Effective Bose–Hubbard interaction with enhanced nonlinearity in an array of coupled cavities*

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An array of coupled cavities, each of which contains an N four-level atom, is investigated. When cavity fields dispersively interact with the atoms, an effective Bose–Hubbard model can be achieved. By numerically comparing the full Hamiltonian with the effective one, we find that within the parameters region, the effective Hamiltonian can completely account for the Mott-insulator as well as the phase transition from the similar Mott-insulator to superfluid. Through jointly adjusting the classical Rabi frequency and the detuning, the nonlinearity can be improved.

Keywords: nonlinearity, Bose–Hubbard model, quantum phase transition

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1. Introduction

Over the past ten years, quantum entanglement[1−7] and quantum nonlinearity[8−16] have aroused people's interest dramatically. Strong nonlinear interaction between photons yields an attracting phenomenon, photon blockade,[10,17] where a single photon in a cavity can block the injection of the second one. Unfortunately, photons tend to interact only weakly, thus enhancement of photon–photon interaction at the few-photon level is a challenge in quantum optics. To improve the nonlinearity of the photon, Imamoğlu *et al.*[9,10] have proposed a scheme for atoms with a level structure considered in electromagnetically induced transparency (EIT)(see Fig. 1, $\varepsilon = \delta = 0$). Grangier *et al.*[12] clarified it when this is strictly a photon–photon interaction. Soon after, Werner and Imamoğlu[13] reinvestigated the system based on the exact eigenstates and found that large detuning δ affects the photon blockade when the EIT phenomenon emerges.

As is well known, controlling the interactions of solid state material in experiment is very difficult owing to their microscopic property. Thus, simulation of the microscopic interactions with artificial structure is a hot research field because this kind of artificial structure can be accessed and manipulated individually. Most recently, interests have arisen for an array of coupled cavities because spin chain,[18−21] Bose–Hubbard interaction and superfluid-to-Mott-insulator quantum phase transition can be simulated in these structures.[14,17,22] In Ref. [17], Hartmann *et al.* employed a polariton technique to investigate the nonlinearity of photon and to simulate Bose–Hubbard interaction. Under strong classical driven conditions, the dark state polariton (joint atomic-photonic excitation) becomes photon. They obtained effective photon nonlinearity which is exactly the same with the resonant case in Ref. [10]. Thus, Imamoğlu relaxed a restricting assumption and increase achievable photon–photon interaction by at least one order of magnitude. However, due to the limitation of the dark state polariton method, the effective Hamiltonian (expression of U) in Ref. [17] still has no relation with detuning δ, although in their numerical simulation they compare the full Hamiltonian with large δ with the effective Hamiltonian without the parameters δ.[14,17]

The detuning δ definitely affects Kerr nonlinearity as it has been shown in Refs. [12] and [13] that the dispersion can affect the photon antibunching so as to affect the photon blockade.[13] In this paper, we aim at achieving an effective Hamiltonian corresponding to dispersive interaction in a coupled cavity system which has not been studied before. By careful adiabatic elimination of all atomic degrees of freedom, we obtain an effective Bose–Hubbard model which contains all parameters and corresponds to the one and the only full Hamiltonian. We analyse the effect of the detuning δ on the Kerr nonlinearity and find that the large value of the detuning can slightly enhance the

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Kerr nonlinearity which is reverse to the nonlinearity decreasing with the increasing of classical Rabi frequency. By comparing the full Hamiltonian with the effective one, we find that within the parameters region the effective Hamiltonian can completely account for the Mott-insulator as well as the phase transition from Mott-insulator to superfluid.

2. Model and the effective Hamiltonian

We consider an array of cavities which are coupled via the exchange of photons. Each of the cavities contains N four-level atoms. The atomic configuration is shown in Fig. 1, which is of the same structure with EIT.[10,11,17] The cavity mode couples to the transitions $|b\rangle \leftrightarrow |a\rangle$ and $|d\rangle \leftrightarrow |c\rangle$ while the classical laser drives the atomic transitions $|c\rangle \leftrightarrow |b\rangle$ with Rabi frequencies Ω. Here, we still consider $\varepsilon = 0$, which shows that it is the two-photon resonance and the EIT phenomenon emerges. In a rotating frame, with respect to $H_0 = \sum_{j=1}^{N}\Big[\sum_{s=a,b,c,d}\omega_s(|s_j\rangle\langle s_j|) - \delta|b_j\rangle\langle b_j| - \Delta|d_j\rangle\langle d_j|\Big] + \omega a^\dagger a$, the Hamiltonian within one cavity is

