Avoiding the decay of entanglement for coupling two-qubit system interacting with a non-Markov environment*

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The entanglement evolution of the coupled qubits interacting with a non-Markov environment is investigated in terms of concurrence. The results show that the entanglement of the quantum systems depends not only on the initial state of the system but also on the coupling between the qubit and the environment. For the initial state $(|00\rangle \pm |11\rangle)/\sqrt{2}$, the coupled qubits will always been in the maximum entangled state under an asymmetric coupling. For the initial state $(|01\rangle \pm |10\rangle)/\sqrt{2}$, in contrast, the entangling degree of the coupled qubits is always equal to unity and does not depend on the evolving time under the symmetric coupling. We find that the stronger the interaction between the qubits is, the better the struggle against the entanglement sudden death is.

Keywords: entanglement, symmetry, non-Markov process, concurrence

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1. Introduction

One of the most remarkable features of quantum physics is the quantum entanglement. In recent years, theoretical and experimental progresses indicate that it is an essential resource for quantum information processing,[1−3] such as quantum teleportation[4,5] and quantum dense coding.[6] Therefore, the quantum entanglement is not only the most important primary resource for quantum information[7−9] but also the cornerstone for the realization of quantum communication and quantum computer. In comparison with the classical information science, the quantum information science based on the quantum entanglement has many advantages. It can not only greatly enhance the speed and the capacity of the information processing, but also resolve information processing functions that might not be achieved in the past. In one word, it is one of the key technologies to implement the quantum information. Thus, searching for stable entanglement is an important issue in the quantum information study.

The phenomenon, in which the quantum systems fully get rid of the entanglement in a limited time due to the effect of environmental dissipation, is known as entanglement sudden death (ESD).[10−12] In order to ensure the realization of quantum logic and quantum computing, researchers have done a lot of research to avoid the ESD and proposed new schemes to achieve the entanglement. However, in a quantum open system, there are still a lot of problems related to the disentanglement.[13,14]

Using the master equation of the quantum open system, many authors have successfully investigated the decoherence of the quantum system interacting with its environment. While the decoherence of the quantum system consisted of coupling subsystems was studied, the coupling effects on the irreversible time evolution were ignored in many cases. As pointed out by Kubo *et al.*, the coupling effects are essential to derive the correct irreversible time evolution of a quantum system.[15] Ban showed that in the Markov approximation, the interaction between subsystems significantly affects the dephasing of the two-qubit system.[16] In the real physical world, the systems are referred to as non-Markov open quantum systems,[17,18] in which the non-Markov dynamics is characterized by the existence of a memory time scale so that the energy or the information would transfer from the system to the environment and then feed

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back into the system. Therefore, it is interesting to study the impact of the coupling model between the qubits and the environment on the entanglement dynamics in a non-Markov process. We will show that the interactions between the qubits and the environment significantly affect the decay of the entanglement. We find that through controlling the coupling ways between the qubits and the environment, we may make the entangling degree of the quantum system be always equal to unity for the given initial state with a maximal entanglement.

2. Model

Let us consider two coupled qubits interacting with the environment. The typical Hamiltonian for this model is ($k_{\rm B} = \hbar = 1$)[19]

$$H_{\rm total} = H_{\rm qb} + H_{\rm B} + H_{\rm qb-qb} + H_{\rm qb-B}, \tag{1}$$

with

$$H_{\rm qb} = \frac{\omega}{2}(\sigma_{1z} + \sigma_{2z}), \tag{2}$$

$$H_{\rm qb-qb} = g(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+), \tag{3}$$

where ω is the independent fluctuation of the qubit, g is the strength of the qubit–qubit interaction and represents an external field. The environment is described by a Boson harmonic oscillator with Hamiltonian $H_{\rm B}$. The interaction between the two-qubit system and the environment is assumed to be

