Effects of self-fields on electron trajectory and gain in two-stream electromagnetically pumped free-electron laser with ion channel guiding

S. Saviz[a)†], Farzin M. Aghamir[b)], H. Mehdian[c)], and M. Ghorannevis[a)]

[a)] *Plasma Physics Research Center, Science and Research Branch, Islamic Azad University, Tehran, P.O. Box.* 14665-678, *Iran*

[b)] *Department of Physics, University of Tehran, N. Kargar Ave, Tehran* 14399, *Iran*

[c)] *Department of Physics and Institute for Plasma Research, Tarbiat Moallem University,* 49, *Dr. Mofatteh avenue, Tehran* 15614, *Iran*

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A theory for the two-stream free-electron laser with an electromagnetic wiggler (EMW) and an ion channel guiding is developed. In the analysis, the effects of self-fields have been taken into account. The electron trajectories and the small signal gain are derived. The stability of the trajectories, the characteristics of the linear gain and the normalized maximum gain are studied numerically. The dependence of the normalized frequency $\hat{\omega}$ corresponding to the maximum gain on the ion-channel frequency is presented. The results show that there are seven groups of orbits in the presence of the self-fields, which are similar to those reported in the absence of the self-fields. It is also shown that the normalized gains of 2 groups decrease while the rest increase with the increasing normalized ion-channel frequency. Furthermore, it is found that the two-stream instability and the self-field lead to a decrease in the maximum gain except for group 4.

Keywords: free electron laser, two-stream

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1. Introduction

The free-electron laser (FEL) makes use of the unstable interaction of a relativistic electron beam with a transverse wiggler magnetic field to generate coherent electromagnetic radiation. The electromagnetic wigglers can lead to the product of a shorter wavelength radiation in comparison with magneto-static wigglers. As a result, the electromagnetic wiggler has advantages over the magneto-static wiggler for the generation of very short wavelength radiation in the case that a limitation on the beam energy is encountered.[1−3] The two-stream free-electron laser amplifiers have been studied extensively during the last few decades.[4−7] Botton *et al.*[8] calculated the gain of an electromagnetic wave in a two-stream free-electron laser. They demonstrated that the gain per pass is more than two orders of magnitudes stronger than the conventional one.

The ion-channel guiding may be employed to facilitate the transport of intense relativistic electron beams. It was first proposed for use in a free-electron laser by Takayama and Harimatsu.[9] The technique of ion-channel guiding of an electron beam in an FEL has become an area of great interest, as it eliminates the need for conventional quadra pole or solenoid focusing magnets, thereby reducing the capital and running cost. There are several ways to create the ion channel. One is called the blowout regime.[10,11] In that regime, the initial beam density is greater than the plasma density (Budker condition). To overcome this condition, we can use high power lasers to create an ion channel. Simulation studies have shown that effective gain enhancement can be obtained by introducing ion-channel focusing in an FEL.[12]

Although there is considerable literature on the theory of two-stream free-electron lasers, most treatments heretofore neglected the influence of the equilibrium self-fields of the electron beams. In the case of low energy beams, an axial magnetic field is usually used to guide the electron beam. In such configuration, the equilibrium self-electric and self-magnetic fields generated by the charge and the current densities of the beams respectively can have considerable effects on the equilibrium orbits. The self-field effects on the electron trajectory and on the electron gain in a two-stream free-electron laser with an electromagnetic wave wiggler and an axial guide field have been

†Corresponding author. E-mail: s_saviz@tmu.ac.ir

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investigated by Mehdian and Saviz.[13]

The purpose of this paper is to investigate the effects of self-fields on the electron trajectory and on the electron gain in a two-stream free-electron laser with an electromagnetic wave wiggler and an ion-channel guiding. In Section 2, the analysis of single-particle trajectories of the two streams in the presence of the ion-channel field, the electromagnetic wiggler (EMW) and the self-fields are given. Section 3 is devoted to the derivation of the small-signal gain. The numerical results and the conclusion are given in Section 4.

2. Single particle orbits

The physical configuration consists of a two-stream relativistic electron beam propagating through a circularly polarized electromagnetic wave wiggler in the presence of the ion-channel guiding. The electromagnetic wave wiggler can be characterized by

$$\begin{aligned}\boldsymbol{B}_{\rm w} &= B_{\rm w}[\hat{\boldsymbol{x}}\ \cos(k_{\rm w}z+\omega_{\rm w}t)\\ &\quad +\hat{\boldsymbol{y}}\ \sin(k_{\rm w}z+\omega_{\rm w}t)], \qquad (1)\\ \boldsymbol{E}_{\rm w} &= -E_{\rm w}[\hat{\boldsymbol{x}}\ \sin(k_{\rm w}z+\omega_{\rm w}t)\\ &\quad -\hat{\boldsymbol{y}}\ \cos(k_{\rm w}z+\omega_{\rm w}t)], \qquad (2)\end{aligned}$$

with

$$E_{\rm w}=\frac{\omega_{\rm w}B_{\rm w}}{ck_{\rm w}}.$$

The transverse electrostatic field generated by the ion channel can be written as

$$\boldsymbol{E}_{\rm i}=2\pi en_i(x\,\hat{\boldsymbol{e}}_x+y\,\hat{\boldsymbol{e}}_y). \qquad (3a)$$