$$H_{\rm fullc} = H_1 + H_2, \tag{1}$$

with

$$\begin{aligned} H_1 &= \sum_{j=1}^{N}[\delta|b_j\rangle\langle b_j| + \Omega(|b_j\rangle\langle c_j| + \text{h.c.}) \\ &\quad +\Delta|d_j\rangle\langle d_j|], \end{aligned} \tag{2}$$

$$H_2 = \sum_{j=1}^{N}[g_1 a^\dagger|a_j\rangle\langle b_j| + g_2 a^\dagger|c_j\rangle\langle d_j| + \text{h.c.})], \tag{3}$$

where g_i is the coupling strength between the cavity and the atoms, Δ and δ represent the detuning shown in Fig. 1, and a is the annihilation operator of the cavity field. In the strong classical driving regime, i.e., $|\Omega| \gg |g_i|$, we switch into the dressed state picture determined by H_1. The eigenstates are

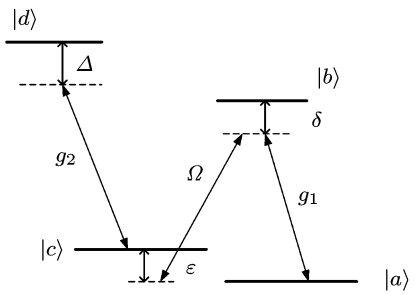


Fig. 1. The configuration of the four-level atom.

$$|\pm_j\rangle = \frac{1}{\sqrt{\Omega^2 + \lambda_\pm^2}}(\lambda_\pm|b_j\rangle + \Omega|c_j\rangle), \tag{4}$$

where

$$\lambda_\pm = \frac{\delta}{2} \pm \frac{\sqrt{\delta^2 + 4\Omega^2}}{2}. \tag{5}$$

Performing the transformation

$$H_3 = \mathrm{e}^{\mathrm{i}H_1 t} H_2 \,\mathrm{e}^{-\mathrm{i}H_1 t},$$

we have

$$\begin{aligned} H_3 = \sum_{j=1}^{N}\{ & g_1 a^\dagger|a_j\rangle\Big(\frac{\sqrt{\lambda_+^2+\Omega^2}}{\lambda_+-\lambda_-}\,\mathrm{e}^{-\mathrm{i}\lambda_+ t}\langle +_j| \\ & - \frac{\sqrt{\lambda_-^2+\Omega^2}}{\lambda_+-\lambda_-}\,\mathrm{e}^{-\mathrm{i}\lambda_- t}\langle -_j|\Big) \\ & + g_2 a^\dagger[\frac{-\lambda_-\sqrt{\lambda_+^2+\Omega^2}}{\Omega(\lambda_+-\lambda_-)}\,\mathrm{e}^{\mathrm{i}(\lambda_+-\Delta)t}|+_j\rangle \\ & + \frac{\lambda_+\sqrt{\lambda_-^2+\Omega^2}}{\Omega(\lambda_+-\lambda_-)}\,\mathrm{e}^{\mathrm{i}(\lambda_--\Delta)t}|-_j\rangle]\langle d_j| + \text{h.c.}\}, \end{aligned} \tag{6}$$

where

$$\sqrt{\lambda_\pm^2+\Omega^2}/(\lambda_+-\lambda_-)$$

and

$$\lambda_\pm\sqrt{\lambda_\pm^2+\Omega^2}/[\Omega(\lambda_+-\lambda_-)]$$

are the probability amplitude of wave function so that they are less than one. If $|\lambda_\pm| \gg g_1$, $|\lambda_\pm - \Delta| \gg g_2$, with the method of effective Hamiltonian theory,[23] the Hamiltonian effectively equals

$$\begin{aligned} H_4 = \sum_{j=1}^{N} & -Va^\dagger a|a_j\rangle\langle a_j| - Paa^\dagger|d_j\rangle\langle d_j| \\ & + \bar{\Omega}g_1 g_2(a^{\dagger 2}|a_j\rangle\langle d_j|\,\mathrm{e}^{-\mathrm{i}\Delta t} + \text{h.c.}), \end{aligned} \tag{7}$$

