Entanglement distribution using three-mode separable state with imperfect equipments*

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We show a scheme to distribute the entanglement by using three-mode separable Gaussian state prepared with imperfect equipments. The scheme achieves the aim that the entanglement is distributed between two distant parties with only Gaussian operations and linear optics elements. Moreover, we analyse the logarithmic negativity of the entanglement shared between the two parties when the systems are imperfect and arrive at the conclusion that the logarithmic negativity is asymptotically stable with fluctuations within a certain space range.

Keywords: entanglement distribution, separable state, imperfect equipments

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1. Introduction

The entanglement is the root of Bell theorem, which has various applications in several fields of quantum information, such as quantum key distribution,[1−7] and quantum teleportation.[8−12] Quantum key distribution based on entanglement swapping[6,7] uses the entangled states previously shared to transfer a key or takes the entangled states as the quantum key to encode and decode the classical information. Quantum teleportation[10−12] utilizes the entangled states shared in advance to transmit an unknown quantum state between two distant parties, Alice and Bob. Besides these, many other quantum protocols also require sharing of the entanglement.

It is currently a basic research topic to establish an entanglement between two distant parties. Generally, the two parties can prepare the entangled states together and share between them the entanglement after their departure. However, the entanglement shared in this way is insufficient for the practical needs. This problem has attracted much attention and several methods are proposed to deal with it. The easiest one is to distribute the entangled states directly between the two distant parties before they carry out the scheme. However, most communication needs to distribute the entanglement between the two systems as everybody would intuitively expect. For example, some one wants to perform operations on a quantum network, such as increasing nodes and removing nodes, because the quantum network has many nodes, it is impossible to distribute a new entangled state among all nodes. In that case, it is significant to distribute the entanglement between original systems by using separable states. Compared with distributing entangled states directly, distributing the entanglement by using separable states can make two separable quantum systems share the entangled channel. In other words, two distant systems can be in one entangled channel by using the method of distributing the entanglement with separable states. So the research on utilising separable states to distribute the entanglement is of great worth. Cubitt *et al.*[13] proposed a protocol based on qubits, in which two distant particles can be entangled by sending a third particle separable from the other two. Mista and Korolkova[14,15] introduced the idea into continuous variable (CV) and

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suggested distributing the entanglement with Gaussian separable states. Then they also explained the origin of the entanglement. The authors of Ref. [15] specified how to distribute the entanglement between two distant parties by transmitting another separable auxiliary mode. The two distant separable modes, the sender's one and the receiver's one, are a squeezed state and a vacuum state, respectively. The auxiliary mode is also a squeezed state with squeezing parameter the same as that of the sender's mode. As a result of optics elements and channel noise, the experiment distributing the entanglement by using separable states is influenced by the parameter's accuracy and the channel type. Obviously, the channel loss is the most important type of error, which has already been studied by many authors.[16−19] Though the errors induced by equipments are much smaller than the channel noise, the effect of the imperfection of the equipments on the entanglement distribution is also important. In this paper, we follow a simple recipe and only analyse the distribution process with errors in squeezing parameters introduced by the imperfect equipments. Moreover, from the comparison of the entanglement between the protocols with and without fluctuations in the squeezing parameters, we can find the errors induced by the equipments.

This paper is organized as follows. Section 2 introduces the essence of a Gaussian state. We give a brief description of the original protocol[15] in Section 3. Our scheme is presented for establishing the entanglement with imperfect equipments in Section 4 and Section 5 is our conclusion.

2. The Gaussian states

In fact, Gaussian states are the most important CV states that can be prepared in the laboratory currently. Before describing the scheme in Ref. [15], we will introduce some notations for Gaussian states.

We consider n distinguishable quantum systems and each system is called a mode. All the n modes are described by canonical operators $\boldsymbol{R} = (X_1, P_1, X_2, P_2, \ldots, X_n, P_n)^{\mathrm{T}} = (R_1, R_2, \ldots, R_{2n})^{\mathrm{T}}$ with the commutation relation $[R_j, R_k] = -\mathrm{i}\,\Omega_{jk}$, where $j, k = 1, 2, \ldots, 2n$ and

$$\boldsymbol{\Omega} = \bigoplus_{i=1}^{n} \boldsymbol{J}, \quad \boldsymbol{J} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{1}$$