$$H_{\rm qb-B} = SB, \tag{4}$$

where

$$S = \chi_1\sigma_{1z} + \chi_2\sigma_{2z},$$

parameter χ_1 (χ_2) represents the strength of the coupling between the qubit 1 (2) and the environment, B is the Hermitian operator of the environment. When $\chi_1 = \chi_2$, the coupling interaction is symmetric, otherwise it is asymmetric. In the following, we use the two-qubit product basis, i.e., $\{|00\rangle = |1\rangle, |01\rangle = |2\rangle, |10\rangle = |3\rangle, |11\rangle = |4\rangle\}$, where $|0\rangle$ and $|1\rangle$ are the eigenstates of the spin-1/2 operator σ_{iz} ($i = 1, 2$) with $\sigma_{iz}|0\rangle = \frac{1}{2}|0\rangle$, $\sigma_{iz}|1\rangle = -\frac{1}{2}|1\rangle$.

The density matrix is introduced to describe the quantum open system, because the state of the open system is not completely known. The time evolution of the reduced density operator ρ described by the integro-differential quantum Liouville equation of motion gives us information about the dynamics of the system. In the interaction picture, we have

$$\frac{\rm d}{{\rm d}t}\rho_{\rm I}^{\rm T}(t) = \frac{1}{{\rm i}\hbar}[H_{\rm I}(t), \rho_{\rm I}^{\rm T}(t)], \tag{5}$$

where

$$H_{\rm I}(t) \equiv {\rm e}^{{\rm i}H_0 t}H_{\rm qb-B}\,{\rm e}^{-{\rm i}H_0 t},$$
$$H_0 = H_{\rm qb} + H_{\rm B} + H_{\rm qb-qb}.$$

Because the capacity of the heat storage R is very large, we may assume that during the dynamic evolution, the quantum system has little effect on the heat storage, i.e., ${\rm Tr}_{\rm B}[H_{\rm I}, \rho(0)\otimes\rho_{\rm B}] = 0$. The $\rho_{\rm B}$ is the thermal equilibrium state of the environment. The evolving equation for the density operator may be rewritten as

$$\frac{\rm d}{{\rm d}t}\rho_{\rm I}(t) = -\int_0^t {\rm d}\tau\,{\rm Tr}_{\rm B}([H_{\rm I}(t), [H_{\rm I}(\tau), \rho_{\rm I}(t)\otimes\rho_{\rm B}]]).$$

After some calculation, the time-convolutionless quantum master equation in the Schrödinger picture is given by

$$\frac{\rm d}{{\rm d}t}\rho(t) = \frac{1}{{\rm i}\hbar}[H_0(t), \rho(t)] + \rho_{\rm L}(t) \tag{6}$$

with

$$\begin{aligned}\rho_{\rm L}(t) =\ & \alpha(t)[S(I - |2\rangle\langle 2| - |3\rangle\langle 3|)\rho(t), S] \\ & + \alpha^*(t)[S, \rho(t)(I - |2\rangle\langle 2| - |3\rangle\langle 3|)S] \\ & + \beta(t)[S(|2\rangle\langle 2| + |3\rangle\langle 3|)\rho(t), S] \\ & + \beta^*(t)[S, \rho(t)(|2\rangle\langle 2| + |3\rangle\langle 3|)S] \\ & + {\rm i}\,\gamma(t)[S(|2\rangle\langle 3| + |3\rangle\langle 2|)\rho(t), S] \\ & - {\rm i}\,\gamma^*(t)[S, \rho(t)(|2\rangle\langle 3| + |3\rangle\langle 2|)S],\end{aligned} \tag{7}$$

where

$$\alpha(t) = \int_0^t {\rm d}\tau\,\langle R(\tau)R(0)\rangle_{\rm R},$$
$$\beta(t) = \int_0^t {\rm d}\tau\,\langle R(\tau)R(0)\rangle_{\rm R}\cos(2g\tau),$$
$$\gamma(t) = \int_0^t {\rm d}\tau\,\langle R(\tau)R(0)\rangle_{\rm R}\sin(2g\tau).$$