where

$$\begin{aligned} V &= \frac{g_1^2}{(\lambda_+-\lambda_-)^2}\Big(\frac{\lambda_+^2+\Omega^2}{\lambda_+} + \frac{\lambda_-^2+\Omega^2}{\lambda_-}\Big), \\ P &= \frac{g_2^2}{\Omega^2(\lambda_+-\lambda_-)^2}\Big[\frac{\lambda_-^2(\lambda_+^2+\Omega^2)}{\lambda_+-\Delta} \\ &\quad + \frac{\lambda_+^2(\lambda_-^2+\Omega^2)}{\lambda_--\Delta}\Big], \\ \bar{\Omega} &= \frac{1}{2\Omega(\lambda_+-\lambda_-)^2}\Big[(\lambda_+^2+\Omega^2)\Big(\frac{\lambda_-}{\lambda_+} + \frac{\lambda_+}{\lambda_--\Delta}\Big) \\ &\quad + (\lambda_-^2+\Omega^2)\Big(\frac{\lambda_+}{\lambda_-} + \frac{\lambda_-}{\lambda_+-\Delta}\Big)\Big]. \end{aligned} \tag{8}$$

We assume that initially the atoms are not populated at $|\pm\rangle$ (i.e. no population at the bare states $|b\rangle$ and

$|c\rangle$). In Eq. (7), we have omitted the terms taken action on the atomic state $|\pm\rangle$. The first line in Eq. (7) stands for Stark shifts on the levels $|a_j\rangle$ and $|d_j\rangle$ and the second line means that the jumping from $|d_j\rangle$ to $|a_j\rangle$ creats two photons, *vice versa*. Thus, we see that the nonlinear process exists in the Hamiltonian.

Because of the large value of Ω, $|\Delta| \gg \bar{\Omega} g_1 g_2$ ($\bar{\Omega} \approx 1/\Omega$) can be satisfied. Therefore, we can use the effective Hamiltonian theory once more and adiabatically eliminate the excited state $|d\rangle$ in Eq.(7). If the atoms are initially in its ground state $|a\rangle$, finally we obtain the effective Hamiltonian as

$$H_{\rm effc} = U a^{\dagger 2} a^2 + NV a^\dagger a, \tag{9}$$

with

$$U = -\frac{N(\bar{\Omega} g_1 g_2)^2}{\Delta}. \tag{10}$$

If $\Delta < 0$, U will be the onsite repulsive potential and for $\Delta > 0$, U stands for attractive interaction. The effective Hamiltonian (9) contains all parameters, which means that it can be the unique correspondence to the full Hamiltonian Eqs. (1)–(3) at least in the aspect of containing all the parameters. We will show the validity of the effective Hamiltonian by comparing it with the full Hamiltonian numerically in the next section.

If $\delta = 0$, from Eq. (8), when $|\Omega| \gg |\Delta|$, $\bar{\Omega} \approx 1/\Omega$; then we can deduce $U = -N(g_1 g_2)^2/\Delta\Omega^2$, which is exactly the same as that in Refs. [10] and [17]. Therefore, our result can be reduced to the resonance case. During the deduction of dispersive Hamiltonian (7), adiabatic elimination conditions are summarized as

$$\begin{aligned} &|\Omega| \gg |g_i|, \\ &|\lambda_\pm| \gg g_1, \quad |\lambda_\pm - \Delta| \gg g_2, \\ &|\Delta| \gg \bar{\Omega} g_1 g_2, \end{aligned} \tag{11}$$

which are the parameters region. Hereafter, we will work on the parameters region.

The variations of repulsive potential U with δ and Ω are shown in Fig. 2. We see that with the increase of Ω, the value of U decreases, which corresponds to that in Refs. [10] and [17]. However, with the increase of δ, the repulsive potential increases a little. Thus, large value of δ will not reduce the Kerr nonlinearity, on the contrary, it can enhance U a little. Undoubtedly, before we verify the effective Hamiltonian, the correctness of the effect of detuning δ should be illustrated from the full interaction rather than from the expression of U. We will show it in the next section.

