An optical rotation sensor based on dispersive slow-light medium*

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Slow light supported by electromagnetically induced transparency effect in dispersive medium is extremely susceptible with respect to Doppler detuning. In this paper, the Doppler effect induced by rotating dispersive medium was considered and the effect of the velocity of rotating dispersive medium on the group velocity was studied. Based on a dispersive slow-light medium, a high sensitive optical rotation sensor for measuring absolute rotation is proposed and analysed. The sensitivity of the rotation sensor is the group delay between the counterpropagationed wave packets in the device, and scales directly with square of the group index which can reach 10^2–10^8 orders of magnitude by selecting a proper dispersive medium.

Keywords: slow light, dispersive medium, absolute rotation, group delay

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1. Introduction

Optical rotation sensor is a high accuracy inertial rotation sensor based on the Sagnac effect and it plays a decisive role in many military and industrial applications, such as aircrafts, satellites, and inertial navigation systems. With the booming development of the slow-light technique, much attention has been paid to using the concept of slow light to enhance the sensitivity of the optical rotation sensor. At present, the significant reduction of the group velocity of light can be achieved by dispersive structure[1−8] or dispersive medium.[9−18] The slow-light mechanism of the dispersive structures (e.g., ring resonators[1−5] and photonic crystals[6−8]) is optical resonance and that of the dispersive mediums (e.g., atomic vapours[9−12], doped crystals,[13−15] photorefractive crystals[16,17] and semiconductors[18]) is electronic resonance. Studies on optical rotation sensor based on dispersive structure have achieved remarkable progress.[19−22] It can realize highly compact integration and miniaturization, but the sensitivity of the miniature rotation sensor is merely equivalent to that of an optical-fibre rotation sensor with several hundred meters of fibre coil.[20]

On the other hand, based on the dispersive mediums, Leonhardt and Piwnicki[23] proposed a slow-light-enhanced interferometer, where the slow light can be generated by electromagnetically induced transparency (EIT) and coherent population trapping (CPT). They used an equivalent time-space metric in moving media to deal with the Sagnac effect. The premise of their theory is Doppler detuning. Since the Sagnac effect in its pure form is in no way related to the Doppler effect,[24] as Shahriar *et al.*[25] pointed out, the dispersive medium can only enhance the relative rotation-induced Sagnac phase shift, where the Doppler detuning is induced by the relative rotation between the medium and the interferometer. Wang *et al.*[26] proposed a new detection method for a relative rotation sensor based on dispersive medium. All the above studies show that the optical rotation sensor based on dispersive medium cannot be employed in sensing absolute rotation. This is a serious drawback of the traditional rotation sensor based on dispersive medium.

By adding some appropriate attachments (for example, a series of magnet pedestals), the drawback can be overcome for our proposed rotation sensor based on dispersive medium. In this paper, we first report the absolute rotation sensing based on a dispersive

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slow-light medium. Moreover, the detection method of the rotating rate of our proposed rotation sensor is different from those of the traditional interferometric optical rotation sensor and resonant optical rotation sensor. The rotating rate in our study is obtained by detecting the group delay[26] between the counter-propagational wave packets in the device. Furthermore, the relations among the detection range of rotating rate, the structure parameters and the sensitivity of the rotation sensor are studied in detail.

2. Slow light in rotating dispersive medium

As a premise, we assume that the slow light in a dispersive medium is supported by the EIT effect. The schematic illustration of the typical atomic three-level Λ system for EIT is shown in Fig. 1. When a strong coupling light is applied, the medium becomes transparent for the probe light with high dispersion and the slow light phenomenon happens. It is worth noting that the transparency of the probe light, the high dispersion of medium and the slow light induced by the EIT effect occur only in a narrow window of a two-photon resonance,[27]

$$|\omega - \omega_{12} - \omega_{\rm c}| = |\omega - \omega_0| < \varepsilon v_{\rm g}\omega_0/c, \qquad (1)$$

where ε is of the order of 10^{-3},[27] $v_{\rm g}$ is the group velocity of probe light, ω is the frequency of the probe light, ω_{12} is the frequency difference between level 1 and level 2, $\omega_{\rm c}$ is the frequency of the coupling light and ω_0 is the resonance frequency of the slow-light beam for the medium, i.e., $\omega_0 = \omega_{12} + \omega_{\rm c}$.