The beams equilibrium self-electric and self-magnetic fields are

$$\boldsymbol{E}_s(\boldsymbol{x})=-2\pi\sum_k n_k e(x\ \hat{\boldsymbol{x}}+y\ \hat{\boldsymbol{y}}), \qquad (3b)$$

$$\boldsymbol{B}_s(\boldsymbol{x})=-2\pi\sum_k n_k e\ \beta_{zk}(-y\ \hat{\boldsymbol{x}}+x\ \hat{\boldsymbol{y}}). \qquad (3c)$$

The small amplitude radiation fields are

$$\delta\boldsymbol{E}=-\omega\,\delta A\,[\hat{\boldsymbol{x}}\ \sin(kz-\omega t)+\hat{\boldsymbol{y}}\ \cos(kz-\omega t)], \quad (4a)$$

$$\delta\boldsymbol{B}=k\delta A\,[\hat{\boldsymbol{x}}\ \cos(kz-\omega t)-\hat{\boldsymbol{y}}\ \sin(kz-\omega t)]. \quad (4b)$$

The relativistic equations of motion for an electron with rest mass m and charge $-e$ moving with velocity $\boldsymbol{v}$ in these combined fields are

$$\frac{{\rm d}\boldsymbol{v}}{{\rm d}t}=-\frac{e}{m\gamma}\left[\boldsymbol{E}-\frac{1}{c^2}-\boldsymbol{v}(\boldsymbol{v}\cdot\boldsymbol{E})+\frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\right], \qquad (5a)$$

$$\frac{{\rm d}\gamma}{{\rm d}t}=-\frac{e}{mc^2}\boldsymbol{v}\cdot\boldsymbol{E}. \qquad (5b)$$

With the aid of the Eqs. (5a) and (5b) and the field Eqs. (1)–(3), the normalized constant transverse velocity of each beam is given by

$$\begin{aligned}\beta_{{\rm w}i}=\hat{\Omega}_{\rm w}(\beta_{z0i}+\beta_{\rm p})^2[&-\hat{\omega}_{\rm p1}^2-\hat{\omega}_{\rm p2}^2+\beta_{z\,0i}(\hat{\omega}_{\rm p1}^2\beta_{z0\,1}\\ &+\hat{\omega}_{\rm p2}^2\beta_{z0\,2})+\hat{\omega}_i^2-\gamma_i(\beta_{z0i}+\beta_{\rm p})^2]^{-1},\\ &i=1,2, \qquad (6a)\end{aligned}$$

where $\hat{\Omega}_{\rm w}=eB_{\rm w}/m_{\rm e}c^2k_{\rm w}$, $\hat{\omega}_i^2=2\pi e^2n_i/mc^2k_{\rm w}^2=\omega_i^2/c^2k_{\rm w}^2$, $\hat{\omega}_{\rm bi}^2=2\pi e^2n_{\rm bi}/mc^2k_{\rm w}^2$, and $\beta_{\rm p}=\omega_{\rm w}/ck_{\rm w}$ denote the normalized wiggler frequency, the normalized ion-channel frequency, the normalized beam frequency and the normalized phase velocity of the EMW, respectively. Since the energy is a constant of motion for the steady-state trajectory, we require that $\beta_{{\rm w}i}$ and β_{zi} are related via

$$\beta_{{\rm w}0i}^2+\beta_{z0i}^2+\gamma_{0i}^{-2}-1=0,\quad i=1,\,2. \qquad (6b)$$

Using Maxwell equations along with Eq. (6), the dispersion relation becomes

$$\begin{aligned}&\beta_{\rm p}^2-1+2\,\hat{\omega}_{\rm p1}^2(\beta_{z\,01}+\beta_{\rm p})^2[-\hat{\omega}_{\rm p1}^2-\hat{\omega}_{\rm p2}^2+\beta_{z0\,1}\\ &\times(\hat{\omega}_{\rm p1}^2\beta_{z0\,1}+\hat{\omega}_{\rm p2}^2\beta_{z\,02})+\hat{\omega}_i^2-\gamma_1(\beta_{z0\,1}+\beta_{\rm p})^2]^{-1}\\ &+2\,\hat{\omega}_{\rm p2}^2(\beta_{z\,2}+\beta_{\rm p})^2[-\hat{\omega}_{\rm p2}^2-\hat{\omega}_{\rm p1}^2+\beta_{z\,02}(\hat{\omega}_{\rm p2}^2\beta_{z\,02}\\ &+\hat{\omega}_{\rm p1}^2\beta_{z0\,1})+\hat{\omega}_i^2-\gamma_2(\beta_{z0\,2}+\beta_{\rm p})^2]^{-1}=0. \qquad (7)\end{aligned}$$