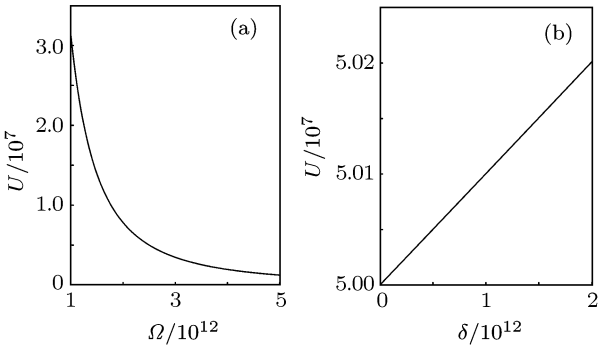


Fig. 2. The variations of onsite repulsion U (a) with Rabi frequency of classical pumping for $\delta = 10^{11}$ Hz, (b) with δ for $\Omega = 10\sqrt{N} g_1$. The parameters are $N = 1000$, $g_1 = g_2 = 2.5 \times 10^9$ Hz, $\Delta = -1.25 \times 10^9$ Hz.

3. Numerical results for coupled cavities

Now, we consider a coupled array of cavities with the photon hopping term

$$H_h = J \sum_s (a_s^\dagger a_{s+1} + \text{h.c.}), \tag{12}$$

where $\sum_s$ is the sum of all cavities and J is the overlap integral over the nearest neighboring cavity. If every cavity is with the same frequency and the condition $\min\{|\Omega|,\ |\lambda_\pm|,\ |\lambda_\pm - \Delta|, \Delta\} \gg J$ is satisfied, the above deduction procedures are still held. Combining Eq. (9) with Eq. (12), we finally have the total effective Hamiltonian of the coupled array system

$$\begin{aligned} H_{\rm eff} = &\sum_s U a_s^{\dagger 2} a_s^2 + NV a_s^\dagger a_s \\ &+ J \sum_s (a_s^\dagger a_{s+1} + \text{h.c.}). \end{aligned} \tag{13}$$

We see that the effective Hamiltonian is the Bose–Hubbard model.

To verify the correctness of our effective Hamiltonian, we present a numerical simulation by comparing the effective Hamiltonian Eq. (13) with the full Hamiltonian, $H_{\rm fulls} = \sum_s H_{\rm fullc} + H_h$ (the summation over Eq. (1) plus Eq. (12)). We consider two coupled cavities, each of which contains one photon and all the atoms are in their ground state, i.e., the system state is $|1a, 1a\rangle$ (the nonlinearity will have effects) . The dynamic evolution of effective Hamiltonian Eq. (13) in

subspace $|2,0\rangle$, $|1,1\rangle$, $|0,2\rangle$ can be done easily. However, for $H_{\rm fulls}$, we need to find the subspace of $H_{\rm fulls}$. Because the two excitations are involved in the evolution process, the subspace bases become a bit more complicated. We give the bases and the matrix of Hamiltonian in Appendix A.

We first discuss the effects of detuning δ and the classical Rabi frequency Ω on the validity of the Hamiltonian Eq. (13). Figure 3(a) shows the mean photon number in cavity one $n_1 = \langle a_1^\dagger a_1\rangle$ for effective Hamiltonian as well as for the full Hamiltonian with several values of δ. For all the lines in Fig. 3(a), $\Omega = 20\sqrt{N}g_1$. For effective Hamiltonian Eq. (13), n_1 always keeps one. Only when the detuning $\delta = 10^{11}$ Hz, the full model can be equal to the effective one. For $\delta = 10^6$ Hz, we see that the detuning changes the behavior of the system greatly, which can be interpreted as the repulsive interaction decrease due to the decreasing of detuning δ, corresponding to that shown in Fig. 2(b). Figure 3(b) shows the effect of Ω on the effectiveness of the effective Hamiltonian Eq. (13). For all the lines in Fig. 3(b), $\delta = 10^6$ Hz. If the detuning keeps its value of 10^6 Hz, we must increase Ω greatly so as to keep the validity of the effective Hamiltonian.

