Landau damping of collective mode in a quasi-two-dimensional repulsive Bose–Einstein condensate*

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We investigate the Landau damping of the collective mode in a quasi-two-dimension repulsive Bose–Einstein condensate by using the self-consistent time-dependent Hatree–Fock–Bogoliubov approximation and a complete and orthogonal eigenfunction set for the elementary excitation of the system. We calculate the three-mode coupling matrix element between the collective mode and the thermal excited quasi-particles and the Landau damping rate of the collective mode. We discuss the dependence of the Landau damping on temperature, on atom number in the condensate, on transverse trapping frequency and on the length of the condensate. The energy width of the collective mode is taken into account in our calculation. With little approximation, our theoretic calculation results agree well with the experimental ones and are helpful for deducing the damping mechanics and the inter-particle interaction.

Keywords: Bose–Einstein condensate, Landau damping, Hatree–Fock–Bogoliubov approximation, energy level width

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1. Introduction

Elementary excitation of Bose–Einstein condensate (BEC) in a magnetic trap is a good subject for studying the physics of many-body system.[1−16] The damping, which is a main character of the elementary excitation, is the most basic phenomenon of the trapped BEC and is widely observed in experiments.[17−24] The precise calculation of the damping may be used to explain many experimental phenomena, to test and to develop the quantum many-body theory for condensed matter; the precise measurement of the damping can be inversely used to deduce the inter-particle interaction and the superfluid property of the system.

Although there are a lot of studies on the Landau damping of the collective excitations in the trapped BEC, which are conducted in many famous laboratories and theoretical groups (the Beliaev damping can be ignored for low-energy collective modes in a trapped Bose gas because of the discretization of the energy levels), the damping of the condensate excitation is not yet very well understood theoretically.

In the collision-less region, which takes place at low temperature and low density, the Bogoliubov approach of mean field theory, which demonstrates the condensate collective excitation at $T = 0$, can be well generalized to finite temperature to study the interaction between the collective excitation and the thermal excited quasi-particle.[25−39]

To calculate the damping, we need to solve the Gross–Piteaviskii (GP) equation and the Bogoliubov–de Gennes (BdG) equation. The analytic solution of BdG equation for three dimension condensation in a harmonic trap is obtained by using the Thomas–Fermi approximation (TFA).[1] The TFA neglects the quantum fluctuations of the condensate, which are not an important aspect of the elementary excitation in the case of a large number repulsive atoms in condensation at low temperature. In addition, the eigen modes of the elementary excitation are not complete and their orthogonality have never been proved in the TFA. Another serious deficit of the TFA is the uncontrollable divergency in the calculation of the three-mode coupling matrix element, which is important for calculating the damping. So nearly all works on Landau

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damping of the three dimension condensate in a magnetic trap are base on numerical simulations.[25−36] References [37 − 43] put forward a method beyond the TFA to analytically calculate the three-mode coupling matrix element and the Landau damping. The new method brings in more approximation by introducing a variational parameter q.

In contrast, the quasi-two-dimension condensate is uniform in the transverse direction, the eigenfunction set of the BdG equation provided in Ref. [1] is complete and orthogonal. There are no more approximations in the study of the damping of the collective mode in the low dimension condensate. Therefore we investigate the Landau damping of the collective modes in a quasi-two-dimension repulsive Bose–Einstein condensate in search of a credible analytical method.

2. The Hatree–Fock–Bogoliubov mean field theory for Landau damping

We adopt the Hartree–Fock–Bogoliubov (HFB) mean field approximation,[25] which is beyond the Popov approximation,[1] and the complete and orthogonal eigenfunction set of the BdG equation, which was deduced in Ref. [1] for the quasi-two-dimension repulsive condensate. We take a brief review in Sections 2.1 and 2.2. In Section 2.3, we introduce our calculation method for the Landau damping rate.

2.1. Eigenfunction set of BdG equation

The grand-canonical Hamiltonian of a dilute Bose-condensed atomic gas trapped in an anisotropic external potential $V_{\rm ext}(\boldsymbol{r})$ in terms of the boson field operator $\psi(\boldsymbol{r},t)$ reads

$$K = H - \mu N = \int {\rm d}\boldsymbol{r}\psi^{\dagger}(\boldsymbol{r},t)\Big[- \frac{\hbar^2\nabla^2}{2m} + V_{\rm ext}(\boldsymbol{r}) - \mu + \frac{g}{2}\psi^{\dagger}(\boldsymbol{r},t)\psi(\boldsymbol{r},t)\Big]\psi(\boldsymbol{r},t), \quad (1)$$

where μ is the chemical potential and m is the atomic mass. The parameter $g = 4\pi\hbar^2 a_{\rm sc}/m$ describes a contact interaction, with $a_{\rm sc}$ being the s-wave scattering length.