For a CV system, it is convenient to describe a state $\boldsymbol{\rho}$ by its Wigner function[20]

$$W_{\boldsymbol{\rho}}(\boldsymbol{\xi}) = \frac{1}{(2\pi)^n \sqrt{\det \boldsymbol{\gamma}}} \times \exp\left[-\frac{1}{2}(\boldsymbol{\xi} - \boldsymbol{d})^{\mathrm{T}} \boldsymbol{\gamma}^{-1} (\boldsymbol{\xi} - \boldsymbol{d}) \right], \tag{2}$$

where $\boldsymbol{\xi}$ is a $2n$ phase-space vector, $\boldsymbol{d}$ is a $2n$ real displacement vector and $\boldsymbol{\gamma}$ is a $2n \times 2n$ covariance matrix (CM). CM $\boldsymbol{\gamma}$ satisfies the condition $\boldsymbol{\gamma} - \mathrm{i}\,\boldsymbol{\Omega} \geq 0$.

For tripartite Gaussian systems, Giedke *et al*[21] classified them into five different entanglement classes: fully inseparable states, one-mode biseparable states, two-mode biseparable states, three-mode biseparable states and fully separable states. Biseparable tripartite state is a system consisting of two particles that is separable from the third particle. In order to judge the separability, we can apply the positive partial transpose (PPT) criterion[22] to Gaussian states.[23−25] With regard to CM $\boldsymbol{\gamma}$, the CM for the partial transpose of mode x is $\boldsymbol{\gamma}^{(\mathrm{T}_x)} = \boldsymbol{\Lambda}_x \boldsymbol{\gamma} \boldsymbol{\Lambda}_x$, where $x = 1, 2, 3$, $\boldsymbol{\Lambda}_1 = \boldsymbol{\sigma}_z \oplus \boldsymbol{I} \oplus \boldsymbol{I}$, $\boldsymbol{\Lambda}_2 = \boldsymbol{I} \oplus \boldsymbol{\sigma}_z \oplus \boldsymbol{I}$ and $\boldsymbol{\Lambda}_3 = \boldsymbol{I} \oplus \boldsymbol{I} \oplus \boldsymbol{\sigma}_z$.

Now we will show how to verify a three-mode system is one-mode biseparable. The statement is that if there exist the inequalities

$$\boldsymbol{\gamma}^{(\mathrm{T}_1)} - \mathrm{i}\,\boldsymbol{\Omega} \geq 0, \ \boldsymbol{\gamma}^{(\mathrm{T}_2)} - \mathrm{i}\,\boldsymbol{\Omega} \ngeq 0, \ \boldsymbol{\gamma}^{(\mathrm{T}_3)} - \mathrm{i}\,\boldsymbol{\Omega} \ngeq 0, \tag{3}$$

then mode 1 is separable from mode $(2, 3)$, mode 2 is entangled with mode $(1, 3)$ and mode 3 is entangled with mode $(1, 2)$. In practice, there are three ways to verify whether a three-mode system satisfies the inequalities. One[23] is to calculate the eigenvalues of the matrix $\boldsymbol{\gamma}^{(\mathrm{T}_1)} - \mathrm{i}\,\boldsymbol{\Omega}$ (denoted by $\{x_1, x_2, x_3\}$). If and only if $x_j \geq 0$ for any j, mode 1 is separable from mode $(2, 3)$ and vice versa. Another one[26] is to calculate the eigenvalues of $\boldsymbol{\Omega}\boldsymbol{\gamma}^{(\mathrm{T}_1)}$, which are $\{\pm\mathrm{i}\,u_1, \pm\mathrm{i}\,u_2, \pm\mathrm{i}\,u_3\}$, with $u_j \geq 0$, $j = 1, 2, 3$. If and only if the eigenvalue u_j satisfies $u_j \geq 1$ for any j, the system is biseparable splitting and mode 1 is separable from mode $(2, 3)$ and vice versa. The third one[27] is to calculate the invariants I_1, I_2 and I_3 about $\boldsymbol{\gamma}^{(\mathrm{T}_1)}$, where I_1, I_2 and I_3 satisfy the equation

$$\det(\boldsymbol{\Omega}\boldsymbol{\gamma}^{(\mathrm{T}_1)} - \lambda \boldsymbol{I}) = \lambda^6 + I_1 \lambda^4 + I_2 \lambda^2 + I_3. \tag{4}$$