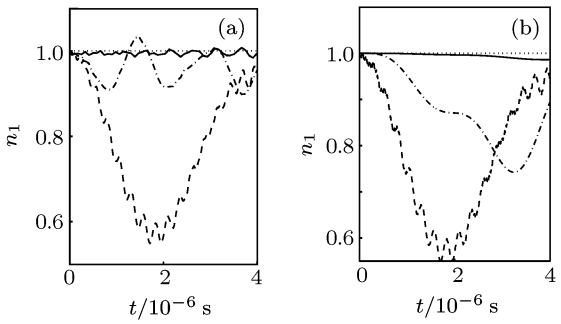


Fig. 3. Comparison of mean photon number n_1 between the effective Hamiltonian (dotted line) and full Hamiltonian for several value of δ (a) as well as Ω (b). The parameters are $N = 1000$, $g_1 = g_2 = 2.5 \times 10^9$ Hz, $\Delta = -1.25\times10^9$ Hz, $J = 6\times10^5$ Hz. (a) The $\delta = 10^{11}$ Hz, 10^7 Hz and 10^6 Hz are corresponding to the solid, dash-dot and dashed lines; (b) the $\Omega = 280\sqrt{N}g_1$, $100\sqrt{N}g_1$, $20\sqrt{N}g_1$ are corresponding to the solid, dash-dot and dashed lines.

We divided the function of δ and Ω into two cases: 1) If the Rabi frequency Ω is extremely large (larger than g (Δ) for four orders or more), for small value of δ (even if $\delta = 0$), the nonlinear two-photon process can still exist, as shown in Fig. 3(b). In our numerical calculation, we find that if $\delta = 0$, only when Ω is as high as $400\sqrt{N}g_1$ does the effective model equal the full model. This case is discussed in Ref. [10]. 2) If Ω is moderately large (larger than g (Δ) for two or three orders), for example $20\sqrt{N}g_1$ as shown in Fig. 3(a) and in Ref. [17], large value of δ can still keep the correctness of effective Hamiltonian. In the other words, large detuning favors the happening of two-photon process. We now explain it from a mathematical as well as a physical point of view. See Eqs. (5) and (11), for extremely large Ω, $\lambda_\pm \approx \pm\Omega$ (for zero or small δ), the condition $|\lambda_\pm - \Delta| \gg g_2$ can be satisfied very well. However, for not large enough Ω, if $\delta = 0$, both $\lambda_+ - \Delta$ and $\lambda_- - \Delta$ cannot be much larger than g_2, while the large value of δ can help to improve λ_+ so that $\lambda_+ - \Delta \gg g_2$ will be satisfied very well (nonlinear process needs at least one of the terms associated with $e^{i(\lambda_+ - \Delta)t}$ and $e^{i(\lambda_- - \Delta)t}$ being treated adiabatically, see Eq. (6)). From physical point of view, no population at levels $|b\rangle$ and $|c\rangle$ is the condition for the existence of two-photon nonlinear process. Large Ω and δ can cooperate to do it. Very high Rabi frequency Ω can make no population at levels $|b\rangle$ and $|c\rangle$. When the classical Rabi frequency Ω is not extremely large, if δ is not large enough, atoms may populate at level $|b\rangle$ so that the jumping from $|d\rangle$ to $|a\rangle$ may be divided into two one-photon process; but if δ are very high, the large detuning can also make no population at level $|b\rangle$; thus the nonlinear two-photon process definitely happen and we have large value of U. We conclude that large δ and Ω is desirable for the happening of the nonlinear process. However, they play different roles in affecting the nonlinearity interaction U. As is well known, large value of Ω decreases U while large value of δ reversely increases it. If we improve Ω for one order, U will decrease two orders (based on the parameters of Fig. 3). However, if we keep Ω moderately large and increase δ, we can increase U at least two orders. Thus, in order to enhance the nonlinearity, we should choose moderately large Ω and δ together.

Since we have made clear the role of detuning and classical pumping, we choose reasonable parameters Ω and δ to show the Mott phase in a coupled cavities system. If the onsite repulsion is larger than the hopping rate, i.e., $U \gg J$, photons are isolated in their own cavities, that is to say, the system is in Mott-like phase. Adopting the parameters in Fig. 4 (dashed line), we have the onsite repulsion $U = 1.25 \times 10^7$ Hz. The condition $U \gg J$ is approxi-

mately satisfied, thus in Fig. 4, we see that the mean photon number $n_1 = \langle a_1^\dagger a_1 \rangle$ from full Hamiltonian for $\delta = 10^{11}$ Hz (solid line) is almost one and the fluctuation $F_1 = \langle (a_1^\dagger a_1)^2 \rangle - \langle a_1^\dagger a_1 \rangle^2$ is relatively small, which means that the double occupancies of a cavity are strongly suppressed and the system is in the Mott insulator phases. Noticing that there is little difference between the full model and the effective one, we know that the effective Hamiltonian can account for the interaction of the full system.