The stability of orbits can be determined by applying small perturbations to the steady-state orbits. To investigate the orbital stability, we assume $\beta_{1i}=\beta_{1i}+\delta\beta_{1i}$, $\beta_{2i}=\delta\beta_{2i}$, $\beta_{zi}=\beta_{z0i}+\delta\beta_{zi}$, $X_{1i}=\delta X_{1i}$, $X_{2i}=X_{{\rm w}0i}+\delta X_{{\rm w}i}$, $\beta_{\rm p}=\beta_{\rm p0}+\delta\beta_{\rm p}$, and $\gamma_{1i}=\gamma_{0i}+\delta\gamma_{1i}$, where $i=1,2$, $\boldsymbol{X}$ represents the transverse displacement. The orbit equations for each of the streams ($i=1,2$) are

$$\frac{\rm d}{{\rm d}t}\delta\beta_{1i}=-B_{1i}\,\delta X_{1i}+A_{1i}\,\delta\beta_{2i}, \qquad (8)$$

$$\begin{aligned}\frac{\rm d}{{\rm d}t}\delta\beta_{2i}=&\ D_{2i}\,\delta X_{2i}+A_{2i}\,\delta\beta_{1i}\\ &+C_{2i}\,\delta\gamma_i+B_{2i}\delta\beta_{zi}, \qquad (9)\end{aligned}$$

$$\frac{\rm d}{{\rm d}t}\delta\beta_{zi}=-B_{3i}\,\delta X_{1i}+A_{3i}\,\delta\beta_{2i}, \qquad (10)$$

$$\frac{{\rm d}\delta\gamma_i}{{\rm d}t}=A_{4i}\,\delta\beta_{2i}+B_{4i}\,\delta X_{1i}, \qquad (11)$$

$$\frac{{\rm d}\delta X_{1i}}{{\rm d}t}=A_{6i}\,\delta\beta_{1i}+B_{6i}\delta\beta_{zi}+C_{6i}\,\delta X_{2i}, \qquad (12)$$

$$\frac{{\rm d}\delta X_{2i}}{{\rm d}t}=A_{5i}\delta\beta_{2i}+B_{5i}\,\delta X_{1i}, \qquad (13)$$

where

$$\begin{aligned}A_{11i}=&\ (\beta_{\rm p}+\beta_{z0i})-\beta_{{\rm w}0i}\,X_{{\rm w}0i}\,\alpha_i\\ &+\frac{\beta_{{\rm w}0i}\,\beta_{\rm p}\,\hat{\Omega}_{\rm w}}{\gamma_i}, \qquad (14a)\end{aligned}$$

$$\alpha_1 = \frac{1}{\gamma_1}(\hat{\omega}_{\rm p1}^2 + \hat{\omega}_{\rm p2}^2 - \hat{\omega}_i^2),$$

$$\alpha_2 = \frac{1}{\gamma_2}(\hat{\omega}_{\rm p2}^2 + \hat{\omega}_{\rm p1}^2 - \hat{\omega}_i^2), \tag{14b}$$

$$B_{1i} = (-\beta_{\rm w0i}^2 + 1)\alpha_i - \beta_{z0i}P_i, \tag{14c}$$

$$P_1 = \frac{1}{\gamma_1}(\hat{\omega}_{\rm p1}^2\,\beta_{z01} + \hat{\omega}_{\rm p2}^2\,\beta_{z02}), \tag{14d}$$

$$P_2 = \frac{1}{\gamma_2}(\hat{\omega}_{\rm p2}^2\,\beta_{z02} + \hat{\omega}_{\rm p1}^2\,\beta_{z01}), \tag{14e}$$

$$A_{2i} = -(\beta_{\rm p} + \beta_{z0i}), \tag{15a}$$

$$B_{2i} = -\beta_{\rm w0i} - \frac{\hat{\Omega}_{\rm w}}{\gamma_i} - P_i X_{\rm w0i}, \tag{15b}$$

$$C_{2i} = \frac{1}{\gamma_{0i}}\left[\frac{1}{\gamma_{0i}}(\hat{\omega}_i^2 - \hat{\omega}_{\rm p1}^2 - \hat{\omega}_{\rm p2}^2)\chi_{{\rm w}i} + \frac{\hat{\Omega}_{\rm w}}{\gamma_{0i}}(\beta_{||i} + \beta_{\rm p0}) + \beta_{z0i}(\hat{\omega}_{\rm p1}^2\,\beta_{z01} + \hat{\omega}_{\rm p2}^2\beta_{z02})\chi_{{\rm w}i}\right], \tag{15c}$$

$$D_{2i} = \alpha_i - \beta_{z0i}P_i, \tag{15d}$$

$$A_{31} = \frac{1}{\gamma_1}[\hat{\Omega}_{\rm w}(1 + \beta_{z01}\beta_{\rm p}) + \hat{\omega}_{\rm p2}^2 X_{\rm w01}(\beta_{z02} - \beta_{z01}) + \hat{\omega}_i^2 X_{\rm w01}\beta_{z01}], \tag{16a}$$

$$A_{32} = \frac{1}{\gamma_2}[\hat{\Omega}_{\rm w}(1 + \beta_{z02}\beta_{\rm p}) + \hat{\omega}_{\rm p1}^2 X_{\rm w02}(\beta_{z01} - \beta_{z02} + \hat{\omega}_i^2 X_{\rm w02}\beta_{z02})], \tag{16b}$$