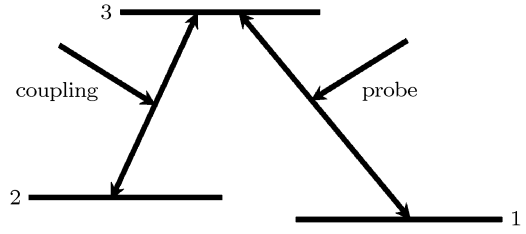


Fig. 1. Schematic illustration of atomic three-level system for EIT.

For the consideration of slow light in the dispersive medium with rotating rate Ω, we assume that the light source is resting in a laboratory frame and the dispersive medium has a relative velocity, ΩR, with respect to the light source, where R is the rotating radius. In this case, a rotating frame is constructed, in which the dispersive medium is at rest. That is to say, the constructed frame corotates with the dispersive medium. Obviously, the light frequency in the rotating frame is different from the frequency of the light source based on the Doppler effect. Assume that the coupling light exactly resonates with the $2\leftrightarrow3$ transition, i.e., $\omega_{\rm c} = \omega_{32}$, and its polarization is perpendicular to that of the probe light. Thus, it is not affected by the Doppler effect induced from the rotating dispersive medium. In addition, we can obtain the frequency ω' of probe light in the rotating frame based on the Doppler effect

$$\omega' = \omega - \Omega R k, \qquad (2)$$

where ω and k are the frequency and wave number of the probe light in laboratory frame, respectively. Therefore, the narrow window of a two-photon resonance for the rotating medium can be obtained in the laboratory frame

$$|\omega' - \omega_0| < \varepsilon v'_{\rm g}\omega_0/c, \qquad (3)$$

where $v'_{\rm g}$ is the group velocity in the rotating frame. We define the detuning of the three-level system induced by the rotation as

$$\delta' = \omega_0 - \omega'. \qquad (4)$$

In the narrow window $|\delta'| < \varepsilon v'_{\rm g}\omega_0/c$, the group velocity in the laboratory frame can be written as[28]

$$v_{\rm g}^{\pm} = \pm v'_{\rm g}\sqrt{1 + 2c\delta/v'_{\rm g} + \Omega^2 R^2/v'^2_{\rm g}}, \qquad (5)$$

where the "+" indicates that the propagation direction of light is in the same direction of the rotating medium, the "−" indicates that the propagation direction of light is in the reverse direction of the rotating medium and δ is the normalized detuning, namely,

$$\delta = (\omega - \omega_0)/\omega_0. \qquad (6)$$

3. Optical rotation sensor based on dispersive medium

Figure 2 shows the schematic illustration of the proposed optical rotation sensor based on a dispersive medium. It includes mainly three parts: the discal dispersive medium, magnet pedestals (MPs) and the interferometer. The dispersive medium and MPs are fixed on the detected object. The interferometer, which is loaded on MPs, is composed of the following parts: the light source (LS), the 50:50 beam splitter (BS), the mirrors (Ms), the bilateral mirror (BM),

the polarizing beam splitters (PBS) and the detectors (D). It is noteworthy that MPs just translate with the detected object, but do not rotate with the detected object. As a result, the interferometer loaded on MP just translates with the detected object, but does not rotate with the detected object. In addition, it is evident that the dispersive medium not only translates with the detected object, but also rotates with the detected object. That is to say, the dispersive medium moves together with the detected object. In a word, the MPs can realize the relative rotation between the dispersive medium and the interferometer. Obviously, the rotating rate of the dispersive medium is that of the detected object. All in all, the relative rotation between the dispersive medium and the interferometer in the proposed optical rotation sensor is consistent with that in the previous optical rotation sensor based on dispersive medium[24] and the proposed rotation sensor can be employed in sensing absolute rotation because the dispersive medium is attached to the detected object, but the interferometer is not.

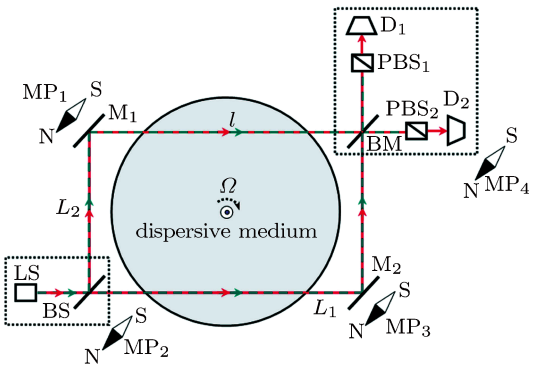


Fig. 2. (colour online) Schematic illustration of proposed optical rotation sensor.