We consider a quasi-two-dimension repulsive Bose-atomic gas condensed in a trap with tight axial confinement and with negligible trapping potential in the transverse direction. The trapping potential is written as $V_{\rm ext} = \frac{1}{2}m\omega_{\rm ho}^2 z^2$, where $\omega_{\rm ho}$ is the axial harmonic frequency.

By taking $\psi(\boldsymbol{r},t) = \psi(x,y,t)\psi(z)$, where $\psi(x,y,t)$ and $\psi(z)$ are the wavefunctions in the transverse and in the axial direction of the system, respectively, we separate the Hamiltonian (1) into two parts $K_{\rm tr}$, $K_{\rm ax}$, $K = K_{\rm tr} + K_{\rm ax}$, where

$$K_{\rm ax} = N_0 \int {\rm d}z\psi^{\dagger}(z)\Big(- \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \frac{1}{2}m\omega_{\rm ho}^2 z^2\Big)\psi(z) \quad (2)$$

is the Hamiltonian in the axial direction of the system and

$$K_{\rm tr} = \int {\rm d}x{\rm d}y\psi^{\dagger}(x,y,t)\Bigg\{\Big[- \frac{\hbar^2}{2m}\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\Big) - \mu\Big] \times\Big[\int {\rm d}z\psi^{\dagger}(z)\psi(z)\Big] + \frac{g}{2}\psi^{\dagger}(x,y,t)\psi(x,y,t) \times\Big[\int {\rm d}z\psi^{\dagger}(z)\psi^{\dagger}(z)\psi(z)\psi(z)\Big]\Bigg\}\psi(x,y,t) \quad (3)$$

is the Hamiltonian in the transverse direction of the system. The normalization of the transverse wavefunction is $\int {\rm d}x{\rm d}y\psi^{\dagger}(x,y,t)\psi(x,y,t) = N_0$, where N_0 is the atomic number.

According to Hamiltonian (2), we get the axial direction ground state wavefunction

$$\psi(z) = \Big(\frac{1}{\sqrt{\pi}a_{\rm ho}}\Big)^{1/2}{\rm e}^{-z^2/2a_{\rm ho}^2}$$

with $a_{\rm ho} = \sqrt{\hbar/m\omega_{\rm ho}}$ being the radius of the Gauss axial distribution of the condensate. Taking into account the interatomic interactions, the ground state wavefunction in the axial direction should be modified to be $\psi(z) = (1/\sqrt{\pi}a_{\perp})^{1/2}{\rm e}^{-z^2/2a_{\perp}^2}$, where the axial semi-axes of the condensate

$$a_{\perp} = a_{\rm ho}\Big(\frac{\sqrt{2\pi}N_0 a_{\rm sc}a_{\rm ho}}{L^2}\Big)^{1/3}$$

is obtained by equating the potential energy $\frac{1}{2}m\omega_{no}^2 a_{\perp}^2$ with the interaction energy $gN_0/(4\sqrt{2\pi}a_{\perp}L^2)$ of a particle, with L being the length of the condensate. The $\int {\rm d}z\psi^{\dagger}(z)\psi^{\dagger}(z)\psi(z)\psi(z) = 1/\sqrt{2\pi}a_{\perp}$ is the average axial particle probability density. The $\sqrt{2\pi}a_{\perp}$ is the axial equivalent radius of the condensate.

Then the Hamiltonian in the transverse direction of the system can be rewritten as

$$K_{\rm tr} = \int {\rm d}x{\rm d}y\psi^{\dagger}(x,y,t)\Big[- \frac{\hbar^2}{2m}\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\Big) - \mu + \frac{g'}{2}\psi^{\dagger}(x,y,t)\psi(x,y,t)\Big]\psi(x,y,t), \quad (4)$$

with $g' = (2\sqrt{2\pi}\hbar^2 a_{\rm sc})/ma_{\rm ho}$. The Heisenberg equation of motion for $\psi(x,y,t)$ can be written as

$$\begin{aligned}\mathrm{i}\hbar\frac{\partial\psi(x,y,t)}{\partial t} &= [\psi(x,y,t), K_{\rm tr}]\\ &= \Big[-\frac{\hbar^2}{2m}\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\Big)-\mu\\ &\quad + g'\psi^\dagger(x,y,t)\psi(x,y,t)\Big]\psi(x,y,t). \quad (5)\end{aligned}$$

We decomposed the field operator into a condensed component $\phi = \langle\psi\rangle$ and a non-condensed component $\widetilde{\psi}$, $\psi(x,y,t) = \phi(x,y,t) + \widetilde{\psi}(x,y,t)$ and $\langle\widetilde{\psi}\rangle = 0$. The condensate is also decomposed into a stationary component $\phi_0 = \langle\psi\rangle_0$ and an excited component $\delta\phi$, $\phi(x,y,t) = \phi_0(x,y) + \delta\phi(x,y,t)$, where the symbol $\langle\cdots\rangle$ ($\langle\cdots\rangle_0$) represents nonequilibrium (equilibrium) average.

