we concretely investigate the interface effects for coexisting phases of one-dimensional orthorhombic quasicrystal-isotropic crystal and three-dimensional icosahedral quasicrystal-cubic crystal. The phason strain fields which play an important role in some processes are determined. Some factors affecting the strain fields, e.g., the material constants of phonon, phason, phonon–phason coupling of the quasicrystal and the elastic modulus and the size of the crystal are also explored.

Keywords: icosahedral quasicrystal, interface defect, phase transition, Fourier transform

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1. Introduction

Quasicrystal is a new solid phase structure as well as a novel material and, like other solid phases and materials, it has defects such as dislocations and cracks. On the dislocations, Edagawa[1] made a review, and on the cracks Fan and Mai[2] gave a review, separately. Apart from dislocations and cracks, the interface also is a kind of defect in the solid phase and is particularly important. All quasicrystals observed to date are alloys, but in the alloys the quasicrystalline phases often coexist with the relevant crystalline phases.[3−8] In addition, in Refs. [9]–[12], this problem was studied experimentally and researchers have considered the long-period structure in magnesium alloys and the precipitation of quasicrystalline phase to be a very important part of this research. In the case where structural transitions appear,[4,5] it was pointed out that the phase transitions are mainly induced by the phason strain field. Some further discussion on this topic was given in Refs. [13]–[15]. Furthermore, the continuous phase transition between icosahedral quasicrystal and body-centre cubic crystal was observed,[16,17] which is induced by the phason strain field. This implies the importance of the phason strain fields of quasicrystals in the coexisting phase between quasicrystals and crystals. Because the crystal–quasicrystal phase transition is a continuous phase transition, there must be an interface between quasicrystal and crystal in the process, and the effect of the interface is significant. There are many research results on dislocations and cracks in quasicrystals,[18−25] while the interface has not been well studied. We propose a simple model of the coexisting phase, and devote it to analysing the effect of the interface. The phason strain fields of the quasicrystals in the coexisting phase are evaluated for some coexisting systems, which may be beneficial for studies of crystal–quasicrystal phase transition.

2. Physical model

The above description shows that the coexisting phase and phase transition between quasicrystals and crystals is a very complicated phenomenon. However, quantitative analysis is only available for some simple physical models. Due to this, we consider the anti-plane deformation state alternatively. Assume that a quasicrystal lies in the upper half-plane (i.e., $y > 0$), which is characterized by the material constants including phonon, phason and phonon–phason coupling elastic constants C_{ij}, K_i and R_i, and the crystal coexisting with the quasicrystal lies in the lower half-plane with finite thickness h(i.e., it is located in the region $-h < y < 0$) characterized by phonon elastic constants $C_{ij}^{(c)}$, then the plane $y = 0$ is the interface between the quasicrystal and crystal (see Fig. 1).

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†Corresponding author. E-mail: tyfan2006@yahoo.com.cn
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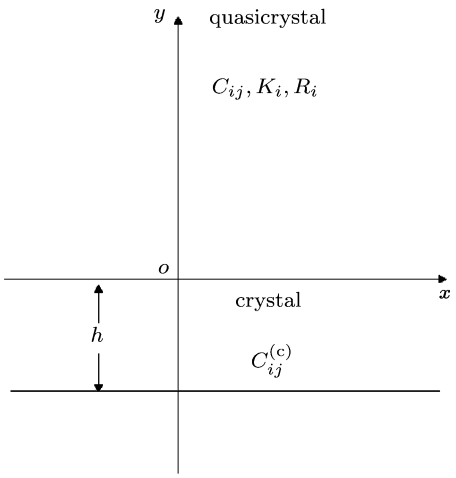


Fig. 1. Physical model of coexisting quasicrystal and crystal.

In what follows, we will study a one-dimensional orthorhombic quasicrystal coexisting with an isotropic crystal, and three-dimensional icosahedral quasicrystals coexisting with a body-centre cubic crystal, respectively. The strain analysis will be given by the theories of elasticity of quasicrystals and the Fourier transformation.