As shown in Fig. 2, the lights are constrained to propagate in a rectangle path with the length L_1 and the width L_2. Both the distances from M_1 to BM and from BS to M_2 in a dispersive medium are l. The probe light (red line) and the coupling light (cyan line) are cross-linearly polarized each other and pass around the device respectively in both clockwise and counterclockwise directions. We assume that the rotating rate of the detected object (i.e., that of the dispersive medium) is Ω. In order to accurately measure the group delay of probe light, the coupling light is filtered by two PBSs. The remaining probe light is respectively accepted by detector 1 (D_1) and detector 2 (D_2), and then feeds into a two-channel digital oscilloscope for comparison. Thus, the group delay can be obtained. The $v = (L_1^2+L_2^2)^{1/2}\Omega$ is the tangent velocity of rotation at BS, M and BM. The $v_{n1} = L_2\Omega/2$ and $v_{n2} = L_1\Omega/2$ are the projections of v along the propagation directions of light in L_2 and L_1, respectively. For the clockwise (+) and counterclockwise (−) directions, the effective distances $L^{\pm}$ and the propagation time $t^{\pm}$ of wave packets from BS to BM in laboratory frame are

$$L^{\pm} = L_{1a}^{\pm} + L_{1b}^{\pm} + L_2^{\pm}, \quad t^{\pm} = t_{1a}^{\pm} + t_{1b}^{\pm} + t_2^{\pm}, \quad (7)$$

where $L_2^{\pm} = L_2 + L_1\Omega t_2^{\pm}/2$, $t_2^{\pm} = L_2^{\pm}/v_2^{\pm}$ and $v_2^{\pm} = c/n(\omega^{\pm})$ are the effective distance, the time, the group velocity of wave packets for traveling from BS to M_1 and from M_2 to BM, respectively; $L_{1a}^{\pm} = L_1 - l + L_2\Omega t_{1a}^{\pm}/2$, $t_{1a}^{\pm} = L_{1a}^{\pm}/v_{1a}^{\pm}$ and $v_{1a}^{\pm} = c/n(\omega^{\pm})$ is the effective distance, the time, the group velocity of wave packets for traveling from M_1 to BM and from BS to M_2 out of the dispersive medium, respectively; $L_{1b}^{\pm} = l + L_2\Omega t_{1b}^{\pm}/2$, $t_{1b}^{\pm} = L_{1b}^{\pm}/v_{1b}^{\pm}$, and $v_{1b}^{\pm} = c/n_g(\omega^{\pm})$ are the effective distance, the time, the group velocity of wave packets for traveling from M_1 to BM and from BS to M_2 in the dispersive medium, respectively. Wherein, $n(\omega^{\pm})$ is the refractive index out of the dispersive medium and $n_g(\omega^{\pm})$ is the group index of the dispersive medium. In a highly dispersive medium, $n_g(\omega^{\pm}) \gg n(\omega^{\pm})$, the group delay can be obtained as

$$\Delta t = t^+ - t^- \approx \frac{l(L_2\Omega + v_{1b}^- - v_{1b}^+)}{(v_{1b}^+ - L_2\Omega/2)(v_{1b}^- + L_2\Omega/2)}. \quad (8)$$

According to Eq. (5), $v_{1b}^{\pm}$ can be written as

$$v_{1b}^{\pm} = \pm\sqrt{v'_{1b}(\omega^{\pm})^2 + 2c\delta v'_{1b}(\omega^{\pm}) + \Omega^2 L_2^2/4}, \quad (9)$$

where $v'_{1b}(\omega^+)$ and $v'_{1b}(\omega^-)$ are the clockwise and the counterclockwise group velocity in the rotating frame, respectively.

We consider that a pulse with a carrier frequency $\omega = \omega_0$ propagates in laboratory frame. In this case, the pulse frequency $\omega^{\pm}$ in the rotating frame can be obtained according to Eq. (2),

$$\omega^{\pm} = \omega_0(1 \mp \Omega L_2/2c), \quad (10)$$

and the detuning $\delta'^{\pm}$ can be obtained according to Eq. (4),

$$\delta'^{\pm} = \omega_0 - \omega^{\pm} = \pm\omega_0\Omega L_2/2c. \quad (11)$$

Obviously, the normalized detuning δ equals zero at $\omega = \omega_0$ according to Eq. (6). By substituting $\delta = 0$ into Eq. (9), $v_{1b}^{\pm}$ can be obtained as

$$v_{1b}^{\pm} = \pm\sqrt{v'_{1b}(\omega^{\pm})^2 + \Omega^2 L_2^2/4}. \quad (12)$$

Note that the group velocity expressed by Eq. (12) is tenable if only the detuning expressed by Eq. (11) just satisfies Eq. (3). Therefore, substituting Eq. (11) into Eq. (3), we can deduce the detection range of the rotating rate Ω as

$$-2\varepsilon v'_{1b}/L_2 < \Omega < 2\varepsilon v'_{1b}/L_2. \tag{13}$$

It shows that the detection range of the rotating rate Ω is constrained by the dispersive medium and the length of L_2 and decreases with the decreasing group velocity v'_{1b} and increasing L_2. Only in this range can the two-photon resonance happen and then we can obtain the effective group velocity in a laboratory frame.