We take the HFB approximation:

(i)

$$\langle\widetilde{\psi}^\dagger(x,y,t)\widetilde{\psi}(x,y,t)\rangle = \widetilde{n}(x,y,t)$$

and

$$\langle\widetilde{\psi}(x,y,t)\widetilde{\psi}(x,y,t)\rangle = \widetilde{m}(x,y,t),$$

where $\widetilde{m}$ and $\widetilde{n}$ denote the normal and the anomalous thermal particle densities, respectively;

(ii)

$$\begin{aligned}&\langle\widetilde{\psi}^\dagger(x,y,t)\widetilde{\psi}(x,y,t)\widetilde{\psi}(x,y,t)\rangle\\ &= \langle\widetilde{\psi}^\dagger(x,y,t)\widetilde{\psi}^\dagger(x,y,t)\widetilde{\psi}(x,y,t)\rangle = 0;\end{aligned}$$

(iii)

$$\begin{aligned}&\langle\widetilde{\psi}^\dagger(x,y,t)\widetilde{\psi}^\dagger(x,y,t)\widetilde{\psi}(x,y,t)\widetilde{\psi}(x,y,t)\rangle\\ &= 4\widetilde{n}(z,t)\widetilde{\psi}^\dagger(z,t)\widetilde{\psi}(z,t) + \widetilde{m}(x,y,t)\widetilde{\psi}^\dagger(x,y,t)\\ &\quad\times\widetilde{\psi}^\dagger(x,y,t) + \widetilde{m}^*(x,y,t)\widetilde{\psi}(x,y,t)\widetilde{\psi}(x,y,t);\end{aligned}$$

(iv)

$$\widetilde{n}(x,y,t) = \widetilde{n}^0(x,y) + \delta\widetilde{n}(x,y,t)$$

and

$$\widetilde{m}(x,y,t) = \widetilde{m}^0(x,y) + \delta\widetilde{m}(x,y,t),$$

where

$$\widetilde{n}^0(x,y) = \langle\widetilde{\psi}^\dagger(x,y,t)\widetilde{\psi}(x,y,t)\rangle_0$$

and

$$\widetilde{m}^0(x,y) = \langle\widetilde{\psi}(x,y,t)\widetilde{\psi}(x,y,t)\rangle_0;$$

(v) $\widetilde{n}^0(x,y) = 0$ and $\widetilde{m}^0(x,y) = 0$.

Using the above approximation, we get the equation of motion for the condensate wavefunction

$$\begin{aligned}&\mathrm{i}\hbar\frac{\partial\phi(x,y,t)}{\partial t}\\ &= -\frac{\hbar^2}{2m}\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\Big)\phi(x,y,t) - \mu\phi(x,y,t)\\ &\quad + g'|\phi(x,y,t)|^2\phi(x,y,t) + 2g'\phi(x,y,t)\delta\widetilde{n}(x,y,t)\\ &\quad + g'\phi(x,y,t)^*\delta\widetilde{m}(x,y,t). \quad (6)\end{aligned}$$

Neglecting $\delta\widetilde{n}$ and $\delta\widetilde{m}$ in Eq. (6), we get the GP equation for the stationary distribution of the condensate

$$\begin{aligned}&\Big[-\frac{\hbar^2}{2m}\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\Big)-\mu\\ &\quad + g'\phi_0^2(x,y)\Big]\phi_0(x,y) = 0. \quad (7)\end{aligned}$$

The solution of Eq. (7) is the groundstate wavefunction $\phi_0 = \sqrt{N_0}/L$ with the chemical potential $\mu = g'\phi_0^2$ and the density of condensate $n_0 = N_0/L^2$.

Taking the Bogoliubov transformation

$$\widetilde{\psi}(x,y,t) = \sum_k [u_k(x,y)\alpha_k^\dagger(t) + v_k^*(x,y)\alpha_k(t)]$$

with the normalization

$$\int \mathrm{d}x\,\mathrm{d}y[u_k^*(x,y)u_{k'}(x,y) - v_k(x,y)v_{k'}^*(x,y)] = \delta_{kk'},$$

where α_k is the Bose operator satisfying $[\alpha_k^\dagger, \alpha_{k'}] = \delta_{kk'}$, Hamiltonian (4) can be diagonalized as