3. Formulation and mathematical solution

According to the Landau[26] phenomenological theory on the elementary excitation of condensed matter, the density function of quasicrystals can be expressed as

$$\rho(\boldsymbol{r}) = \sum \rho_{\boldsymbol{G}} \exp(\mathrm{i}\,\boldsymbol{G}\cdot\boldsymbol{r}),$$
$$\rho_{\boldsymbol{G}} = |\rho_{\boldsymbol{G}}| \exp(\mathrm{i}\,\varphi_{\boldsymbol{G}}). \tag{1}$$

Bak[27] suggested that

$$\rho_{\boldsymbol{G}} = \boldsymbol{G}\cdot\boldsymbol{u} + \boldsymbol{G}'\cdot\boldsymbol{w}, \tag{2}$$

where $\boldsymbol{G}$ is the reciprocal vector in the physical space, and $\boldsymbol{G}'$ the conjugate one in the perpendicular space, $\boldsymbol{u}$ and $\boldsymbol{w}$ are the phonon and phason vectors, respectively. Therefore, there are two strain tensors[2]

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad w_{ij} = \frac{\partial w_i}{\partial x_j}, \tag{3}$$

where $i, j = 1, 2$, $x_1 = x$, $x_2 = y$, $\partial/\partial x_3 = \partial/\partial z = 0$ for plane and anti-plane deformation, ε_{ij} is the phonon strain tensor similar to that in the crystals, and w_{ij} is the phason strain tensor, an unusual field that appears only in this new solid phase. The appearance of phason displacement vectors and phason strain tensors results in the essential difference between quasicrystals and crystals. In the following subsections we will discuss two different kinds of coexisting phases between quasicrystals and crystals.

3.1. One-dimensional orthorhombic quasicrystal-isotropic crystal coexistence

We will focus on the interface between crystals and quasicrystals when they coexist. As mentioned above, in quasicrystals there are two elementary excitations, i.e., phonons $\boldsymbol{u}(u_x, u_y, u_z)$ and phasons $\boldsymbol{w}(w_x, w_y, w_z)$. For one-dimensional orthorhombic quasicrystals, if we consider the anti-plane elasticity problems and assume the z-axis to be the quasiperiodic direction, then the non-zero displacement components are only u_z and w_z, and all field variables are independent of coordinate z, i.e.,

$$\frac{\partial}{\partial x_3} = \frac{\partial}{\partial z} = 0. \tag{4}$$

The final governing equation of their deformation equilibrium is

$$\left(a_1\frac{\partial^4}{\partial x^4} + a_3\frac{\partial^4}{\partial x^2\partial y^2} + a_5\frac{\partial^4}{\partial y^4}\right)F = 0, \tag{5}$$

where $F(x, y)$ denotes the displacement potential and a_1, a_3, a_5 are the material constants composed of C_{ij}, K_i and R_i discussed by Liu *et al.*[28] We have assumed that the crystal coexisting with the quasicrystal lies in the lower half-plane with thickness h ($-h < y < 0$), then the plane $y = 0$ is the interface between the quasicrystal and crystal. For simplicity, suppose that the crystal is an isotropic material, characterized by elastic constants $C_{44}^{(c)} = \mu^{(c)}$ (see Fig. 1). At the interface, there are the following boundary conditions:

$$y = 0, \quad -\infty < x < \infty, \quad \sigma_{zy} = \tau f(x) + ku(x),$$
$$H_{zy} = 0, \tag{6}$$

where σ_{zy} and H_{zy} represent the phonon stress component and the phason stress component, τ is the applied constant shear stress, $f(x)$ denotes the distribution function of applied stress, $u(x) = u_z(x, 0)$ is the value of displacement component of phonon field at the interface, and k is a material constant taken as

$$k = \frac{\mu^{(c)}}{h}, \tag{7}$$

where $\mu^{(c)}$ and h are the shear modulus and thickness of the crystal, respectively. In addition, the outer boundaries are assumed to be stress-free.

The above formulas are for quasicrystal, while for crystal there are the following formulas:

$$\boldsymbol{\nabla}^2 u_z^{(c)} = 0, \tag{8}$$

$$\sigma_{zy}^{(c)} = \sigma_{yz}^{(c)} = 2\mu^{(c)}\varepsilon_{yz}^{(c)}, \quad \sigma_{zx}^{(c)} = \sigma_{xz}^{(c)} = 2\mu^{(c)}\varepsilon_{xz}^{(c)}, \tag{9}$$

because only the phonon field exists, and

$$\varepsilon_{ij}^{(c)} = \frac{1}{2}\left(\frac{\partial u_i^{(c)}}{\partial x_j} + \frac{\partial u_j^{(c)}}{\partial x_i}\right) \tag{10}$$

is similar to that of the first one of Eq. (3).