Moreover, for a three-level system, the general expressions of refractive index $n(\omega_p)$ is given as[29]

$$n(\omega_p) = 1 + \frac{N\mu_{13}^2}{2\varepsilon_0\hbar}\frac{\Delta\left(\Delta^2 - \Omega_c^2/4 + \gamma_{12}^2\right)}{\left(\Delta^2 - \Omega_c^2/4 - \gamma_{12}\gamma_{13}\right)^2 + \Delta^2\left(\gamma_{12}+\gamma_{13}\right)^2}, \tag{14}$$

where N is the number density of atoms, μ_{13} is the atomic dipole moments, Ω_c is the Rabi frequency characterizing the coupling light at the 2↔3 transition, Ω_p is the Rabi frequency characterizing the probe light at the 1↔3 transition, γ_{12} and γ_{13} are respectively the damping rates of level $|1\rangle$ to level $|2\rangle$ and to level $|3\rangle$, and $\Delta = \omega_{31} - \omega_p$ is the detuning of the probe light from the 1↔3 transition. As we assumed $\omega_c = \omega_{32}$ in Section 2, we can deduce that $\omega_{31} = \omega_0 = \omega_c + \omega_{12}$, i.e., Δ is equivalent to δ' of the three-level system in the rotating medium. Therefore, Δ in Eq. (14) can be substituted by δ' as

$$n(\omega_p) = 1 + \frac{N\mu_{13}^2}{2\varepsilon_0\hbar}\frac{\delta'\left(\delta'^2 - \Omega_c^2/4 + \gamma_{12}^2\right)}{\left(\delta'^2 - \Omega_c^2/4 - \gamma_{12}\gamma_{13}\right)^2 + \delta'^2\left(\gamma_{12}+\gamma_{13}\right)^2}. \tag{15}$$

Then the group index can be obtained as

$$n_g(\delta') \cong \frac{N\omega_p\mu_{13}^2}{2\varepsilon_0\hbar}\frac{\delta'^6 + A\delta'^4 + B\delta'^2 + C}{\left[\left(\delta'^2 - \Omega_c^2/4 - \gamma_{12}\gamma_{13}\right)^2 + \delta'^2 D\right]^2}, \tag{16}$$

where

$$\begin{aligned} A &= 2\gamma_{12}^2 - \gamma_{13}^2 - \Omega_c^2/4, \\ B &= (\gamma_{12}^2 - \Omega_c^2/4)(\gamma_{12}^2 + \gamma_{13}^2 - \Omega_c^2/2) \\ &\quad - 3(\Omega_c^2/4 + \gamma_{12}\gamma_{13})^2, \\ C &= (\Omega_c^2/4 - \gamma_{12}^2)(\Omega_c^2/4 + \gamma_{12}\gamma_{13})^2, \\ D &= (\gamma_{12} + \gamma_{13})^2, \end{aligned}$$

and the frequency ω_p of probe light in the rotating frame is $\omega_p = \omega_0 - \delta'$. According to Eq. (10) and $\Omega L_2/2c \ll 1$, we can easily know that $\omega_p \approx \omega_0$, thus $n_g(\delta')$ can be approximately considered to be an even function of δ', i.e., $n_g(\delta'^+) = n_g(\delta'^-)$ according to Eq. (11).