$$K_{\rm tr} = K_0 + K_2^{(1)} - \sum_j \epsilon_j \int \mathrm{d}x\,\mathrm{d}y|v_j|^2 + \sum_j \epsilon_j\alpha_j^\dagger\alpha_j,$$

with

$$K_0 = \int \mathrm{d}x\,\mathrm{d}y\phi^*\left[-\frac{\hbar^2}{2m}(\partial^2/\partial x^2 + \partial^2/\partial y^2) + g'n_0\right]\phi,$$

and

$$\begin{aligned}K_2^{(1)} &= \sum_{jj'}\int \mathrm{d}x\,\mathrm{d}y g'\phi_0\{[2(\delta\phi+\delta\phi^*)u_j^*u_{j'}\\ &\quad + \delta\phi u_j^*v_{j'} + \delta\phi^*u_{j'}v_j^*]\alpha_j^\dagger\alpha_{j'}\\ &\quad + [2(\delta\phi+\delta\phi^*)v_jv_{j'}^* + \delta\phi v_ju_{j'}^*\\ &\quad + \delta\phi^*v_{j'}^*u_j]\alpha_j\alpha_{j'}^\dagger\\ &\quad + [2(\delta\phi+\delta\phi^*)u_j^*v_{j'}^* + \delta\phi u_j^*u_{j'}^*\\ &\quad + \delta\phi^*v_j^*v_{j'}^*]\alpha_j^\dagger\alpha_{j'}^\dagger\\ &\quad + [2(\delta\phi+\delta\phi^*)v_ju_{j'} + \delta\phi v_jv_{j'}\\ &\quad + \delta\phi^*u_ju_{j'}]\alpha_j\alpha_{j'}\}.\end{aligned}$$

We have used the BdG equations

$$\begin{aligned}\mathscr{L}u_k(x,y) + g'n_0v_k(x,y) &= \epsilon_k u_k(x,y),\\ \mathscr{L}v_k(x,y) + g'n_0u_k(x,y) &= -\epsilon_k v_k(x,y), \quad (8)\end{aligned}$$

where $\mathscr{L} = -\frac{\hbar^2}{2m}(\partial^2/\partial x^2 + \partial^2/\partial y^2) + g'n_0$.

Searching for a solution of the form $u_k(z) = u_k\,\mathrm{e}^{\mathrm{i}(k_x x+k_y y)}/L$ and $v_k(z) = v_k\,\mathrm{e}^{\mathrm{i}(k_x x+k_y y)}/L$ with the elementary excitation spectrum

$$\epsilon_k = \sqrt{\frac{\hbar^2k^2}{2m}\left(\frac{\hbar^2k^2}{2m}+g'\frac{N_0}{L^2}\right)}, \tag{9}$$

Eq. (8) becomes

$$\begin{aligned}&\left(\frac{\hbar^2k^2}{2m}+g'\frac{N_0}{L^2}-\epsilon_k\right)u_k+g'\frac{N_0}{L^2}v_k=0,\\ &\left(\frac{\hbar^2k^2}{2m}+g'\frac{N_0}{L^2}+\epsilon_k\right)v_k+g'\frac{N_0}{L^2}u_k=0.\end{aligned} \tag{10}$$

We obtain the following complete and orthogonal quasiparticle wavefunction

$$\begin{aligned}u_k(x,y)=&\frac{1}{L}\sqrt{\frac{1}{2}\left(\frac{1+\dfrac{g''N_0}{k^2L^2}}{\sqrt{1+\dfrac{2g''N_0}{k^2L^2}}}+1\right)}\,\mathrm{e}^{\mathrm{i}(k_x x+k_y y)},\\ v_k(x,y)=&\frac{1}{L}\sqrt{\frac{1}{2}\left(\frac{1+\dfrac{g''N_0}{k^2L^2}}{\sqrt{1+\dfrac{2g''N_0}{k^2L^2}}}-1\right)}\,\mathrm{e}^{\mathrm{i}(k_x x+k_y y)},\end{aligned} \tag{11}$$

and rewrite the eigenvalue in Eq. (9) as

$$\epsilon_k = \frac{\hbar^2}{2m}\sqrt{k^2\left(k^2+2g''\frac{N_0}{L^2}\right)}, \tag{12}$$

with $g'' = 4\sqrt{2\pi}a_{\mathrm{sc}}/a_{\mathrm{ho}}$.

From Eq. (6) we can also get the equation of motion

$$\begin{aligned}&\mathrm{i}\,\hbar\frac{\partial\delta\phi(x,y,t)}{\partial t}\\ =&\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial x^2}\right)+g'n_0\right]\\ &\times\delta\phi(x,y,t)+g'n_0\delta\phi^*(x,y,t)\\ &+2g'\phi_0\delta\widetilde{n}(x,y,t)+g'\phi_0^*\delta\widetilde{m}(x,y,t)\end{aligned} \tag{13}$$

for the excitation of the condensate.