The $\langle\cdots\rangle_{\rm R}$ denotes ${\rm Tr}_{\rm B}(\ldots\rho_{\rm B})$. In order to simplify our calculation and without loss of generality, we assume that the correlative function of the environment decays exponentially with time t, i.e.,

$$\langle R(\tau)R(0)\rangle_{\rm R} = \Delta^2\,{\rm e}^{-t/\tau}, \tag{8}$$

where τ is the relaxation time of the reservoir. After a simple calculation, we find

$$\alpha(t) = \Delta^2\tau\left(1 - {\rm e}^{-t/\tau}\right), \tag{9}$$

$$\begin{aligned}\beta(t) =\ & \frac{\Delta^2\tau}{1 + 4g^2\tau^2}[1 - {\rm e}^{-t/\tau}\cos(2gt) \\ & + 2g\tau\,{\rm e}^{-t/\tau}\sin(2gt)],\end{aligned} \tag{10}$$

$$\begin{aligned}\gamma(t) =\ & \frac{\Delta^2\tau}{1 + 4g^2\tau^2}[2g\tau - 2g\tau\,{\rm e}^{-t/\tau}\cos(2gt) \\ & - {\rm e}^{-t/\tau}\sin(2gt)].\end{aligned} \tag{11}$$

3. The solutions of quantum master equation

In the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the two qubits are initially prepared in an X-state, which includes the Bell state, the Werner state and the maximally entangled mixed state as special cases,[20,21] and we have

$$\rho(0) = \begin{pmatrix} \rho_{11}(0) & 0 & 0 & \rho_{14}(0) \\ 0 & \rho_{22}(0) & \rho_{23}(0) & 0 \\ 0 & \rho_{32}(0) & \rho_{33}(0) & 0 \\ \rho_{41}(0) & 0 & 0 & \rho_{44}(0) \end{pmatrix}. \quad (12)$$

Substituting Eq. (12) into quantum master Eq. (6), we find

$$\dot{\rho}_{11}(t) = 0, \quad (13)$$

$$\dot{\rho}_{22}(t) = \mathrm{i}\, g[\rho_{22}(t) - \rho_{33}(t)], \quad (14)$$

$$\dot{\rho}_{33}(t) = -\mathrm{i}\, g[\rho_{22}(t) - \rho_{33}(t)], \quad (15)$$

$$\dot{\rho}_{14}(t) = [-\mathrm{i}\, 2\omega - (\chi_1 + \chi_2)^2 \times \Delta^2\tau(1 - \mathrm{e}^{-t/\tau})]\rho_{14}(t), \quad (16)$$

$$\begin{aligned} \dot{\rho}_{23}(t) = & -\frac{\Delta^2\tau(\chi_1 - \chi_2)^2}{2(1 + 4g^2\tau^2)}[1 - \mathrm{e}^{-t/\tau}\cos(2gt) \\ & + \mathrm{e}^{-t/\tau}\sin(2gt)]\,[\rho_{23}(t) + \rho_{32}(t)] \\ & + \Big\{ -\mathrm{i}\, g + \frac{(\chi_1 - \chi_2)^2\Delta^2\tau}{2(1 + 4g^2\tau^2)}[1 - \mathrm{e}^{-t/\tau}\cos(2gt) \\ & + 2g\tau\,\mathrm{e}^{-t/\tau}\sin(2gt)] \Big\}\,[\rho_{23}(t) - \rho_{32}(t)] \\ & - \frac{(\chi_1 - \chi_2)^2\Delta^2\tau}{2(1 + 4g^2\tau^2)}[2g\tau - 2g\tau\,\mathrm{e}^{-t/\tau}\cos(2gt) \\ & - \mathrm{e}^{-t/\tau}\sin(2gt)]\,[\rho_{22}(t) - \rho_{33}(t)]. \end{aligned} \quad (17)$$