The third rule can be expressed as follows: mode 1 is separable from mode $(2, 3)$ when

$$\Sigma = I_3 - I_2 + I_1 - 1 = \Pi_{i=1}^{3}(s_i^2 - 1) > 0. \tag{5}$$

3. The MK protocol

Mista and Korolkova[15] proposed a protocol (i.e. MK protocol) for establishing an entanglement between two distant parties using separable ancilla and explained the physical mechanism underlying the phenomenon. The logarithmic negativity[28] $E_{\rm n}(\boldsymbol{\rho}) \equiv \log_2 \|\boldsymbol{\rho}^{\rm T_A}\|_1$, which changes to zero for unentangled states and corresponds to the absolute value of the sum of the negative eigenvalues of $\boldsymbol{\rho}^{\rm T_A}$, can be used to quantify the degree of the entanglement in composite systems. The logarithmic negativity distributed by the MK protocol is more than one order of magnitude higher than that of the previous ones. In the following, we will show the particular process of the MK protocol. The protocol is based on two steps: (i) preparation of the quantum system and (ii) distributing the entanglement.

In the first phase, the sender Alice prepares mode 1 and auxiliary mode 3 in momentum and position-squeezed vacuum states, respectively. The receiver Bob prepares mode 2 in a vacuum state. The three modes are fully separable.

In the distributing phase of the protocol, the distant parties operate local random displacements. Then the entire states transform under the beam splitter operation on modes 1 and 3. Through proper displacements, the entire system (1,2,3) will be in a two-mode biseparable state, in which mode 1 is entangled with mode (2,3), mode 2 is separable from mode (1,3) and mode 3 is separable from mode (1,2). Then Alice distributes mode 3 to Bob. Bob mixes modes 2 and 3 by using beam splitter S_{23}. After all the operations, the system is changed into a one-mode biseparable state, i.e., mode 1 is entangled with mode (2,3), mode 2 is entangled with mode (1,3) and mode 3 is separable from mode (1,2). In this way, Alice and Bob share the entanglement between their original systems.

Essentially, the two parties achieve the scheme only with Gaussian operations and linear optics elements and can distribute more entanglement. The distributed entanglement originates from the entanglement between mode 1 and auxiliary mode 3 after Alice mixes modes 1 and 3. Then the entanglement is destroyed by the local displacements that make the system a two-mode biseparable state. After that, only part of the entanglement between modes 1 and 3 can be restored by the receiver Bob via mixing modes 2 and 3. And then the system is changed into a one-mode biseparable state. In the next section, we construct a protocol, in which the participating states have errors in squeezing parameters and then give a detailed investigation of the process.

4. Distributing entanglement with imperfect equipments

The previous section described how to distribute the entanglement between a squeezed vacuum mode and a vacuum mode by transmitting a squeezed mode. All the squeezed modes have the same squeezing parameters. In practice, it is challenging for techniques to keep the squeezing parameter of mode 3 the same as that of mode 1. So before applying the MK[15] protocol, we must improve the theory, i.e., to understand better the influence of practical conditions on the entanglement. The channel losses have been studied before. We will consider the imperfection of the equipments in this section. The particular process is given in the following.

(i) The first phase is the preparation of the quantum system. Like the MK protocol, the distant parties, Alice and Bob, want to distribute the entanglement between their systems. Alice prepares mode 1 and auxiliary mode 3 in momentum and position-squeezed vacuum states, respectively. Bob prepares a vacuum state denoted by mode 2. Considering the influence of the imperfect equipments on the parameters of the squeezing quadrature of the two modes 1 and 3, we know that unknown random fluctuations in the squeezing parameter lead to states that are mixtures of Gaussian states with different covariance matrices. The problem with a mixture of Gaussian states is too complicated and goes beyond the scope of our paper. So we use the following CM of the initial system,

$$\begin{aligned}\boldsymbol{\gamma}' &= \boldsymbol{\gamma}_1 \oplus \boldsymbol{\gamma}_2 \oplus \boldsymbol{\gamma}_3 \\ &= \begin{pmatrix} {\rm e}^{2t} & 0 \\ 0 & {\rm e}^{-2t} \end{pmatrix} \oplus \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \oplus \begin{pmatrix} {\rm e}^{-2t(1+\delta)} & 0 \\ 0 & {\rm e}^{2t(1+\delta)} \end{pmatrix}, \end{aligned} \tag{6}$$

where δ is a stochastic variable obeying the standard normal distribution. The variance σ^2 shows the deviation of the squeezing parameter in practice to the ideal situation. The parameter σ would be obtained after Alice tests the equipment many times.