Neglecting $\delta\widetilde{n}$ and $\delta\widetilde{m}$ in Eq. (13), it is easy to see that the collective excitation in the condensate $\delta\phi(x,y,t) = u_{k_{\mathrm{cl}}}(x,y)\exp(-\mathrm{i}\,\epsilon_{k_{\mathrm{cl}}}t)$ and $\delta\phi^*(x,y,t) = v_{k_{\mathrm{cl}}}(x,y)\exp(-\mathrm{i}\,\epsilon_{k_{\mathrm{cl}}}t)$ are also determined by the BdG equations, namely $u_{k_{\mathrm{cl}}}(x,y)$ and $v_{k_{\mathrm{cl}}}(x,y)$ obey Eq. (8).

2.2. coupling matrix elements and energy correction of the collective mode

With the Bogoliubov transformation, we have

$$\delta\widetilde{n} = \sum_{ij}[(u_i^*u_j+v_i^*v_j)f_{ij}+u_i^*v_j^*g_{ij}^*+u_iv_jg_{ij}],$$

$$\delta\widetilde{m} = \sum_{ij}(2u_iv_j^*f_{ij}+u_i^*v_j^*g_{ij}^*+u_iv_jg_{ij}),$$

where $f_{ij} = \langle\alpha_i^\dagger\alpha_j\rangle - f_i^0$ and $g_{ij} = \langle\alpha_i\alpha_j\rangle$ are the normal and the anomalous quasiparticle distribution functions, respectively, $f_j^0 = \langle\alpha_j^\dagger\alpha_j\rangle_0 = 1/[\exp(\epsilon_j/k_{\mathrm{B}}T)-1]$ is the equilibrium density of the quasiparticles, T is the temperature and k_{B} is the Boltzman constant. So the time evolutions of $\delta\widehat{n}$ and $\delta\widehat{m}$ are reflected by the following equations of motion

$$\begin{aligned}&\mathrm{i}\,\hbar\frac{\partial f_{ij}(t)}{\partial t}\\ =&\langle[\alpha_i^\dagger\alpha_j,K_{\mathrm{tr}}]\rangle\\ =&(\epsilon_j-\epsilon_i)f_{ij}(t)+2g'[f_i^0(t)-f_j^0(t)]\\ &\times\int\mathrm{d}x\,\mathrm{d}y\phi_0\{[\delta\phi(x,y,t)+\delta\phi^*(x,y,t)]\\ &\times[u_i(x,y)u_j^*(x,y)+v_i(x,y)v_j^*(x,y)]\\ &+\delta\phi(x,y,t)v_i(x,y)u_j^*(x,y)\\ &+\delta\phi^*(x,y,t)u_i(x,y)v_j^*(x,y)\}\end{aligned} \tag{14}$$

and

$$\begin{aligned}&\mathrm{i}\,\hbar\frac{\partial g_{ij}(t)}{\partial t}\\ =&\langle[\alpha_i\alpha_j,K_{\mathrm{tr}}]\rangle\\ =&(\epsilon_j+\epsilon_i)g_{ij}(t)+2g'[1+f_i^0(t)+f_j^0(t)]\\ &\times\int\mathrm{d}x\,\mathrm{d}y\phi_0\{[\delta\phi(x,y,t)+\delta\phi^*(x,y,t)]\\ &\times[u_i^*(x,y)v_j^*(x,y)+v_i^*(x,y)u_j^*(x,y)]\\ &+\delta\phi(x,y,t)u_i^*(x,y)u_j^*(x,y)\\ &+\delta\phi^*(x,y,t)v_i^*(x,y)v_j^*(x,y)\},\end{aligned} \tag{15}$$

respectively. And the Eq. (6) is rewritten as

$$\begin{aligned}&\mathrm{i}\,\hbar\frac{\partial\delta\phi(x,y,t)}{\partial t}\\ =&\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+g'n_0\right]\delta\phi(x,y,t)\\ &+g'n_0\delta\phi^*(x,y,t)+g'\phi_0\sum_{ij}\{2[u_i^*(x,y)u_j(x,y)\\ &+v_i^*(x,y)v_j(x,y)+v_i^*(x,y)u_j(x,y)]f_{ij}(t)\\ &+[2v_i(x,y)u_j(x,y)+u_i(x,y)u_j(x,y)]g_{ij}(t)\\ &+[2u_i^*(x,y)v_j^*(x,y)+v_i^*(x,y)v_j^*(x,y)]g_{ij}^*(t)\}.\end{aligned} \tag{16}$$