For the given initial condition in Eq. (12), the solutions of Eqs. (13)–(17) can be expressed as

$$\rho_{11}(t) = \rho_{11}(0), \quad (18)$$

$$\rho_{22}(t) = \frac{1}{2}[1 + f_1(t)]\rho_{22}(0) + \frac{1}{2}[1 - f_1(t)]\rho_{33}(0) - \frac{g}{\Omega}f_3(t)\,\mathrm{Im}\,[\rho_{23}(0)], \quad (19)$$

$$\rho_{33}(t) = \frac{1}{2}[1 - f_1(t)]\rho_{22}(0) + \frac{1}{2}[1 + f_1(t)]\rho_{33}(0) + \frac{g}{\Omega}f_3(t)\,\mathrm{Im}\,[\rho_{23}(0)], \quad (20)$$

$$\rho_{14}(t) = \rho_{14}(0)\exp\left[-\Delta^2\tau(\chi_1 + \chi_2)^2(1 - \mathrm{e}^{-t/\tau})t\right] \times [\cos(2\omega t) - \mathrm{i}\sin(2\omega t)], \quad (21)$$

$$\rho_{23}(t) = \frac{1}{2}[\,\mathrm{e}^{-2ut} + f_2(t)]\rho_{23}(0) + \frac{1}{2}[\,\mathrm{e}^{-2ut} - f_2(t)]\rho_{32}(0) + \mathrm{i}\,\frac{g + v}{\Omega}f_3(t)\,[\rho_{22}(0) - \rho_{33}(0)], \quad (22)$$

$$\rho_{32}(t) = \rho_{23}^*(t), \quad (23)$$

$$\rho_{41}(t) = \rho_{14}^*(t), \quad (24)$$

with

$$f_1(t) = \mathrm{e}^{-ut}\left[\cosh(\Omega t) + \frac{u}{\Omega}\sinh(\Omega t)\right], \quad (25)$$

$$f_2(t) = \mathrm{e}^{-ut}\left[\cosh(\Omega t) - \frac{u}{\Omega}\sinh(\Omega t)\right], \quad (26)$$

$$f_3(t) = \mathrm{e}^{-ut}\sinh(\Omega t), \quad (27)$$

$$u = \frac{\Delta^2\tau(\chi_1 - \chi_2)^2}{2(1 + 4g^2\tau^2)}[1 - \mathrm{e}^{-t/\tau}\cos(2gt) + \mathrm{e}^{-t/\tau}\sin(2gt)], \quad (28)$$

$$v = \frac{\Delta^2\tau(\chi_1 - \chi_2)^2}{2(1 + 4g^2\tau^2)}[2g\tau - 2g\tau\,\mathrm{e}^{-t/\tau}\cos(2gt) - \mathrm{e}^{-t/\tau}\sin(2gt)], \quad (29)$$

$$\Omega = \sqrt{u^2 - 4g(g + v)}. \quad (30)$$

It is necessary to emphasize that Eqs. (18)–(24) include the non-Markov effects of the composite quantum system. In the limit $t \to \infty$, it gives the quantum features of Markovian.

4. Decay of two-qubit entanglement

In order to describe the evolution of the quantum entanglement, we adopt the concurrence defined by Wootters to quantify the degree of entanglement for any bi-partite system. It is well-known that any reliable measurement of the entanglement will yield the same result. The concurrence $C(t)$ is defined as[22]

$$C(t) = \max\{0,\ \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (31)$$

where λ_j are the eigenvalues of $W = \rho(t)\tilde{\rho}(t)$ in nonincreasing order of magnitude. The $\tilde{\rho}(t) = \left(\sigma_y^{(1)} \otimes \sigma_y^{(2)}\right)\rho^*(t)\left(\sigma_y^{(1)} \otimes \sigma_y^{(2)}\right)$. In our case, the concurrence with the time evolution is given by

$$C(t) = 2\max\{0,\ |\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)}, |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}\}, \quad (32)$$

which is obviously dependent on the initial conditions. Our goal is to preserve the entanglement of the two-qubit system. Therefore, the maximum entanglement state is prepared as the initial state.