(ii) Alice and Bob operate local random displacements on their states according to CM $\boldsymbol{M}$. Then the state transforms under the beam splitter operation on modes 1 and 3. After this, the CM of the entire system reads

$$\begin{aligned}\boldsymbol{\gamma}'' &= \boldsymbol{U}_{13}(\boldsymbol{\gamma}' + x\boldsymbol{M})\boldsymbol{U}_{13}^{\rm T} = \boldsymbol{U}_{13}\boldsymbol{\gamma}'\boldsymbol{U}_{13}^{\rm T} + x\boldsymbol{N} \\ &= \boldsymbol{\gamma}''_{123} + x\boldsymbol{N} = \boldsymbol{\gamma}''_{13} \oplus \boldsymbol{\gamma}_2 + x\boldsymbol{N}, \end{aligned} \tag{7}$$

where $\boldsymbol{N} = \boldsymbol{U}_{13}\boldsymbol{M}\boldsymbol{U}_{13}^{\rm T}$ and $\boldsymbol{U}_{13}$ describes the beam splitter S_{13},

$$\boldsymbol{U}_{13} = \begin{pmatrix} \frac{1}{\sqrt{2}}\boldsymbol{I} & 0 & \frac{1}{\sqrt{2}}\boldsymbol{I} \\ 0 & \boldsymbol{I} & 0 \\ \frac{1}{\sqrt{2}}\boldsymbol{I} & 0 & -\frac{1}{\sqrt{2}}\boldsymbol{I} \end{pmatrix}. \tag{8}$$

The inverse operation of $\boldsymbol{U}_{13}$ is $\boldsymbol{U}_{13}^{-1} = \boldsymbol{U}_{13}^{\mathrm{T}} = \boldsymbol{U}_{13}$. Similar to the MK protocol, we construct the noise matrix $\boldsymbol{N}$ in Eq. (7). If and only if the matrix

$$\boldsymbol{\gamma}_{13}'' - \mathrm{i}\,\boldsymbol{J} \oplus (-\boldsymbol{J}) = \begin{pmatrix} \frac{\mathrm{e}^{2t} + \mathrm{e}^{-2t(1+\delta)}}{2} & \mathrm{i} & \frac{\mathrm{e}^{2t} - \mathrm{e}^{-2t(1+\delta)}}{2} & 0 \\ -\mathrm{i} & \frac{\mathrm{e}^{-2t} + \mathrm{e}^{2t(1+\delta)}}{2} & 0 & \frac{\mathrm{e}^{-2t} - \mathrm{e}^{2t(1+\delta)}}{2} \\ \frac{\mathrm{e}^{2t} - \mathrm{e}^{-2t(1+\delta)}}{2} & 0 & \frac{\mathrm{e}^{2t} + \mathrm{e}^{-2t(1+\delta)}}{2} & -\mathrm{i} \\ 0 & \frac{\mathrm{e}^{-2t} - \mathrm{e}^{2t(1+\delta)}}{2} & \mathrm{i} & \frac{\mathrm{e}^{-2t} + \mathrm{e}^{2t(1+\delta)}}{2} \end{pmatrix} \tag{9}$$

has a negative eigenvalue, the system (1,3) is entangled. The negative eigenvalue of the matrix given in Eq. (9) is

$$\lambda = \frac{\mathrm{e}^{-2t} + \mathrm{e}^{-2t(1+\delta)} - \sqrt{(\mathrm{e}^{-2t} - \mathrm{e}^{-2t(1+\delta)})^2 + 4}}{2}.$$