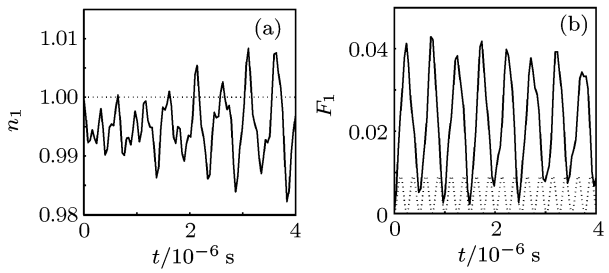


Fig. 4. Comparison of mean photon number n_1 and the fluctuation F_1 between the effective Hamiltonian (dotted line) and full Hamiltonian (solid line), where we choose $\delta = 10^{11}$ Hz, $\Omega = 20\sqrt{N}g_1$, and the other parameters are the same as that in Fig. 3.

Now, we discuss the case of quantum phase transition. In Fig. 5(a), we let the Rabi frequency of classical field Ω change from $25\sqrt{N}g_1$ to $100\sqrt{N}g_1$ with time evolution. Figure 5(b) shows the time evolution of the effective repulsion U as well as the hopping rate J. We see that with the increasing of Ω, the value of U decreases (corresponding to Fig. 2(a)) and the relation between U and J changes from $U > J$ to $U < J$. Although the mean photon number approximately keeps fixed, its fluctuation becomes larger when the value of U is decreased. Thus, the photon can freely jump between adjacent cavities which means it is now in a superfluid phase. Therefore, by adjusting the value of Rabi frequency, the system can receive a phase transition similar to the phase transition from Mott-insulator to superfluid. Little difference between full model and effective model is shown in Figs. 5(c) and 5(d), which can further verify the correctness of our effective Hamiltonian.

During the above discussion, to compare the effective model with the full model easily, we have neglected the loss of the cavity and atoms. Owing to strong classical driving and large detuning, no population is at the excited states, so the atomic spontaneous emissions are much less and can be ignored. The loss of the system mainly results from the cavity decays. To generate sufficiently larger nonlinearity (exceeding the decays rate), from Eq. (10), we know that the strong coupling g_i is still needed, the larger the coupling, the larger the nonlinearity. For the parameters of Fig. 3, the coupling g_i should be 3 or 4 orders of magnitude higher than the cavity decay rates if we need the repulsion U to be 1 or 2 orders of magnitude higher than the cavity loss rate. For toroidal microcavities, the Q factor can be as high as 10^8,[24–27] for which $g_i/\Gamma_c \sim 10^4$;[17] thus the repulsive potential U being 1 or 2 orders of magnitude higher than the cavity loss rate can be achieved. In addition, larger number of atoms can also improve the nonlinearity.

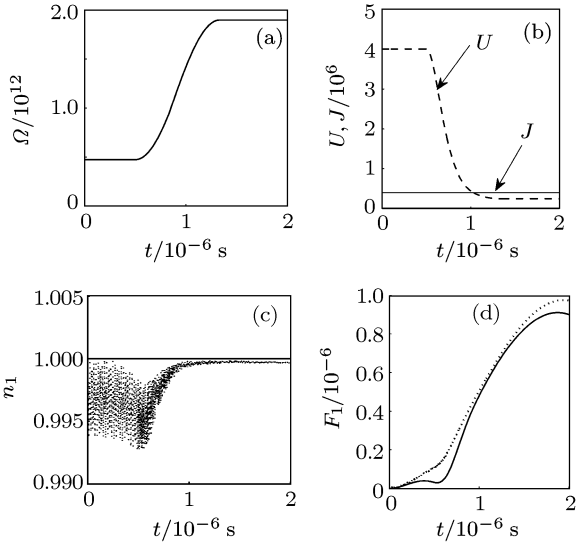


Fig. 5. The time variations of (a) Rabi frequency Ω, (b) the effective repulsive potential U and hopping rate J as well as their relation, (c) the mean photon number and (d) their fluctuation for full model (dotted line) as well as for effective model (solid line). The parameters are $N = 1000$, $g_1 = g_2 = 2.5 \times 10^9$ Hz, $\Delta = -2.5 \times 10^9$ Hz, $J = 0.4 \times 10^6$ Hz, $\delta = 10^{11}$ Hz.