First, we assume that the two qubits are initially at the two-excitation Bell state, i.e.,

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right), \quad (33)$$

which may be written in the density matrix representation as

$$\rho(0) = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & \pm 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pm 1 & 0 & 0 & 1 \end{pmatrix}.$$

Inserting $\rho(0)$ into Eqs. (18)–(24), we obtain

$$\begin{aligned}
\rho_{11}(t) &= \rho_{44}(t) = \frac{1}{2}, \\
\rho_{14}(t) &= \pm\frac{1}{2}\exp\left[-\Delta^2\tau(\chi_1+\chi_2)^2(1-\mathrm{e}^{-t/\tau})t\right] \\
&\quad \times [\cos(2\omega t) - \mathrm{i}\sin(2\omega t)], \\
\rho_{41}(t) &= \frac{1}{2}\exp\left[-\Delta^2\tau(\chi_1+\chi_2)^2(1-\mathrm{e}^{-t/\tau})t\right] \\
&\quad \times [\cos(2\omega t) + \mathrm{i}\sin(2\omega t)],
\end{aligned}$$

the others are zero. Note that the time evolution of the density matrix preserves the form of the Bell state. Thus we can easily obtain the concurrence function as

$$C(t) = \max\{0,\ \tilde{C}(t)\},$$

with

$$\tilde{C}(t) = \exp\left[-\Delta^2\tau(\chi_1+\chi_2)^2(1-\mathrm{e}^{-t/\tau})t\right].$$

Under the asymmetric coupling with $\chi_1 + \chi_2 = 0$, obviously, the degree of the entanglement is always 1 and the decoherence does not occur. By keeping the interaction between the quantum bits and the environment at an asymmetric coupling, we can make the quantum system always be in the maximum entangled state for the initial state $(|00\rangle \pm |11\rangle)/\sqrt{2}$. Furthermore, we find that the concurrence is independent of the interacting constant g between the coupled qubits, which means that the interaction does not contribute to the entanglement.

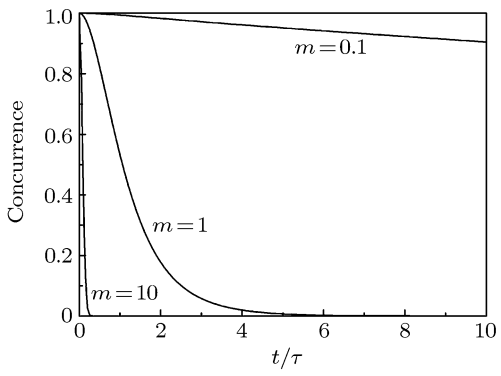


Fig. 1. Concurrence as a function of t/τ for different m. The two qubits are initially in the state $(|00\rangle \pm |11\rangle)/\sqrt{2}$.

By setting $\Delta\tau$ as a constant and $m = \Delta\tau/(\chi_1 + \chi_2)$, the concurrence $\tilde{C}(t)$ as a function of t/τ is shown in Fig. 1 for different values of m. The rate m actually reflects the environmental impact (when m is fixed). The bigger the rate m ($\Delta\tau$) is, the bigger the dissipative effect is. From Fig. 1, we can see that the bigger the dissipation is, the more obvious the decoherent phenomenon is and the ESD is easier to occur.

Then, we consider the case that the initial state of the two qubits is at the one-excitation Bell state, i.e.,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right), \tag{34}$$

which has the following density matrix representation

$$\rho(0) = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \pm 1 & 0 \\ 0 & \pm 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The solution of the master equation may be written as

$$\begin{aligned}
\rho_{22}(t) &= \rho_{33}(t) = \frac{1}{2}, \\
\rho_{23}(t) &= \pm\frac{1}{2}\exp\left\{-\frac{\Delta^2\tau(\chi_1-\chi_2)^2}{(1+4g^2\tau^2)}\right. \\
&\quad \left.\times[1-\mathrm{e}^{-t/\tau}\cos(2gt) + \mathrm{e}^{-t/\tau}\sin(2gt)]t\right\}, \\
\rho_{32}(t) &= \rho_{23}^*(t),
\end{aligned}$$

the others are zero. The concurrence function $\tilde{C}(t)$ is given by

$$\begin{aligned}
\tilde{C}(t) &= \exp\left\{-\frac{\Delta^2\tau(\chi_1-\chi_2)^2}{(1+4g^2\tau^2)}\right. \\
&\quad \left.\times[1-\mathrm{e}^{-t/\tau}\cos(2gt) + \mathrm{e}^{-t/\tau}\sin(2gt)]t\right\}.
\end{aligned}$$