And the corresponding eigenvector is $\boldsymbol{P} = (0,-1,0,-1)^{\mathrm{T}} + \mathrm{i}(-\lambda + \mathrm{e}^{-2\mathrm{t}}, 0, \lambda - \mathrm{e}^{-2\mathrm{t}}, 0)^{\mathrm{T}}$. The noise matrix $\boldsymbol{N}$ can be constructed by extending the real and the imaginary parts of the eigenvector $\boldsymbol{P}$. Suppose $x > 0$ and $\boldsymbol{N} = \boldsymbol{q}_1\boldsymbol{q}_1^{\mathrm{T}} + \boldsymbol{q}_2\boldsymbol{q}_2^{\mathrm{T}}$, where $\boldsymbol{q}_1 = (0,-1,0,2,0,-1)^{\mathrm{T}}$, $\boldsymbol{q}_2 = (1,0,2,0,-1,0)^{\mathrm{T}}$, by adding $x\boldsymbol{N}$ to CM $\boldsymbol{\gamma}_{123}''$, we can destroy the entanglement between modes 1 and 3 and make mode 1 be entangled with mode (2,3), mode 2 be separable from mode (1,3) and mode 3 be separable from mode (1,2). Then the CM of the entire system is changed into the form given in Eq. (7). Now let us calculate the separability. By all appearances, mode 2 is separable from mode (1,3). The separability of mode 3 with mode (1,2) can be calculated by the partial transpose $\boldsymbol{\gamma}''^{(\mathrm{T}_3)}$. The lowest symplectic eigenvalue of $\boldsymbol{\gamma}''^{(\mathrm{T}_3)}$ is

$$u_1 = \sqrt{\frac{A_1 - \sqrt{A_1^2 - 4B_1}}{2}}, \tag{10}$$

with

$$\begin{aligned} A_1 &= (2x + \mathrm{e}^{-2t})[2x + \mathrm{e}^{-2t(1+\delta)}] + 8x + 1, \\ B_1 &= (4x\,\mathrm{e}^{-2t} + \mathrm{e}^{-2t} + 2x) \\ &\quad \times [4x\,\mathrm{e}^{-2t(1+\delta)} + \mathrm{e}^{-2t(1+\delta)} + 2x]. \end{aligned} \tag{11}$$

When $u_1 = 1$, the corresponding value of parameter x is the threshold value x_{sep}. Mode 3 is separable from mode (1,2) if and only if $x \geq x_{\mathrm{sep}}$, where

$$x_{\mathrm{sep}} = \frac{\mathrm{e}^{2t(2+\delta)} - 1}{\mathrm{e}^{2t} + \mathrm{e}^{2t(1+\delta)} + 2}. \tag{12}$$

The formula on the right-hand side of Eq. (12) is a function of variable δ and the function increases as δ increases. We know that about 99.7% of the values drawn from a normal distribution are within three standard deviations (3σ) away from the mean value. Suppose the maximum value of δ is 3σ, so the maximum of the function locates at $\delta = 3\sigma$. Then mode 3 is separable from mode (1,2) if and only if

$$x \geq \frac{\mathrm{e}^{2t(2+3\sigma)} - 1}{\mathrm{e}^{2t} + \mathrm{e}^{2t(1+3\sigma)} + 2}. \tag{13}$$

Now considering the separability of mode 1 with mode (2,3), the lowest symplectic eigenvalue of $\boldsymbol{\gamma}''^{(\mathrm{T}_1)}$ is

$$u_2 = \sqrt{\frac{A_2 - \sqrt{A_2^2 - 4B_2}}{2}}, \tag{14}$$

where

$$\begin{aligned} A_2 &= (2x + \mathrm{e}^{-2t})[2x + \mathrm{e}^{-2t(1+\delta)}] \\ &\quad + 16x^2 + (1 + 4x)^2, \\ B_2 &= (4x\,\mathrm{e}^{-2t} + \mathrm{e}^{-2t} + 2x) \\ &\quad \times [4x\,\mathrm{e}^{-2t(1+\delta)} + \mathrm{e}^{-2t(1+\delta)} + 2x]. \end{aligned} \tag{15}$$

We can see that for $x > 0$, $\delta > 0$ and $t > 0$, the lowest eigenvalue u_2 satisfies $u_2 < 1$. So there exists an entanglement between mode 1 and mode (2,3). Then, we know that the system with CM $\boldsymbol{\gamma}''$ is a two-mode biseparable state, i.e., mode 1 is entangled with mode (2,3), mode 2 is separable from mode (1,3) and mode 3 is separable from mode (1,2).