4. Conclusion

We have achieved an effective Hamiltonian corresponding to dispersive interaction in a coupled cavities system. By careful adiabatic elimination of all atomic degrees of freedom, we obtain an effective Bose–Hubbard model, which contains all parameters and corresponds to the one and the only full Hamiltonian. By comparison between the full Hamiltonian with the effective one, we find that within the parameters region, the effective Hamiltonian can completely account for the Mott-insulator as well as the

phase transition similar to the phase transition from Mott-insulator to superfluid. We analyse the effects of detuning δ and Rabi frequency Ω on the Kerr nonlinearity and find that high Rabi frequency Ω decreases the two-photon nonlinearity while the large value of detuning δ can slightly improve it. Thus, by adjusting Ω and δ together, we can improve the nonlinearity, compared with the resonant case.

Appendix A

We give the detailed calculation of the density matrix of two-mode fields here.

For the zero excitation subspace,

$$|0a\rangle = |0\rangle \otimes \prod_{j=1}^{N} |a_j\rangle. \tag{A1}$$

For the one excitation subspace,

$$\begin{aligned} |1a\rangle &= |1\rangle \otimes \prod_{j=1}^{N} |a_j\rangle, \\ |0b\rangle &= |0\rangle \otimes \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |\cdots b_j \cdots\rangle, \\ |0c\rangle &= |0\rangle \otimes \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |\cdots c_j \cdots\rangle. \end{aligned} \tag{A2}$$

For the two excitation subspace,

$$\begin{aligned} |2a\rangle &= |2\rangle \otimes \prod_{j=1}^{N} |a_j\rangle, \\ |1b\rangle &= |1\rangle \otimes \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |\cdots b_j \cdots\rangle, \\ |1c\rangle &= |1\rangle \otimes \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |\cdots c_j \cdots\rangle, \\ |0bb\rangle &= |0\rangle \otimes \sqrt{\frac{2}{N(N-1)}} \sum_{i\neq j=1}^{N} |\cdots b_i \cdots b_j \cdots\rangle, \\ |0bc\rangle &= |0\rangle \otimes \sqrt{\frac{1}{N(N-1)}} \sum_{i\neq j=1}^{N} |\cdots b_i \cdots c_j \cdots\rangle, \\ |0cc\rangle &= |0\rangle \otimes \sqrt{\frac{2}{N(N-1)}} \sum_{i\neq j=1}^{N} |\cdots c_i \cdots c_j \cdots\rangle, \\ |0d\rangle &= |0\rangle \otimes \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |\cdots d_j \cdots\rangle. \end{aligned} \tag{A3}$$

For the initial state $|1a, 1a\rangle$, the two cavities system will have 23 base vectors. We enumerate these states as $|2a, 0a\rangle$, $|1b, 0a\rangle$, $|1c, 0a\rangle$, $|0bb, 0a\rangle$, $|0bc, 0a\rangle$, $|0cc, 0a\rangle$, $|0d, 0a\rangle$, $|1a, 1a\rangle$, $|0b, 1a\rangle$, $|0c, 1a\rangle$, $|1a, 0b\rangle$, $|0c, 0b\rangle$, $|1a, 0c\rangle$, $|0c, 0c\rangle$, $|0b, 0b\rangle$, $|0b, 0c\rangle$, $|0a, 2a\rangle$, $|0a, 1b\rangle$, $|0a, 1c\rangle$, $|0a, 0bb\rangle$, $|0a, 0bc\rangle$, $|0a, 0cc\rangle$, $|0a, 0d\rangle$,