We are interested in the collective mode with energy ϵ_{cl}. We can get the energy correction to the collective mode in the Landau mechanism by using the Fourier transition of Eqs. (14)–(16),

$$\epsilon = \epsilon_{\mathrm{cl}}+4g'^2\sum_i(f_i^0-f_j^0)\frac{|A_{ij}|^2}{\epsilon_{\mathrm{cl}}+\epsilon_i-\epsilon_j}, \tag{17}$$

where

$$A_{ij} = \int \mathrm{d}x\,\mathrm{d}y\phi_0[u_{\rm cl}(u_i u_j^* + v_i v_j^* + v_i u_j^*) + v_{\rm cl}(u_i u_j^* + v_i v_j^* + u_i v_j^*)] \quad (18)$$

is the three-mode coupling matrix element describing the interaction between the collective mode and the quasi-particle, $\epsilon_{\rm cl}$ is the energy of the collective mode, ϵ_i and ϵ_j are the energies of the quasi-particles. In the right hand side of Eq. (17), the first term is the un-corrected energy of the collective mode, the second term is the energy correction to the collective mode. The quasi-particle transitions satisfying the phase-match condition $\epsilon_{\rm cl}+\epsilon_i-\epsilon_j = 0$ have remarkable contributions to the second term. The quasi-particle transition, in which a quasiparticle with energy ϵ_i is turned into another one with energy ϵ_j by absorbing a collective mode energy $\epsilon_{\rm cl}$, contributes to the energy correction of the collective mode through the Landau mechanism.

2.3. Calculation method for Landau damping rate

In the right hand side of Eq. (17), which is deduced from the perturbation theory, the first term is the zero-order energy of the collective mode and the second term is the perturbation correction to the collective mode, which diverges under the condition $\epsilon_{\rm cl}+\epsilon_i-\epsilon_j = 0$. Considering the relaxation of the collective mode, we replace $\epsilon_{\rm cl}$ with $\epsilon_{\rm cl}+\mathrm{i}\,\hbar\gamma$ in the right hand side of Eq. (17), where γ is the Landau damping rate, we get

$$\begin{aligned}\epsilon &= \epsilon_{\rm cl} + \mathrm{i}\,\hbar\gamma + 4g'^2\sum_i (f_i^0 - f_j^0)\frac{|A_{ij}|^2}{\epsilon_{\rm cl}+\mathrm{i}\,\hbar\gamma+\epsilon_i-\epsilon_j} \\ &= \epsilon_{\rm cl} + 4g'^2\sum_i (f_i^0 - f_j^0)\frac{|A_{ij}|^2(\epsilon_{\rm cl}+\epsilon_i-\epsilon_j)}{(\epsilon_{\rm cl}+\epsilon_i-\epsilon_j)^2+(\hbar\gamma)^2} \\ &\quad + \mathrm{i}\left[\hbar\gamma - 4g'^2\sum_i (f_i^0 - f_j^0)\right. \\ &\quad \left.\times \frac{|A_{ij}|^2\hbar\gamma}{(\epsilon_{\rm cl}+\epsilon_i-\epsilon_j)^2+(\hbar\gamma)^2}\right]. \end{aligned} \quad (19)$$

Comparing with Eq. (17), the imaginary part in Eq. (19) should be zero, we obtain the formula

$$\gamma = 4g'^2\sum_i (f_i^0 - f_j^0)\frac{|A_{ij}|^2\gamma}{(\epsilon_{\rm cl}+\epsilon_i-\epsilon_j)^2+(\hbar\gamma)^2} \quad (20)$$

for calculating the Landau damping rate.

It is necessary to compare our method with the usual one for three dimension systems. In the usual calculation method, the formula is written as a Lorentz distribution function

$$\gamma = 4g'^2\sum_i (f_i^0 - f_j^0)\frac{|A_{ij}|^2\Delta}{(\epsilon_{\rm cl}+\epsilon_i-\epsilon_j)^2+(\hbar\Delta)^2}. \quad (21)$$

The calculated γ is found in the region that it changes slowly with Δ.[27] Thus the significance of the Lorentz width Δ has not been elaborated. In our calculation method, $\hbar\Delta$ in the Lorentz distribution function is understood as the energy width of the collective mode. According to the uncertainty principle of time and energy, namely $\hbar\Delta\times\tau = \hbar$, $\Delta = 1/\tau$ is just the damping rate γ, with τ being the average life time of the collective mode. In addition, the usual method only fits for the case that A_{ij} varies gently with the change of the miss-match $\epsilon_{\rm cl}+\epsilon_i-\epsilon_j$, while A_{ij} in low dimension systems changes like a δ function. Thus the calculational result of Landau damping rate is beyond reliable if we do not consider the energy width $\hbar\Delta$.