$$B_{31} = \frac{1}{\gamma_1}[\beta_{\rm w01}\hat{\omega}_{\rm p2}^2(\beta_{z02} - \beta_{z01}) + \beta_{z01}\beta_{\rm w01}\hat{\omega}_i^2], \tag{16c}$$

$$B_{32} = \frac{1}{\gamma_2}[\beta_{\rm w02}\hat{\omega}_{\rm p1}^2(\beta_{z01} - \beta_{z02}) + \beta_{z02}\beta_{\rm w02}\hat{\omega}_i^2], \tag{16d}$$

$$A_{4i} = -\beta_{\rm p}\hat{\Omega}_{\rm w} + \gamma_i\alpha_i X_{\rm w0i}, \tag{17a}$$

$$B_{4i} = \gamma_i\beta_{\rm w0i}\alpha_i, \tag{17b}$$

$$A_{5i} = 1, \quad A_{6i} = 1, \tag{18a}$$

$$B_{5i} = -(\beta_{z0i} + \beta_{\rm p}), \quad B_{6i} = X_{\rm w0i}, \tag{18b}$$

$$C_{6i} = (\beta_{z0i} + \beta_{\rm p}). \tag{18c}$$

Differentiating Eqs. (9) and (12) with respect to time and employing Eqs. (8)–(18), we obtain two homogeneous higher order differential equations as

$$\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega_{1i}^2\right)\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega_{2i}^2\right)\delta v_{2i} = 0, \tag{19a}$$

$$\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega_{1i}^2\right)\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega_{2i}^2\right)\delta X_{1i} = 0, \tag{19b}$$

where

$$\Omega_{1i}^2 = \frac{(\omega_{1i}^2 + \omega_{2i}^2) - [(\omega_{1i}^2 - \omega_{2i}^2)^2 + 4N_iM_i]^{1/2}}{2}, \tag{20a}$$

$$\Omega_{2i}^2 = \frac{(\omega_{1i}^2 + \omega_{2i}^2) + [(\omega_{1i}^2 - \omega_{2i}^2)^2 + 4N_iM_i]^{1/2}}{2}, \tag{20b}$$

with

$$\omega_{1i}^2 = -(A_{2i}A_{1i} + B_{2i}A_{3i} + D_{2i}A_{5i} + A_{4i}C_{2i}), \tag{21a}$$

$$\omega_{2i}^2 = -(A_{6i}B_{1i} + C_{6i}B_{5i} + B_{6i}B_{3i}), \tag{21b}$$

$$N_i = A_{6i}A_{1i} + B_{6i}A_{3i} + A_{5i}C_{6i}, \tag{21c}$$

$$M_i = A_{2i}B_{1i} + C_{2i}B_{4i} + D_{2i}B_{5i} + B_{2i}B_{3i}. \tag{21d}$$

Let us assume that all the variables oscillate with the same frequency ω, e.g.,

$$\delta\beta_{1i} = A\exp({\rm i}\,\omega t). \tag{22}$$

Using the above assumption, equation (19) becomes

$$\bar{\omega}^2 - (\Omega_{1i}^2 + \Omega_{2i}^2)\bar{\omega} - \Omega_{1i}^2\Omega_{2i}^2 = 0, \tag{23}$$

where $\bar{\omega} = \omega^2$. This equation has two roots for $\bar{\omega}$ ($\bar{\omega}_1 = \Omega_{1i}^2$ and $\bar{\omega}_2 = \Omega_{2i}^2$). The stability of the steady-state two-stream electron trajectory requires that both roots, $\bar{\omega}_1$ and $\bar{\omega}_2$, are real and positive.

3. Small-signal gain

In this section we wish to study the small-signal gain. Substituting Eq. (4) into Eqs. (8)–(19) and then through some straight forward algebra, we obtain

$$\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega_{1i}^2\right)\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega_{2i}^2\right)\delta v_{2i} = (K_i\omega_{2i}^2 + M_iL_i)\frac{\delta A}{\gamma_i m_{\rm e}c^2}\sin\psi_i, \tag{24a}$$

$$\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega_{1i}^2\right)\left(\frac{{\rm d}^2}{{\rm d}t^2} + \Omega_{2i}^2\right)\delta X_{1i} = (L_i\,\omega_{1i}^2 + N_i\,K_i)\frac{\delta A}{\gamma_i m_{\rm e}c^2}\sin\psi_i, \tag{24b}$$

$$\frac{\rm d}{{\rm d}t}\delta\beta_{3i} = -B_{3i}\,\delta X_{1i} + A_{3i}\,\delta\beta_{2i} + (k - \omega\beta_{z0i})\beta_{\rm w0i}\frac{\delta A}{\gamma_i m_{\rm e}c^2}\sin\psi_i, \tag{24c}$$

where

$$\psi_i = [(k + k_{\rm w})\beta_{z0i} - (\omega - \omega_{\rm w})]t,$$

$$K_i = A_{2i}(\omega - k\beta_{z0i} - \omega\,\beta_{\rm w0i}^2) + B_{2i}\beta_{\rm w0i}(k - \omega\beta_{z0i}) + C_{2i}\omega\gamma_i\beta_{\rm w0i}, \tag{25}$$

$$L_i = A_{6i}(\omega - k\beta_{z0i} - \omega\beta_{\rm w0i}^2) + B_{6i}\beta_{\rm w0i}(k - \omega\beta_{\rm w0i}). \tag{26}$$