(iii) Alice transfers mode 3 to Bob. Bob mixes modes 2 and 3 by using beam splitter S_{23}, then the CM is changed into $\boldsymbol{\gamma}''' = \boldsymbol{U}_{23}\boldsymbol{\gamma}''\boldsymbol{U}_{23}^{\mathrm{T}}$, where

$$\boldsymbol{U}_{23}=\begin{pmatrix}\boldsymbol{I} & 0 & 0\\ 0 & \frac{1}{\sqrt{2}}\boldsymbol{I} & \frac{1}{\sqrt{2}}\boldsymbol{I}\\ 0 & \frac{1}{\sqrt{2}}\boldsymbol{I} & -\frac{1}{\sqrt{2}}\boldsymbol{I}\end{pmatrix}. \tag{16}$$

Now we consider the separability of mode 1 with mode 2. First, we can compute $\boldsymbol{\gamma}_{12}^{\prime\prime\prime(\mathrm{T}_2)}$ corresponding to the partial transpose of the reduced state of modes 1 and 2. We have

$$\boldsymbol{\gamma}_{12}^{\prime\prime\prime}=\begin{pmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{B}^{\mathrm{T}} & \boldsymbol{C}\end{pmatrix}, \tag{17}$$

with the submatrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ of the form

$$\begin{aligned}\boldsymbol{A}&=\begin{pmatrix}\frac{\mathrm{e}^{2t}+\mathrm{e}^{-2t(1+\delta)}}{2}+x & 0\\ 0 & \frac{\mathrm{e}^{-2t}+\mathrm{e}^{2t(1+\delta)}}{2}+x\end{pmatrix},\\ \boldsymbol{B}&=\begin{pmatrix}\frac{\mathrm{e}^{2t}-\mathrm{e}^{-2t(1+\delta)}}{2\sqrt{2}}+\frac{x}{\sqrt{2}} & 0\\ 0 & \frac{\mathrm{e}^{-2t}-\mathrm{e}^{2t(1+\delta)}}{2\sqrt{2}}-\frac{x}{\sqrt{2}}\end{pmatrix},\\ \boldsymbol{C}&=\begin{pmatrix}\frac{\mathrm{e}^{2t}+\mathrm{e}^{-2t(1+\delta)}}{4}+\frac{1+x}{2} & 0\\ 0 & \frac{\mathrm{e}^{-2t}+\mathrm{e}^{2t(1+\delta)}}{4}+\frac{1+x}{2}\end{pmatrix}.\end{aligned} \tag{18}$$

The symplectic eigenvalue of $\boldsymbol{\gamma}_{12}^{\prime\prime\prime(\mathrm{T}_2)}$ is[26]

$$u_3=\sqrt{\frac{\alpha-\sqrt{\alpha^2-4\det(\gamma_{12}^{\prime\prime\prime})}}{2}}, \tag{19}$$

where

$$\begin{aligned}&\det(\boldsymbol{\gamma}_{12}^{\prime\prime\prime})=|\boldsymbol{AC}-\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}|\\ &=\frac{1}{16}[4x\,\mathrm{e}^{-2t}+\mathrm{e}^{-2t}+\mathrm{e}^{2t(1+\delta)}+2\,\mathrm{e}^{2t\delta}+2x]\\ &\quad\times[4x\,\mathrm{e}^{-2t(1+\delta)}+\mathrm{e}^{-2t(1+\delta)}+\mathrm{e}^{2t}+2\,\mathrm{e}^{-2t\delta}+2x],\\ \alpha&=\det\boldsymbol{A}+\det\boldsymbol{C}-2\det\boldsymbol{B}\\ &=\left[\frac{\mathrm{e}^{2t}+\mathrm{e}^{-2t(1+\delta)}}{2}+x\right]\left[\frac{\mathrm{e}^{-2t}+\mathrm{e}^{2t(1+\delta)}}{2}+x\right]\\ &\quad+\left[\frac{\mathrm{e}^{2t}+\mathrm{e}^{-2t(1+\delta)}}{4}+\frac{1+x}{2}\right]\\ &\quad\times\left[\frac{\mathrm{e}^{-2t}+\mathrm{e}^{2t(1+\delta)}}{4}+\frac{1+x}{2}\right]\\ &\quad-2\left[\frac{\mathrm{e}^{2t}-\mathrm{e}^{-2t(1+\delta)}}{2\sqrt{2}}+\frac{x}{\sqrt{2}}\right]\\ &\quad\times\left[\frac{\mathrm{e}^{-2t}-\mathrm{e}^{2t(1+\delta)}}{2\sqrt{2}}-\frac{x}{\sqrt{2}}\right].\end{aligned} \tag{20}$$