In the narrow detuning window, n_g often takes account of a large value for the highly dispersive medium and can be considered to be a constant, i.e., $n_g(\omega^+) = n_g(\omega^-) = n_g$. Therefore, the group velocity $v_{1b}(\omega^\pm)$ can be considered to be a constant V_g in the narrow detuning window,

$$v_{1b}\left(\omega^+\right) = v_{1b}\left(\omega^-\right) = V_g. \tag{17}$$

Substituting Eqs. (17) and (12) into Eq. (8), we can obtain the group delay as

$$\Delta t = lL_2\Omega/V_g^2 = lL_2\Omega n_g^2/c^2. \tag{18}$$

In this paper, the group delay Δt is defined as the rotation sensitivity of the proposed optical rotation sensor. It shows that the rotation sensitivity scales directly with the square of the group index n_g which can be as large as 10^2–10^8 orders of magnitude[9–16] for slow light in atomic and solid medium. Moreover, the rotation sensitivity is independent of L_1, but relevant to L_2 and l. Therefore, we should select the highly dispersive medium and increase L_2 and l to enhance the rotation sensitivity. In addition, it is noteworthy that according to Eq.(13) the group velocity and the width L_2 restrict the detection range of the rotating rate Ω. In order to conveniently discuss the detection range of rotating rate, a width W_Ω of detection range is introduced on the basis of Eq. (13),

$$W_\Omega = 4\varepsilon c/n_g L_2. \tag{19}$$

Obviously, the group delay between the counter-propagational wave packets in the device can be detected only if the varied range of rotating rate Ω is smaller than W_Ω.

Based on the above discussion, we give a rough design for the proposed rotation sensor. Assuming that L_2 is 0.2 m, l is 0.2 m. The curve of the group

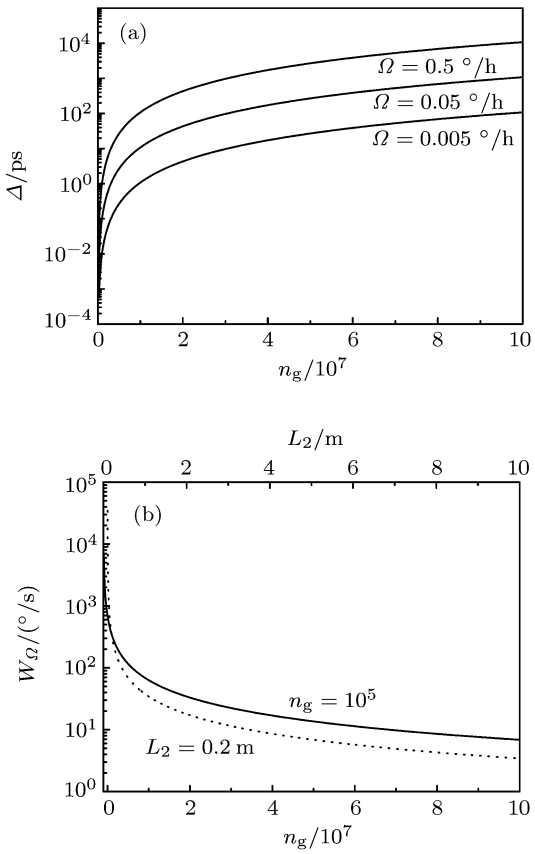


Fig. 3. (a) Group delay Δt versus the group index n_g at rotating rate Ω of 0.5 °/h, 0.05 °/h and 0.005 °/h; (b) width of detected rangeW_Ω versus the group index n_g (dot line) and length L_2 (solid line). The parameters are $L_2 = 0.2$ m, $l = 0.2$ m, $\varepsilon = 10^{-3}$.

delay Δt versus group index n_g is shown in Fig. 3(a) according to Eq. (18) at rotating rate Ω of 0.5 °/h, 0.05 °/h, 0.005 °/h, respectively. As we expected, the group delay increased with n_g, that is to say, the accuracy of rotating rate detection is improved with the light slowing down. In addition, it can be seen that the group delay at the magnitude order shown in the figure is easy to obtain at a slower rotating rate (for example $\Omega = 0.005$ °/h). Therefore, the proposed rotation sensor can be operated as a high accuracy rotation sensor. The curves of W_Ω versus group index n_g and length L_2 are shown in Fig. 3(b) according to Eq. (19). As shown in the figure, the increase of n_g or L_2 causes the decrease of W_Ω, which is just the reverse of the trend of group delay. As a result, the sensitivity enhancement will be accompanied by a reduction in detection range of rotating rate Ω. An effective solution to this problem is the simultaneous decrease of length L_2 and increase of l. According to Eqs. (18) and (19), the decrease of length L_2 will increase W_Ω and decrease Δt and the latter can be overcome by the increase of length l. It is noteworthy that the increase of length l does not affect W_Ω.

4. Conclusion

In this paper, a high sensitive optical rotation sensor based on a dispersive medium is proposed for sensing absolute rotation. The sensitivity of the proposed rotation sensor scales with the square of the group index n_g, the width L_2 and the distance l and is directly detected by the group delay. It will bring a new vigour for optical rotation sensor based on a dispersive medium.

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