We here use Fourier analysis rather than the complex variable function method, which is different from the method in Ref. [28].

Applying the Fourier transform

$$\bar{F}(\xi, y) = \int_{-\infty}^{\infty} F(x, y)\, \mathrm{e}^{\mathrm{i}\,\xi x}\, \mathrm{d}x \tag{11}$$

to Eq. (5), we have

$$\left(a_5 \frac{\mathrm{d}^4}{\mathrm{d}y^4} + a_3\xi^2 \frac{\mathrm{d}^2}{\mathrm{d}y^2} + a_1\xi^4\right)\bar{F} = 0. \tag{12}$$

If the solution of Eq. (12) is given by

$$\bar{F}(\xi, y) = \mathrm{e}^{-\lambda|\xi|y},$$

where λ is a parameter. Substituting the former solution into Eq. (12) leads to

$$a_5\lambda^4 - a_3\lambda^2 + a_1 = 0, \tag{13}$$

with roots

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 = \pm\sqrt{\frac{2a_3 \pm \sqrt{a_3^2 - 4a_1a_5}}{2a_5}}. \tag{14}$$

Then

$$\bar{F}(\xi, y) = A\,\mathrm{e}^{-\lambda_1|\xi|y} + B\,\mathrm{e}^{-\lambda_2|\xi|y} + C\,\mathrm{e}^{-\lambda_3|\xi|y} + D\,\mathrm{e}^{-\lambda_4|\xi|y}.$$

By considering the condition of stress free at $y = \infty$, $C = D = 0$, there must hold

$$\bar{F}(\xi, y) = A\,\mathrm{e}^{-\lambda_1|\xi|y} + B\,\mathrm{e}^{-\lambda_2|\xi|y}, \tag{15}$$

where $A(\xi)$ and $B(\xi)$ are unknown functions to be determined.

According to Ref. [28], we have

$$\begin{aligned}
\sigma_{zy} &= (R_6C_{44} - R_5C_{55})\frac{\partial^3 F}{\partial x^2\partial y},\\
H_{zy} &= -\left(C_{55}\frac{\partial^2}{\partial x^2} + C_{44}\frac{\partial^2}{\partial y^2}\right)\left(K_1\frac{\partial}{\partial x} + K_2\frac{\partial}{\partial y}\right)F,\\
u_z &= \left(R_6\frac{\partial^2}{\partial x^2} + R_5\frac{\partial^2}{\partial y^2}\right)F.
\end{aligned} \tag{16}$$

The Fourier transforms of part of the stresses and displacements are

$$\begin{aligned}
\bar{\sigma}_{zy} &= -(R_6C_{44} - R_5C_{55})\xi^2\frac{\mathrm{d}\bar{F}}{\mathrm{d}y},\\
\bar{H}_{zy} &= -\,\mathrm{i}\,C_{55}K_1\,|\xi|\,\xi^2\bar{F} + C_{55}K_2\xi^2\frac{\mathrm{d}\bar{F}}{\mathrm{d}y}\\
&\quad + \mathrm{i}\,C_{44}K_1\,|\xi|\,\frac{\mathrm{d}^2\bar{F}}{\mathrm{d}y^2} - C_{44}K_2\frac{\mathrm{d}^3\bar{F}}{\mathrm{d}y^3},\\
\bar{u}_z &= \left(-\xi^2R_6 + R_5\frac{\mathrm{d}^2}{\mathrm{d}y^2}\right)\bar{F}.
\end{aligned} \tag{17}$$

Substituting Eq. (15) into the second equation of Eq. (17) and then into the second equation of Eq. (6), we have

$$B = \alpha A, \tag{18}$$

where

$$\begin{aligned}
\alpha &= \frac{-\lambda_1c_2 - \lambda_1^3c_4 + \mathrm{i}(c_1 + \lambda_1^2c_3)}{\lambda_2c_2 + \lambda_2^3c_4 + \mathrm{i}(-c_1 - \lambda_2^2c_3)},\\
c_1 &= C_{55}K_1, \quad c_2 = C_{55}K_2,\\
c_3 &= C_{44}K_1, \quad c_4 = C_{44}K_2.
\end{aligned} \tag{19}$$