With the aid of Eqs. (24a) and (24b), equation (24c) takes the following form:

$$\frac{{\rm d}\delta\beta_{3i}}{{\rm d}t} = \frac{\phi_i}{\Omega_{1i}^2\,\Omega_{2i}^2}\,\frac{\delta A}{\gamma_i m_{\rm e}c^2}\sin\psi_i, \tag{27}$$

where

$$\phi_i = A_{3i}(K_i\omega_{2i}^2 + L_iM_i) + B_{3i}(L\omega_{i1}^2 + N_iK_i)$$

$$+ (k - \omega\beta_{\mathrm{w}0i})\beta_{\mathrm{w}0i}(\omega_{i1}^2\,\omega_{2i}^2 - M_i N_i). \quad (28)$$

The combination of Maxwell equations with Eq. (27) yields

$$2k\frac{\mathrm{d}}{\mathrm{d}z}\delta a = -2\sum_{i=1}^{2}\hat{\omega}_{\mathrm{b}i}^2\,\beta_{\mathrm{w}0i}\langle\sin\psi_i\rangle, \quad (29)$$

where

$$\delta a = \frac{e\delta A}{m_{\mathrm{e}}c},$$

$$\langle\sin\psi_i\rangle = -\frac{(\omega-\omega_{\mathrm{w}})\,\phi_i}{2\,\Omega_{1i}^2\,\Omega_{2i}^2\beta_{z0i}^3\,\gamma_i\,\Delta k_i^2} \times [\sin(\Delta k_i z) - \Delta k_i z\cos(\Delta k_i z)], \quad (30)$$

with

$$\Delta k_i = k + 1 - \frac{\omega-\beta_{\mathrm{p}}}{\beta_{z0i}}.$$

Through the definition of the gain over some length L,

$$G_L \equiv \frac{\delta a(z=L) - \delta a(z=0)}{\delta a(z=0)}, \quad (31)$$

and employing Eqs. (29) and (30), we obtain

$$G_L = -\frac{L^3\hat{\omega}_{b1}^2\beta_{\mathrm{w}01}(1-\omega_{\mathrm{w}}/\omega)\phi_1}{8\Omega_{11}^2\Omega_{21}^2\beta_{z01}^2\gamma_1}\frac{\mathrm{d}}{\mathrm{d}\theta_1}\left(\frac{\sin\theta_1}{\theta_1}\right)^2 - \frac{L^3\hat{\omega}_{b2}^2\beta_{\mathrm{w}02}(1-\omega_{\mathrm{w}}/\omega)\phi_2}{8\,\Omega_{12}^2\Omega_{22}^2\beta_{z02}^2\gamma_2}\frac{\mathrm{d}}{\mathrm{d}\theta_2}\left(\frac{\sin\theta_2}{\theta_2}\right)^2, \quad (32)$$

where

$$\theta_{1,2} = \frac{\Delta k_{1,2}\,L}{2}.$$

4. Numerical results and conclusion

A numerical study of the steady-state relativistic two-stream electron trajectories in the presence of self-fields, EMW and ion-channel guiding has been taken. The typical dependence of the axial velocity on $\bar{\hat{\omega}}_i = \hat{\omega}_i/\gamma_1^{1/2}$ is obtained by the solution of Eqs. (6) and (7). The electron energy factor was taken to be $\gamma_1 = 4$ and$\gamma_2 = 4.1$, the normalized magnetic wiggler and the plasma frequencies of the two beams were chosen to be $\hat{\Omega}_{\mathrm{w}} = 0.2$, $\hat{\omega}_{\mathrm{p}1} = \hat{\omega}_{\mathrm{p}2} = 0.212$, respectively. Figures 1(a) and 1(b) show the plot of the normalized axial velocities of the two beams, β_{z1} and β_{z2}, as a function of the normalized ion-channel frequency ($\bar{\hat{\omega}}_i = \hat{\omega}_i/\gamma_1^{1/2}$). It is evident from these figures that the electron trajectories in the presence of self-fields have seven distinguished group orbits, which are similar to those reported in the absence of the self-fields.[14] The normalized electromagnetic wiggler frequency, $\beta_{\mathrm{p}} = \omega_{\mathrm{w}}/ck_{\mathrm{w}}$, is shown in Fig. 2 as a function of the normalized ion-channel frequency $\bar{\hat{\omega}}_i$. To investigate the stability of the steady-state two-stream electron trajectory, we find the roots of Eq. (23), namely $\bar{\omega}_1$ and $\bar{\omega}_2$, through a numerical analysis (Figs. 3 and 4). As shown in these figures, since both roots are real and positive for groups 1 and 2,