We can see that under the symmetrical coupling ($\chi_1 - \chi_2 = 0$), the concurrence is always equal to 1, so the decoherence does not occur. By keeping the interaction between the qubits and the environment at the symmetric coupling, the degree of the entanglement is independent of the evolving time and the quantum system is always at the maximum entangled state for the initial state $(|01\rangle \pm |10\rangle)/\sqrt{2}$. Even if we cannot make the coupling be completely symmetric, as long as the difference of the coupling coefficients is small, the ESD is difficult to occur. Subsequently, the result is conducive to preserve the entanglement in the quantum information processing. Different from the situation for the initial state $(|00\rangle \pm |11\rangle)/\sqrt{2}$, the degree of the entanglement depends obviously on the coupling constant g between the qubits. Through a

suitable choice of the coupling, it is conducive to the completion of the quantum information processing.

By setting $\Delta\tau$ as a constant and $n = \Delta\tau/(\chi_1 - \chi_2)$, the concurrence $\tilde{C}(t)$ as a function of t/τ is shown in Figs. 2 for different values of n.

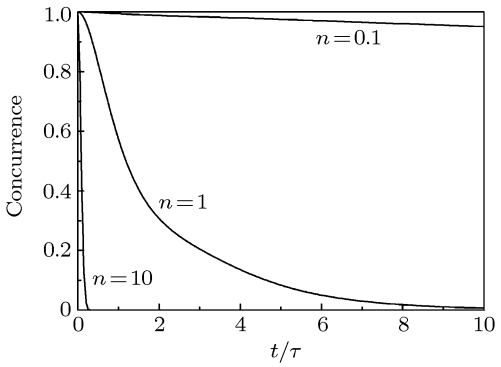


Fig. 2. Concurrence as a function of t/τ for different n. The two qubits are initially in the state $(|01\rangle \pm |10\rangle)/\sqrt{2}$, $2g\tau = 1$.

Comparing Fig. 2 with Fig. 3, we can see that with the increasing interaction between the qubits, it is easier to preserve the entanglement of the quantum system, which indicates that the contribution of the interaction between the qubits is to avoid the entanglement sudden death for certain initial states. The result may be helpful to realize the quantum information processing.

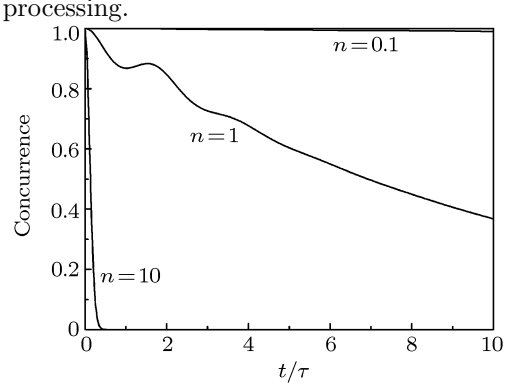


Fig. 3. Concurrence as a function of t/τ for different rates n. The two qubits are initially in the state $(|01\rangle \pm |10\rangle)/\sqrt{2}$, $2g\tau = 3$.

5. Conclusion

In summary, the entangling dynamics has been investigated for the coupled two-qubit system interacting with a non-Markov environment in terms of concurrence. The results show that the degree of the entanglement depends not only on the initial quantum state but also on the coupling between the qubit and the environment and on the interaction between the qubits. Furthermore, we find that by controlling the interaction between the qubit and the environment at an asymmetric coupling, we may preserve the maximum entangled state in the evolution of non-Markov process for the initial state $|\Phi\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$. For the initial state $|\Psi\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$, in contrast, the interaction should be operated at the symmetric coupling in order to preserve the maximum entangled state. In addition, it is helpful to the entanglement of the quantum system by strengthening the interaction between the qubits.

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