$$\begin{aligned} H_{1,2} &= H_{8,9} = H_{8,11} = H_{9,15} = H_{10,12} \\ &= H_{11,15} = H_{13,16} = H_{17,18} = \sqrt{N} g_1, \\ H_{1,8} &= H_{8,17} = \sqrt{2} J, \\ H_{2,2} &= H_{5,5} = H_{9,9} = H_{11,11} = H_{12,12} = H_{16,16} \\ &= H_{18,18} = H_{21,21} = \delta, \\ H_{2,3} &= H_{9,10} = H_{10,10} = H_{11,13} = H_{12,14} \\ &= H_{12,15} = H_{14,16} = H_{15,16} = H_{18,19} = \Omega \\ H_{2,4} &= H_{18,20} = \sqrt{2(N-1)} g_1, \\ H_{2,9} &= H_{3,10} = H_{11,18} = H_{13,19} = J, \\ H_{3,5} &= H_{19,21} = \sqrt{N-1} g_1, \\ H_{3,7} &= H_{19,23} = g_2, \\ H_{4,4} &= H_{15,15} = H_{20,20} = 2\delta, \\ H_{4,5} &= H_{5,6} = H_{20,21} = H_{21,22} = \sqrt{2}\Omega, \\ H_{7,7} &= H_{23,23} = \Delta, \\ H_{8,9} &= H_{8,11} = H_{9,15} = H_{10,12} = H_{11,15} \\ &= H_{13,16} = H_{17,18} = \sqrt{N} g_1. \end{aligned} \tag{A4}$$

Except the complex conjugate of the elements in (A4), the others matrix elements of the two cavities Hamiltonian are all zero.

References

[1] Solano E, Agarwal G S and Walther H 2003 *Phys. Rev. Lett.* **90** 027903

[2] Zubairy M S, Kim M and Scully M O 2003 *Phys. Rev.* A **68** 033820

[3] Zheng S B 2004 *Phys. Rev.* A **69** 055801

[4] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392

[5] Scully M O, Fry E S, Ooi C H R and Wodkiewicz K 2006 *Phys. Rev. Lett.* **96** 010501

[6] Zhou L, Yang G H and Patnaik A K 2009 *Phys. Rev.* A **79** 062102

[7] Zhou L, Xiong H and Zubairy M S 2006 *Phys. Rev.* A **74** 022321

[8] Birnbaum K M, Boca A, Miller R, Boozer A D, Northup T E and Kimble H J 2005 *Nature* **436** 87

[9] Schmidt H and Imamoğlu A 1996 *Opt. Lett.* **21** 1936

[10] Imamoğlu A, Schmidt H, Woods G and Deutsch M 1997 *Phys. Rev. Lett.* **79** 1467 (also see the Erratum: 1998 ibid **81** 2836)

[11] Fleischhauer M, Imamoğlu A and Marangos J P 2005 *Rev. Mod. Phys.* **77** 633

[12] Grangier P, Walls D F and Gehri K M 1998 *Phys. Rev. Lett.* **81** 2833

[13] Werner M J and Imamoğlu A 1999 *Phys. Rev.* A **61** 011801(R)

[14] Hartmann M J, Brandão F G S L and Plenio M B 2006 *Nature Phys.* **2** 849

[15] Su H, Niu Y P and Gong S Q 2007 *Chin. Phys.* **16** 0429

[16] Wu H B, Chang H, Ma J, Xie C D and Wang H 2005 *Acta Phys. Sin.* **54** 3632 (in Chinese)

[17] Hartmann M J and Plenio M B 2007 *Phys. Rev. Lett.* **99** 103601

[18] Hartmann M J, Brandão F G S L and Plenio M B 2007 *Phys. Rev. Lett.* **99** 160501

[19] Cho J, Angelakis D G and Bose S 2008 *Phys. Rev.* A **78** 062338

[20] Zhou L, Yan W B and Zhao X Y 2009 *J. Phys.* B: *At. Mol. Opt. Phys.* **42** 065502

[21] Yang W L, Wei H, Feng M and An J H 2009 *Chin. Phys.* B **18** 3677

[22] Hartmann M J, Brandão F G S L and Plenio M B 2008 *New J. Phys.* **10** 033011

[23] James D F V and Jerke J 2007 *Can. J. Phys.* **85** 625

[24] Armani D K, Kippenberg T J, Spillane S M and Vahala K J 2003 *Nature* **421** 925

[25] Aoki T, Dayan B, Wilcut E, Bowen W P, Parkins A S, keppenberg T J, Vahala K J and Kimble H 2006 *Nature* **443** 671

[26] Spillane S M, Kuooebberg T J, Vahala K T, Goh K W, Wilcut E and Kimble H J 2005 *Phys. Rev.* A **71** 013817

[27] Vahala K J 2003 *Nature* **424** 839