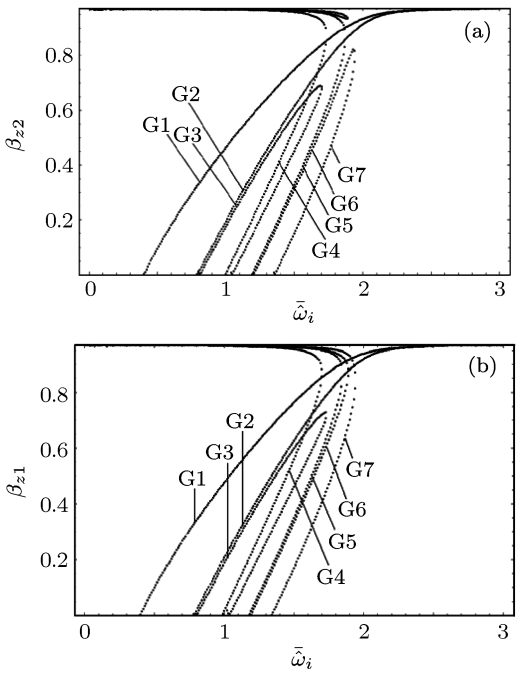


Fig. 1. (a) The normalized axial velocity (β_{z1}) of the steady-state orbits as a function of the normalized cyclotron frequency ($\bar{\hat{\omega}}_i$) in the case of two streams. (b) The normalized axial velocity (β_{z2}) of the steady-state orbits as a function of the normalized cyclotron frequency ($\bar{\hat{\omega}}_i$) in the case of two streams. In this figure and all following figures, the Gi denotes the group i (i = 1, 2, ..., 7) and the chosen parameters are: $\hat{\Omega}_{\mathrm{w}} = 0.2$, $\hat{\omega}_{\mathrm{p}1} = 0.212$, $\hat{\omega}_{\mathrm{p}2} = 0.212$.

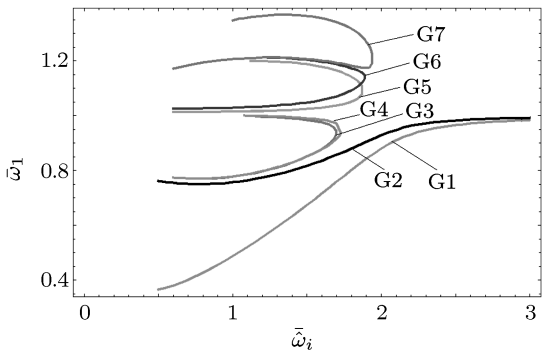


Fig. 2. The normalized wiggler phase velocity ($\hat{\beta}_{\mathrm{p}}$) as a function of the normalized cyclotron frequency ($\bar{\hat{\omega}}_i$) in the case of two streams.

these groups are completely stable. The obtained results for groups 4–7 indicate that these groups

are not completely stable. With the aid of Figs. 3 and 4, it can be seen that these groups are stable except for the following regions: group 4, $\hat{\omega}_i < 1.7$ and $\beta_z < 0.882$; group 5, $\hat{\omega}_i < 1.84$ and $\beta_z < 0.901$; group 6, $\hat{\omega}_i < 1.89$ and $\beta_z < 0.858$ and group 7, $\hat{\omega}_i < 1.94$ and $\beta_z < 0.872$.

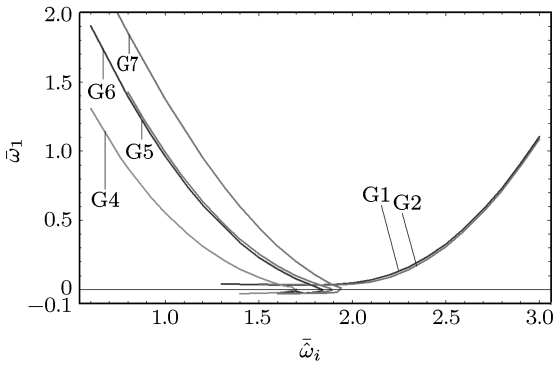


Fig. 3. The $\bar{\omega}_1$ versus $\bar{\bar{\omega}}_i$ corresponding to the group 1–7 orbits.

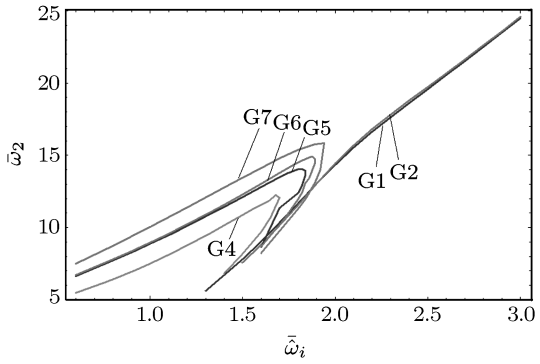


Fig. 4. The $\bar{\omega}_2$ versus $\bar{\bar{\omega}}_i$ corresponding to the groups 1–7 orbits.