Therefore, the number of unknown functions is reduced to only one. The values of the Fourier transforms of the stress and displacement component at the interface are

$$\begin{aligned}
\bar{\sigma}_{zy}(\xi, 0) &= A(\xi)\,|\xi|\,(\lambda_1\,\mathrm{e}^{-\lambda_1|\xi|y} + \alpha\lambda_2\,\mathrm{e}^{-\lambda_2|\xi|y}),\\
\bar{u}_z(\xi, 0) &= A(\xi)\xi^2[(-R_6 + \lambda_1^2R_5)\,\mathrm{e}^{-\lambda_1|\xi|y}\\
&\quad + \alpha(R_6 - \lambda_2^2R_5)\,\mathrm{e}^{-\lambda_2|\xi|y}],
\end{aligned} \tag{20}$$

and the Fourier transform of the first of the boundary conditions (6) is

$$\bar{\sigma}_{zy}(\xi, 0) = \tau\bar{f}(\xi) + k\bar{u}(\xi, 0). \tag{21}$$

From Eqs. (20) and (21), one determines the unknown function

$$A(\xi) = \frac{\tau\bar{f}(\xi)}{|\xi|\,(\lambda_1 + \alpha\lambda_2) - k\xi^2[-R_6 + \lambda_1^2R_5 + \alpha(R_6 - \lambda_1^2R_5)]}. \tag{22}$$

Thus, all stress and displacement components for phonon and phason fields can be evaluated

$$\begin{aligned}
\sigma_{zy} &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{\sigma}_{zy}\,\mathrm{e}^{-\mathrm{i}\,\xi x}\,\mathrm{d}\xi \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}A(\xi)\,|\xi|(\lambda_1\,\mathrm{e}^{-\lambda_1|\xi|y} \\
&\quad +\alpha\lambda_2\,\mathrm{e}^{-\lambda_2|\xi|y})\,\mathrm{e}^{-\mathrm{i}\,\xi x}\,\mathrm{d}\xi, \\
u_z &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{u}_z\,\mathrm{e}^{-\mathrm{i}\,\xi x}\,\mathrm{d}\xi \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}A(\xi)\xi^2[(-R_6+\lambda_1^2R_5)\,\mathrm{e}^{-\lambda_1|\xi|y} \\
&\quad +\alpha(R_6-\lambda_2^2R_5)\,\mathrm{e}^{-\lambda_2|\xi|y}]\,\mathrm{e}^{-\mathrm{i}\,\xi x}\,\mathrm{d}\xi, \\
w_z &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{w}_z\,\mathrm{e}^{-\mathrm{i}\,\xi x}\,\mathrm{d}\xi \\
&= -\frac{1}{2\pi}\int_{-\infty}^{\infty}A(\xi)\xi^2[(-C_{55}+\lambda_1^2C_{66})\,\mathrm{e}^{-\lambda_1|\xi|y} \\
&\quad +\alpha(C_{66}-\lambda_2^2C_{55})\,\mathrm{e}^{-\lambda_2|\xi|y}]\,\mathrm{e}^{-\mathrm{i}\,\xi x}\,\mathrm{d}\xi, \qquad (23)
\end{aligned}$$

and the phason strains are

$$\begin{aligned}
w_{zx} &= \frac{\partial w_z}{\partial x} \\
&= -\,\mathrm{i}\,\frac{1}{2\pi}\int_{-\infty}^{\infty}A(\xi)\xi^3[(-C_{55}+\lambda_1^2C_{66})\,\mathrm{e}^{-\lambda_1|\xi|y} \\
&\quad +\alpha(C_{66}-\lambda_2^2C_{55})\,\mathrm{e}^{-\lambda_2|\xi|y}]\,\mathrm{e}^{-\mathrm{i}\,\xi x}\,\mathrm{d}\xi, \\
w_{zy} &= \frac{\partial w_z}{\partial y} \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}A(\xi)\xi^3[\lambda_1\,|\xi|\,(-C_{55}+\lambda_1^2C_{66})\,\mathrm{e}^{-\lambda_1|\xi|y} \\
&\quad +\lambda_2\,|\xi|\,\alpha(C_{66}-\lambda_2^2C_{55})\,\mathrm{e}^{-\lambda_2|\xi|y}]\,\mathrm{e}^{-\mathrm{i}\,\xi x}\,\mathrm{d}\xi, \qquad (24)
\end{aligned}$$

which is an analytic solution. The effects of all parameters concerning symmetry and geometry of the quasicrystal and crystal and the behaviour of interface are explored.