3. Calculation and result

We consider quasi-two-dimension atoms ^{87}Rb (scattering length $a_{\rm sc} = 5.82\times10^{-9}$ m) condensing in a harmonic axial trapping.

3.1. The Landau damping rate

We take the following parameters: trapping frequency $\omega_{\rm ho} = 800$ Hz (corresponding to $a_{\rm ho} = 9.52522\times10^{-7}$ m), atomic number $N_0 = 6000$, length of condensate $L = 3.4\times10^{-5}$ m. With these parameters, the density of the condensate

$$n_0 = N_0/2\sqrt{2\pi}a_\perp L^2 \quad (a_\perp = 3.96463\times10^{-7}\ \mathrm{m})$$

is the same as the experimental one 2.6×10^{18} m^{-3}.[17]

We take collective modes $k_{{\rm cl}x} = \frac{2\pi}{L}l_x$ and $k_{{\rm cl}y} = \frac{2\pi}{L}l_y$ with $\{l_x, l_y\}$=$\{5,5\}$, $\{7,8\}$, $\{10,11\}$ (corresponding to $\omega_{\rm cl} = 0.9075\omega_{\rm ho}$, $1.889\omega_{\rm ho}$, $3.565\omega_{\rm ho}$, where $\omega_{\rm cl} = \epsilon_{\rm cl}/\hbar$), respectively. The quasiparticles interacting with the collective modes are taken to be $k_{ix} = \frac{2\pi}{L}n_x$ with $n_x = 0, \pm1, \pm2, \ldots$ and $k_{iy} = \frac{2\pi}{L}n_y$ with $n_y = 0, \pm1, \pm2, \ldots$ (except $n_x = n_y = 0$, the case is trivial for the damping mechanics) and $k_{jx} = \frac{2\pi}{L}m_x$ with m_x satisfying $l_x + n_x - m_x = 0, \pm1, \pm2, \ldots$ and $k_{jy} = \frac{2\pi}{L}m_y$ with m_y satisfying $l_y + n_y - m_y = 0, \pm1, \pm2, \ldots$ (except $m_x = m_y = 0$), covering the desirable value of the three-mode coupling matrix A_{ij} around the phase-match condition $k_{{\rm cl}x} + k_{ix} - k_{jx} = 0$ and $k_{{\rm cl}y} + k_{iy} - k_{jy} = 0$ being mentioned in the following text.

Inserting the elementary excitation wavefunction from Eq. (11), there are two integral product factors $\int_{-L/2}^{L/2} \mathrm{d}\alpha\, \mathrm{e}^{\mathrm{i}(k_{\mathrm{cl}\alpha}+k_{i\alpha}-k_{j\alpha})\alpha}$ with $\alpha = x, y$ in Eq. (18) for calculating A_{ij}. The factors equal to $2\pi\delta(k_{\mathrm{cl}\alpha} + k_{\mathrm{i}\,\alpha} - k_{j\alpha})$ as $L \to \infty$, namely it diverges as $L \to \infty$. Thus we replace the integral product factor with another Lorentz distribution function $2\delta_\alpha/((k_{\mathrm{cl}\alpha} + k_{\mathrm{i}\,\alpha} - k_{j\alpha})^2 + \delta_\alpha^2)$, which takes into account the wave vector width δ_α of the collective modes and equals to $2\pi\delta(k_{\mathrm{cl}\alpha} + k_{i\alpha} - k_{j\alpha})$ as $\delta_\alpha \to 0$, where

$$\delta_\alpha = \hbar\Delta/\left[\frac{\hbar^2}{m}(k_{\mathrm{cl}}^2+g''N_0/L^2)/\frac{k_{\mathrm{cl}}}{k_{\mathrm{cl}\alpha}}\sqrt{k_{\mathrm{cl}}^2+2g''N_0/L^2}\right]$$

is evaluated by differentiating Eq. (12). We find that the quasiparticle transition satisfying the phase match condition $k_{\mathrm{cl}\alpha} + k_{i\alpha} - k_{j\alpha} = 0$ has prominent contribution to the Landau damping rate.

Using the results of the three-mode coupling matrix and the elementary excitation eigenfunction and eigenvalue, we calculate the Landau damping rate γ in an iterative way. Figure 1 show the values of γ versus the values of Δ at $T = 100$ nK for collective modes $\{5, 5\}$, $\{7, 8\}$, $\{10, 11\}$. We can find in Fig. 1 that the value of γ decreases with the increase of Δ and there are three points represented by the stars that γ equal to Δ. The Landau damping rates for the three collective modes are read to be 52, 67 and 84 s^{-1}, respectively. As expected, the Landau damping rate increases with the increase of the frequency of the collective mode. The reason is that the low energy mode is more stable than the high one. Our theoretic result is in agreement with the experimental ones.[18,19]