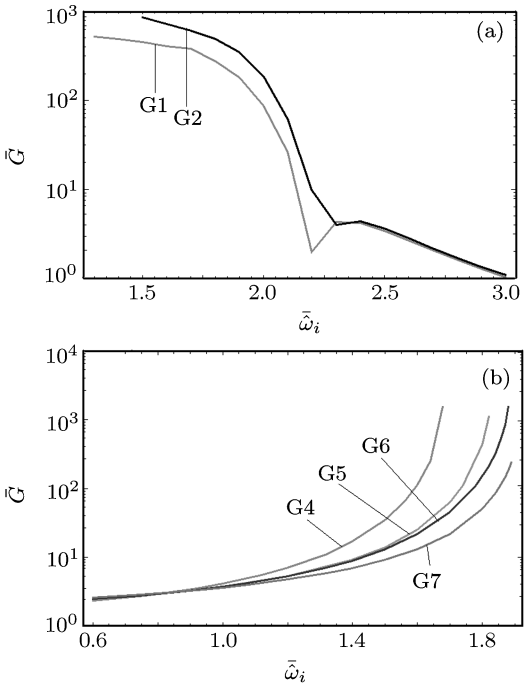


Fig. 5. The normalized maximum gain ($\bar{G}$) versus the ion-channel frequency $\bar{\bar{\omega}}_i$ (logarithmic scale): (a) for groups 1 and 2, (b) for groups 4–7.

The normalized maximum gain, $\bar{G} \equiv G_{\max}/G_0$ (the maximum gain normalized by the maximum gain of the magnetostatic wiggler without the ion-channel guiding and the self-fields), as a function of the normalized ion-channel frequency has been shown in Figs. 5(a) and 5(b). Figure 5(a) shows that the gains of groups 1 and 2 are similar and decrease with the increasing $\bar{\bar{\omega}}_i$ until a minimum is reached. A further increase in frequency leads to an asymptotic behaviour. The behaviour for the gains of both groups in the region $1.4 \leq \bar{\bar{\omega}}_i \leq 2.2$ is that $\bar{G}_2 > \bar{G}_1$ ($\bar{G}_i$ indicates the normalized maximum gain for group i).

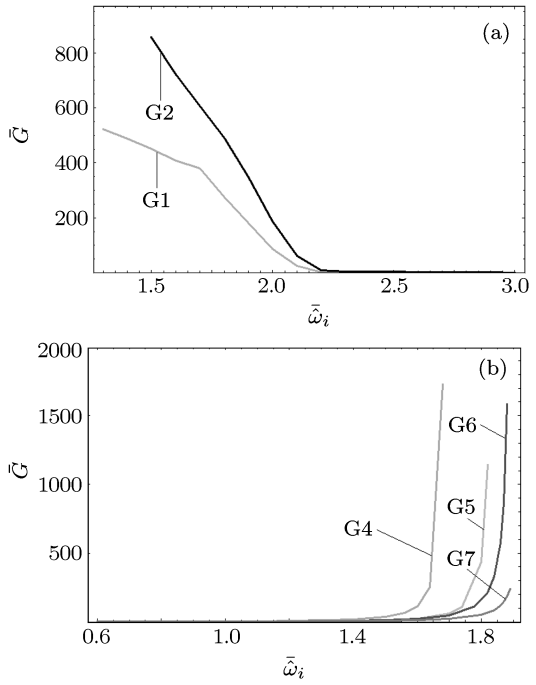


Fig. 6. The normalized maximum gain ($\bar{G}$) versus normalized ion channel frequency $\bar{\bar{\omega}}_i$ (linear scale): (a) for groups 1 and 2, (b) for groups 4–7.

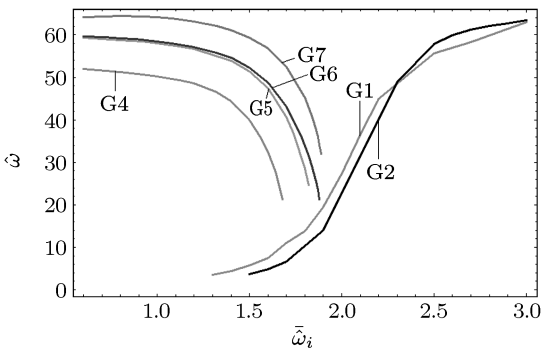


Fig. 7. The normalized maximum frequency $\hat{\omega}$ versus normalized ion channel frequency ($\bar{\bar{\omega}}_i$) for groups 1–7.

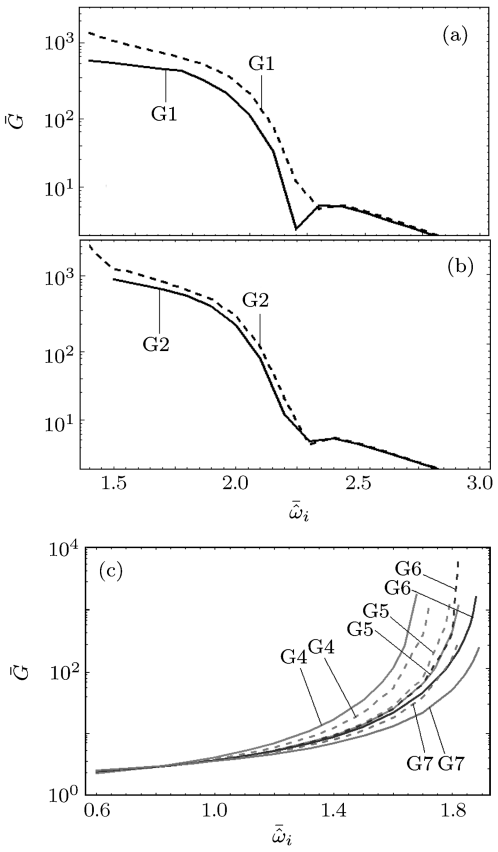


Fig. 8. The normalized maximum gain $\bar{G}$ of two streams with (solid line) and without self-fields (dashed line) as a function of the normalized ion channel frequency $\bar{\omega}_i$ (logarithmic scale): (a) for group 1, (b) for group 2, (c) for groups 4–7.