For practical calculation, we can give concrete values for different $f(x)$ forms and different values of h, τ, C_{44}, C_{55}, C_{66}, K_1, K_2, R_5, R_6 and $\mu^{(\mathrm{c})}$.

3.2. Three-dimensional icosahedral quasicrystal-cubic crystal

The coexisting phase between three-dimensional icosahedral quasicrystal and the cubic crystal is more interesting, which is observed experimentally. For simplicity, we here consider also the anti-plane state. Assume that z-axis is the direction of five-fold symmetry of the icosahedral quasicrystal, then for this case the elastic deformation will be governed by the following equations:[2]

$$\boldsymbol{\nabla}^2u_z=0,\quad \boldsymbol{\nabla}^2w_z=0, \qquad (25)$$

and there are the following stress-strain relations:

$$\begin{aligned}
&\sigma_{zy}=\sigma_{yz}=2\mu\varepsilon_{yz}+Rw_{zy}, \\
&\sigma_{zx}=\sigma_{xz}=2\mu\varepsilon_{xz}+Rw_{zx}, \\
&H_{zy}=R\varepsilon_{yz}+(K_1-K_2)w_{zy}, \\
&H_{zx}=R\varepsilon_{xz}+(K_1-K_2)w_{zx} \qquad (26)
\end{aligned}$$

for quasicrystal, and

$$\begin{aligned}
&\boldsymbol{\nabla}^2u_z^{(\mathrm{c})}=0, \\
&\sigma_{zy}^{(\mathrm{c})}=\sigma_{yz}^{(\mathrm{c})}=2\mu^{(\mathrm{c})}\varepsilon_{yz}^{(\mathrm{c})},\quad \sigma_{zx}^{(\mathrm{c})}=\sigma_{xz}^{(\mathrm{c})}=2\mu^{(\mathrm{c})}\varepsilon_{xz}^{(\mathrm{c})}
\end{aligned}$$

for crystal, where $\mu^{(\mathrm{c})}=C_{44}^{(\mathrm{c})}$. Of course, the strain tensors defined by Eq. (10), and the boundary conditions at the interface (6), respectively, still hold for the present case.

After Fourier transforms, the solution of Eq. (25) is very easy to obtain as

$$\begin{aligned}
u_z(x,y)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}A(\xi)\,\mathrm{e}^{-|\xi|y-\mathrm{i}\,\xi x}\,\mathrm{d}\xi, \\
w_z(x,y)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}B(\xi)\,\mathrm{e}^{-|\xi|y-\mathrm{i}\,\xi x}\,\mathrm{d}\xi,
\end{aligned}$$

and the relevant stresses

$$\begin{aligned}
\sigma_{zy} &= -2\mu\frac{1}{2\pi}\int_{-\infty}^{\infty}|\xi|A(\xi)\,\mathrm{e}^{-|\xi|y-\mathrm{i}\,\xi x}\,\mathrm{d}\xi \\
&\quad -R\frac{1}{2\pi}\int_{-\infty}^{\infty}|\xi|B(\xi)\,\mathrm{e}^{-|\xi|y-\mathrm{i}\,\xi x}\,\mathrm{d}\xi, \\
H_{zy} &= -R\frac{1}{2\pi}\int_{-\infty}^{\infty}|\xi|A(\xi)\,\mathrm{e}^{-|\xi|y-\mathrm{i}\,\xi x}\,\mathrm{d}\xi \\
&\quad -(K_1-K_2)\frac{1}{2\pi}\int_{-\infty}^{\infty}|\xi|B(\xi)\,\mathrm{e}^{-|\xi|y-\mathrm{i}\,\xi x}\,\mathrm{d}\xi, \qquad (27)
\end{aligned}$$

where $A(\xi)$ and $B(\xi)$ are arbitrary functions to be determined.