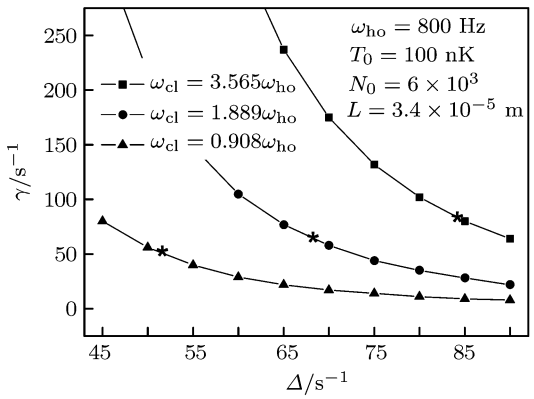


Fig. 1. The γ as a function of Δ at $T = 100$ nK for the Landau damping. We consider collective modes $\{5, 5\}$, $\{7, 8\}$, $\{10, 11\}$, respectively.

3.2. T-, ω_{ho}-, N_0- and L-dependences of the Landau damping

We show the damping rate γ (in unit of s^{-1}) of collective mode $\{7, 8\}$ versus the temperature T (in unit of K) in Fig. 2. The axial trapping frequencies are $\omega_{\mathrm{ho}} = 400$, 800, 1600 Hz, the corresponding frequencies of collective mode $\{7, 8\}$ are 1479, 1512, 1555 Hz, respectively, with $N_0 = 6000$ and $L = 3.4 \times 10^{-5}$ m.

As shown in Fig. 2 and as expected, the Landau damping rate increases with the increase of the temperature and the trapping frequency. Because the available quasiparticle number increases with the increase of T and the atom density increases as the trapping frequency increases.

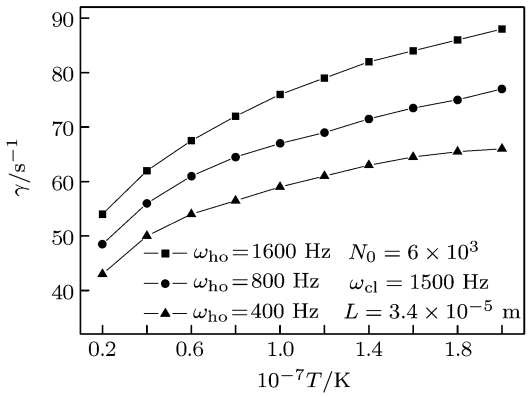


Fig. 2. The Landau damping rate γ of collective mode $\{7, 8\}$ in the condensate as a function of temperature T.

In Fig. 3, we have plotted γ (in unit of ω_{ho}) of collective mode $\omega_{\mathrm{cl}} = 10\omega_{\mathrm{ho}}$ versus the atom number N_0 at $T = 100$ nK. We take the lengthes of the condensate to be $L = 1.7$, 3.4, 6.8×10^{-5} m, respectively, with $\omega_{\mathrm{ho}} = 800$ Hz.

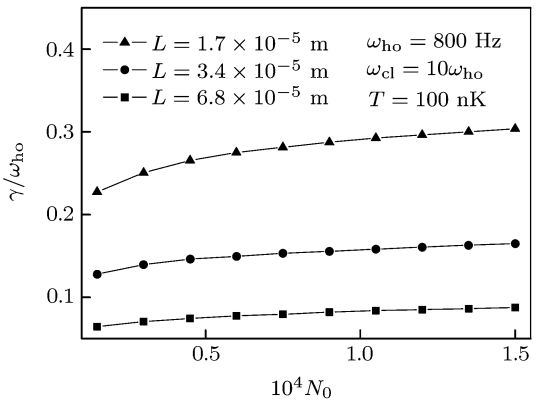


Fig. 3. The Landau damping rate γ of collective mode $\omega_{\mathrm{cl}} = 10\omega_{\mathrm{ho}}$ in the condensate as a function of particle number N_0 at temperature $T = 100$ nK.

We find that, as expected, the Landau damping rates increase with the particle number but decrease

with the length of condensate, which is caused by the change of the atom density in the condensate.

4. Conclusion

We have studied the Landau damping of the collective mode excited in a quasi-two-dimension repulsive Bose–Einstein condensate trapped in an axial harmonic trap. We have calculated the Landau damping rate of the collective mode and discuss the temperature-, atom number-, frequency- and condensate length- dependence of the Landau damping rate. We use the complete and orthogonal eigen-function set of the BdG equation in the quasi-two-dimension condensate. We take into account the spectrum width of the collective mode. Our theoretic calculation result agrees well with the experiment ones. Our theory well pictures the physics of the interatomic interaction and is helpful to test and to develop the quantum many-body theory for condensed matter.

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