In contrary to Fig. 5(a), the normalized gains of groups 4) presented in Fig. 5(b) follow an increasing trend with the increasing $\bar{\omega}_i$ until they reach their asymptotic regions. The order of these normalized gains is $\bar{G}_4 > \bar{G}_5 > \bar{G}_6 > \bar{G}_7$ in the range of $0.6 \leq \bar{\Omega}_0 \leq 1.9$. The normalized gains for groups 1 and 2 and groups 4–7 are shown in linear scale in Figs. 6(a) and 6(b), respectively. The plot of the normalized frequency $\hat{\omega}$ corresponding to the maximum gain as a function of $\bar{\omega}_i$ is shown in Fig. 7. As shown in Fig. 7, the order of the normalized frequency is in reverse order of that of the normalized maximum gain and is given as $\hat{\omega}_1 > \hat{\omega}_2$ for the region $1.4 \leq \bar{\omega}_i \leq 2.2$ ($\hat{\omega}_i$ indicates the normalized maximum frequency for group i). The order of the normalized frequency for groups 4–7 is $\hat{\omega}_7 > \hat{\omega}_6 > \hat{\omega}_5 > \hat{\omega}_4$.

The comparison between the normalized maximum gains for the two streams with and without the self-fields is given in Figs. 8(a) and 8(b). The results show that the self-fields cause the decrease of the maximum gain with the exception of group 4. The comparison between the normalized maximum gains for the two streams with and without the self-fields is shown in linear scale in Fig. 9(a) and 9(b), respectively.

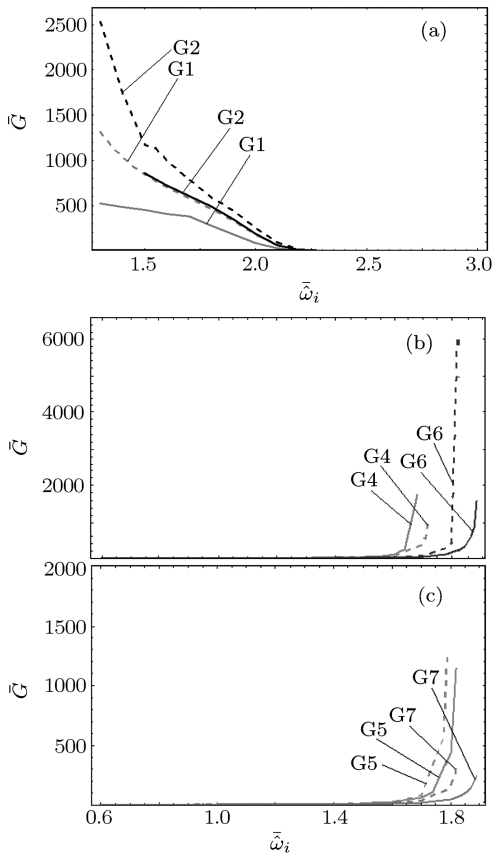


Fig. 9. The normalized maximum gain $\bar{G}$ of two streams with (solid line) and without self-fields (dashed line) as a function of the normalized ion channel frequency $\bar{\omega}_i$ (linear scale): (a) for groups 1 and 2, (b) for groups 4 and 6 and (c) for groups 5 and 7.

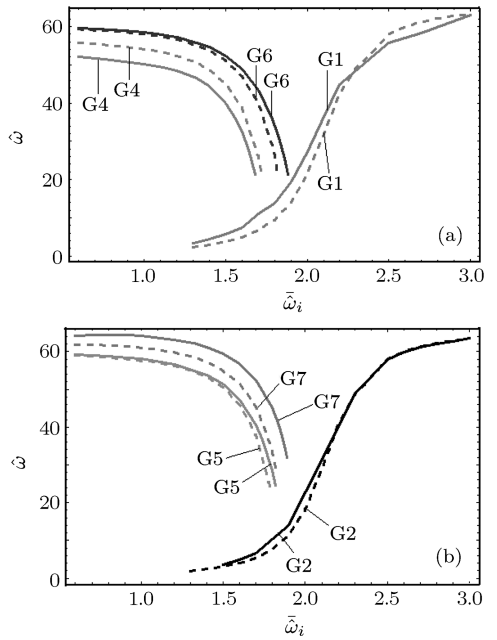


Fig. 10. The normalized maximum frequency $\hat{\omega}$ of two streams with (solid line) and without self-fields (dashed line) as a function of the normalized ion channel frequency ($\bar{\omega}_i$): (a) for groups 1, 4 and 6, (b) for groups 2, 5 and 7.

The comparison between the normalized frequency $\hat{\omega}$ corresponding to the maximum gain for the two streams with and without the self-fields is given in Fig. 10. As shown in this figure, in contrast to the maximum gain, the frequency in similar groups in the presence of the self-fields is greater than that with out the self-fields, except for group 4.

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