According to the second formula of boundary conditions (6), one may find the relation between the two unknown functions to be

$$B(\xi)=-\frac{R}{K_1-K_2}A(\xi). \qquad (28)$$

From Eqs. (27) and (28), we obtain

$$\bar{\sigma}_{zy}=-(2\mu+\frac{R^2}{K_1-K_2})A(\xi)\,|\xi|\,,$$

and the first formula in Eq. (6) determines the unknown function

$$A(\xi)=-\frac{\tau\bar{f}(\xi)}{\left(\dfrac{R^2}{K_1-K_2}+\mu\right)|\xi|+k}, \qquad (29)$$

such that

$$B(\xi)=\frac{R\tau\bar{f}(\xi)}{(K_1-K_2)\left[\left(\dfrac{R^2}{K_1-K_2}+\mu\right)|\xi|+k\right]}, \tag{30}$$

where τ and k are defined by Eqs. (6) and (7) respectively.

Thus, the problem is solved. The phason strain field can be determined as

$$\begin{aligned} w_{zy}(x,y)&=-R\tau\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{|\xi|\,\bar{f}(\xi)}{(R^2-\mu(K_1-K_2))\,|\xi|+k(K_1-K_2)}\,\mathrm{e}^{-|\xi|y-\mathrm{i}\,\xi x}\mathrm{d}\xi,\\ w_{zx}(x,y)&=\mathrm{i}\,R\tau\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\xi\bar{f}(\xi)}{(R^2-\mu(K_1-K_2))\,|\xi|+k(K_1-K_2)}\,\mathrm{e}^{-|\xi|y-\mathrm{i}\,\xi x}\mathrm{d}\xi. \end{aligned} \tag{31}$$

For practical calculation, we can give concrete values for different $f(x)$ forms and different values of h, τ, μ, K_1, K_2, R and $\mu^{(\mathrm{c})}$.

4. Discussion and conclusion

We only analyse the results given by Eq. (31) because the coexistence and the transition between icosahedral quasicrystals and cubic crystals are more interesting based on the experimental observation and their corresponding mathematical solutions are simple.

The phason strain field given by Eq. (31) shows the effects of the applied stress at the interface, which are described by τ and $\bar{f}(\xi)$(the Fourier transforms of the distribution function$f(x)$), and it also reveals the effects of μ, K_1, K_2, R, the phonon, phason and phonon-phason coupling elastic constants of the quasicrystal and $k(=\mu^{(\mathrm{c})}/h)$, the ratio between elastic modulus and size of the crystal. Some factors related to the experimental environment have not been taken into account because we discuss the problem from the point of view of mathematical elasticity.

As an example, assume $f(x)=1$ for $-a/2<x<a/2$, and $f(x)=0$ for $x<-a/2$ and $x>a/2$, further assume a to be very small, then from Eq. (31), we can obtain

$$\begin{aligned} w_{zy}(x,y)&=\frac{\mu^{(\mathrm{c})}}{h}\tau a\frac{R(K_1-K_2)}{[\mu(K_1-K_2)-R^2]^2}\exp\Big(-\frac{\mu^{(\mathrm{c})}(K_1-K_2)}{\mu(K_1-K_2)-R^2}\frac{y}{h}\Big)\cos\left\{\frac{\mu^{(\mathrm{c})}(K_1-K_2)}{\mu(K_1-K_2)-R^2}\left(\frac{a}{2}-\frac{x}{h}\right)\right\},\\ w_{zx}(x,y)&=\frac{\mu^{(\mathrm{c})}}{h}\tau a\frac{R(K_1-K_2)}{[\mu(K_1-K_2)-R^2]^2}\exp\Big(-\frac{\mu^{(\mathrm{c})}(K_1-K_2)}{\mu(K_1-K_2)-R^2}\frac{y}{h}\Big)\sin\left\{\frac{\mu^{(\mathrm{c})}(K_1-K_2)}{\mu(K_1-K_2)-R^2}\left(\frac{a}{2}-\frac{x}{h}\right)\right\}. \end{aligned} \tag{32}$$

For further comparison, assume that $f(x)=\delta(x)$, then from Eq. (31), we have

$$w_{zy}(x,y)=\frac{\mu^{(\mathrm{c})}}{h}\tau a\frac{R(K_1-K_2)}{[\mu(K_1-K_2)-R^2]^2}\exp\Big(-\frac{\mu^{(\mathrm{c})}(K_1-K_2)}{\mu(K_1-K_2)-R^2}\frac{y}{h}\Big)\sin\left\{\frac{\mu^{(\mathrm{c})}(K_1-K_2)}{\mu(K_1-K_2)-R^2}\frac{x}{h}\right\}, \tag{33}$$

$$w_{zx}(x,y)=\frac{\mu^{(\mathrm{c})}}{h}\tau a\frac{R(K_1-K_2)}{[\mu(K_1-K_2)-R^2]^2}\exp\Big(-\frac{\mu^{(\mathrm{c})}(K_1-K_2)}{\mu(K_1-K_2)-R^2}\frac{y}{h}\Big)\cos\left\{\frac{\mu^{(\mathrm{c})}(K_1-K_2)}{\mu(K_1-K_2)-R^2}\frac{x}{h}\right\}. \tag{34}$$