For $t>0$, $t(1+\delta)>0$ and

$$x=\frac{\mathrm{e}^{2t(2+3\sigma)}-1}{\mathrm{e}^{2t}+\mathrm{e}^{2t(1+3\sigma)}+2},$$

we have

$$\begin{aligned}&\det(\boldsymbol{\gamma}_{12}^{\prime\prime\prime})-\alpha+1\\ &=\det(\boldsymbol{\gamma}_{12}^{\prime\prime\prime})-\det\boldsymbol{A}-\det\boldsymbol{C}+2\det\boldsymbol{B}+1<0,\end{aligned} \tag{21}$$

i.e., $u_3<1$, so modes 1 and 2 are entangled. In addition, mode 3 is separable from mode (1,2). Hence after this step, the system is changed into a one-mode biseparable state, i.e., mode 1 is entangled with mode (2,3), mode 2 is entangled with mode (1,3) and mode 3 is separable from mode (1,2).

So far, we have proposed a particular scheme that achieves the aim that two distant parties distribute the entanglement by an auxiliary squeezed state. We give not only the form of the noise matrix $x\boldsymbol{N}$ in the system with errors in the squeezing parameters, but also the threshold value of x. Compared with that of the MK protocol, in our scheme, the squeezing parameter of auxiliary mode 3 is fluctuant and is easier to be carried out. Subsequently, the influence of the errors on the entanglement shared between Alice and Bob is studied.

In order to present the relationship between δ and the logarithmic negativity E_{n} more clearly, we set

$$x=\frac{\mathrm{e}^{2t(2+\delta)}-1}{\mathrm{e}^{2t}+\mathrm{e}^{2t(1+\delta)}+2}.$$

For $t>0$, the numerical results of the logarithmic negativity $E_{\mathrm{n}}=-\log_2 u_3$[26] versus δ values are given in Table 1: $\mathrm{e}^{2t}=2$ corresponds to input squeezing $t=0.34$; and $\mathrm{e}^{2t}=10$ corresponds to the squeezing parameter $t=1.15$.

Table 1. The logarithmic negativity E_{n} versus δ.

	δ	0	1%	3%	5%	10%	15%
$\mathrm{e}^{2t}=2$	u_3	0.6277	0.6269	0.6252	0.6235	0.6193	0.6154
	E_{n}	0.6718	0.6737	0.6777	0.6816	0.6912	0.7005
$\mathrm{e}^{2t}=10$	u_3	0.3565	0.3564	0.3562	0.3561	0.3560	0.3560
	E_{n}	1.4881	1.4885	1.4891	1.4895	1.4900	1.4900

When $\delta=0$, the logarithmic negativity E_{n} is the mean value and equals to the result of the MK protocol, as shown in Fig. 1(a). And E_{n} of $\mathrm{e}^{2t}=10$ is larger than that of $\mathrm{e}^{2t}=2$ when δ is the same. Actually, in our scheme, when the squeezing parameter t satisfies $\mathrm{e}^{2t}=2$, the logarithmic negativity E_{n} can be considered as asymptotic stable with the squeezing error δ. Up to now, we can give no physical explanation for the phenomenon. When $\mathrm{e}^{2t}=2$, E_{n} is equal to 1.111 in the limit of $\delta=20$. And when $\mathrm{e}^{2t}=10$, E_{n} is equal to 1.458 in the limit of $\delta=10$. This does not mean that the new scheme gives larger logarithmic negativity regardless of the squeezing error δ. As shown in Fig. 1(b), the E_{n} is a concave function of the squeezing error δ when the squeezing t satisfies $\mathrm{e}^{2t}=10$.