Figure 2 shows the variations of the phason strains with x or y for different $\mu^{(\mathrm{c})}$ values and the given values of μ, K_1, K_2 and R, and the measured values of these quantities for Al–Pd–Mn icosahedral quasicrystals are provided in Refs. [29] and [30] as $\mu=72.4$ GPa, $K_1=125$ MPa, $K_2=-50$ MPa, and $R=-0.04$ μ.

Owing to space limitations, we do not show the variation of the phason strains for different values of a/h and τ/μ.

It is evident that there are three multiple factors as follows:

1) The principal part, which consists of three factors $\mu^{(\mathrm{c})}/h$, τa, and $[R(K_1-K_2)]/[\mu(K_1-K_2)-R^2]^2$, denoting the crystal property, the applied stress at the interface and the quasicrystal property, respectively.

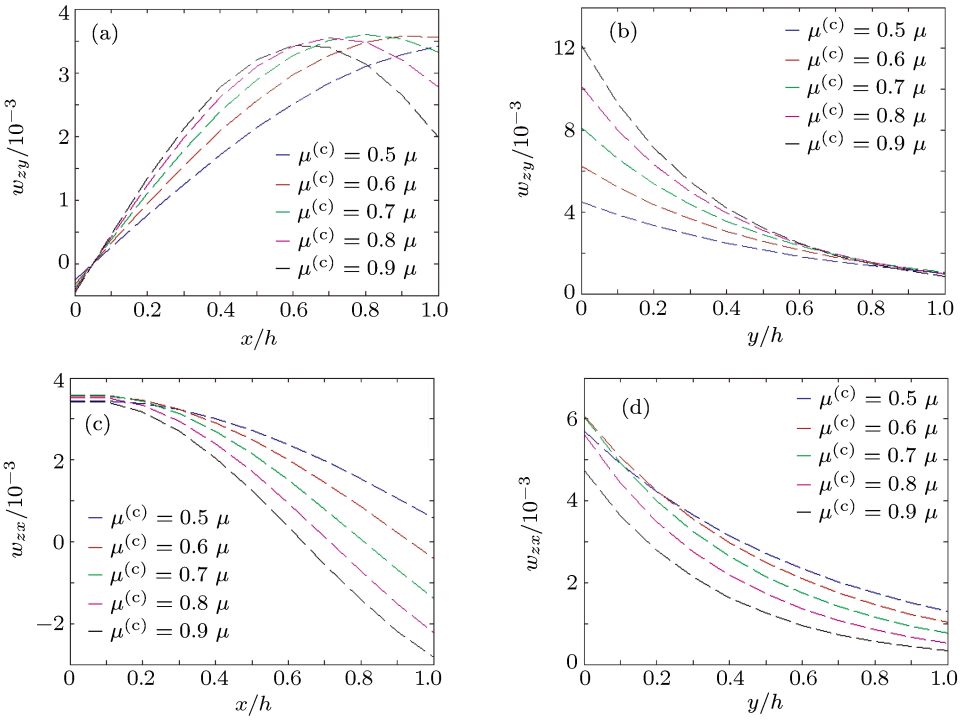


Fig. 2. (colour online) Ratios of shear modulus of crystal-quasicrystal (a) w_{zy} versus x, (b) w_{zy} versus y, (c) w_{zx} versus x, (d) w_{zx} versus y.

2) The exponential function, describing the crystal–quasicrystal interaction.

3) The trigonal function, reflecting the crystal–quasicrystal interaction too. Of course, for other applied stresses, i.e., for different forms of distribution function $f(x)$, the solutions will differ.

In addition, the characteristic of solution (24) is quite different from that of Eq. (31) due to the difference between quasicrystalline systems. We do not further discuss solution (24) due to the limitation of space.

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