When $-1 < \delta < 0$, $E_{\rm n}$ reduces sharply with the decreasing δ. Furthermore, we arrive at another conclusion when the initial system is different from that of the MK protocol, i.e., mode 1 is a vacuum state at Alice's station, mode 2 is a momentum-squeezed vacuum state and mode 3 is a position-squeezed state with errors in the squeezing parameter. After the same process, two distant parties also can distribute the entanglement between the systems when the noise matrix $\tilde{\boldsymbol{N}} = (0,-1,0,2,0,-1)^{\rm T}(0,-1,0,2,0,-1) + (1,0,2,0,-1,0)^{\rm T}(1,0,2,0,-1,0)$ and

$$\tilde{x} \geq \tilde{x}_{\rm sep} = \frac{{\rm e}^{2t} + {\rm e}^{-2t} - {\rm e}^{-2t(2+\delta)} - {\rm e}^{-2t\delta}}{4\,{\rm e}^{-2t(1+\delta)} + 2\,{\rm e}^{2t} + 2\,{\rm e}^{-2t(2+\delta)}}.$$

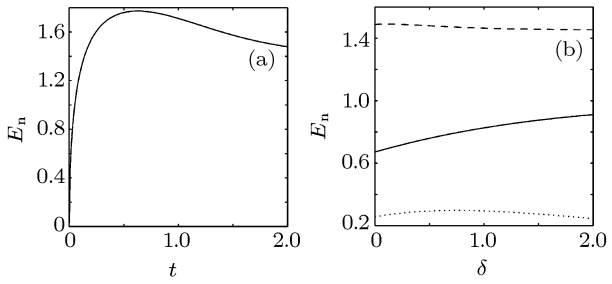


Fig. 1. (a) The logarithmic negativity $E_{\rm n}$ versus the squeezing t, when $\delta = 0$ and $x = ({\rm e}^{2t} - 1)/2$.[15] The $E_{\rm n}$ achieves the maximum value 1.7728 when $t = 0.63$, i.e., ${\rm e}^{2t} \approx 3.52$. The $E_{\rm n}$ approaches to the value of 1.585 as $t \to \infty$. (b) The logarithmic negativity $E_{\rm n}$ versus the squeezing error δ at ${\rm e}^{2t} = 2$ (solid line), ${\rm e}^{2t} = 10$ (dashed line) when Alice holds a squeezed state, Bob holds a vacuum state and $x = \frac{{\rm e}^{2t(2+\delta)}-1}{{\rm e}^{2t}+{\rm e}^{2t(1+\delta)}+2}$; and at ${\rm e}^{2t} = 2$ (dotted line) when Alice possesses a vacuum state, Bob possesses a squeezed state and $\tilde{x} = \frac{{\rm e}^{2t}+{\rm e}^{-2t}-{\rm e}^{-2t(2+\delta)}-{\rm e}^{-2t\delta}}{4\,{\rm e}^{-2t(1+\delta)}+2\,{\rm e}^{2t}+2\,{\rm e}^{-2t(2+\delta)}}$.

Figure 1(b) shows $E_{\rm n}$ as a function of error δ when the squeezing parameter t satisfies ${\rm e}^{2t} = 2$. We know that when the squeezing parameter is the same, the logarithmic negativity distributed by the initial system in which Alice and Bob possess the same states as that of the MK protocol, is larger than that of the system in which Alice possesses a vacuum state and Bob possesses a squeezed state. Then we can conclude in two points. One is that the $E_{\rm n}$ of the shared entanglement between two parties is correlated with the squeezing parameter. Generally speaking, when the squeezing error δ is the same, the logarithmic negativity $E_{\rm n}$ increases with the increasing squeezing t. The other is that the shared entanglement between two parties is correlated with the states possessed by each participant when the squeezing t is the same. Figure 1(b) provides the second point.

5. Conclusion

We have presented a scheme that the distant parties could distribute the entanglement between them by transmitting an auxiliary mode. Compared with the influence of the channel noise, the influence of practical equipments to the entanglement distribution is more complicated. The influence of the errors to the entanglement is added to the CV entanglement distribution in our scheme. The errors mean discrepancies between the fluctuation of the squeezing quadrature of the auxiliary mode and that of the original mode. We analyse the change of the logarithmic negativity shared. For the squeezing $t > 0$ and a certain noise threshold $x_{\rm sep}$, the logarithmic negativity $E_{\rm n}$ is asymptotically stable with δ. The specific relations between the logarithmic negativity and the fluctuation of the squeezing parameter are formulated mathematically. We obtain the entanglement distribution with the errors existed in the optical elements.

Our results may be expanded. The unknown random fluctuations in the squeezing parameter can lead to states that are mixtures of Gaussian states with different covariance matrices. In this paper, we simplify the problem and deal with the parameters of the covariance matrix as random variables. The problem with mixtures of Gaussian states should be discussed in future. Furthermore, the distribution under more general conditions, including channel loss, should also be considered.

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