Entanglement dynamics of a moving multi-photon Jaynes–Cummings model in mixed states*

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(Received 31 October 2010; revised manuscript received 21 March 2011)

Using the algebraic dynamical method, the entanglement dynamics of an atom-field bipartite system in a mixed state is investigated. The atomic center-of-mass motion and the field-mode structure are also included in this system. We find that the values of the detuning and the average photon number are larger, the amplitude of the entanglement is smaller, but its period does not increase accordingly. Moreover, with the increase of the field-mode structure parameter and the transition photon number, the amplitude of the entanglement varies slightly while the oscillation becomes more and more fast. Interestingly, a damping evolution of the entanglement appears when both the detuning and the atomic motion are considered simultaneously.

Keywords: algebraic dynamical method, entanglement, mixed state

PACS: 03.65.Ud, 03.67.Bg, 03.67.Mn **DOI:** 10.1088/1674-1056/20/7/070303

1. Introduction

Entanglement has been widely investigated in quantum information processing[1−8] and plays an important role in many potential applications, such as quantum communication, quantum teleportation, quantum cryptography, entanglement swapping, dense coding and quantum computing.[9−17] Moreover, with the rapid development of quantum information, more accurate quantifications for entanglement are required to produce, manipulate and detect entanglement states. Most earlier studies paid much attention to the bipartite systems that start in pure states, which are presently well understood.[18] Meanwhile, for mixed states, the entanglement is difficult to measure, because it is not easy to define an analog of the Schmidt decomposition. However, it is more significant for studying the systems in mixed states since there exists no system that can be decoupled perfectly from environmental influence for realistic states observed in experiments. Several entanglement measures for mixed states have been proposed, such as the entanglement of formation $E_{\rm F}(\rho)$, the entanglement cost $E_{\rm C}(\rho)$ and the distillable entanglement $E_{\rm D}(\rho)$,[19,20] but these are not effective computational means. To overcome these shortcomings, Peres[21] and Horodecki *et al.*[22] have proposed a new criterion for separability. Based on this criterion, negativity is introduced, which is a good quantity for measuring the entanglement of mixed states[23,24] and is equal to absolute value of the sum of negative eigenvalues of the partial transpose of density operator with a bipartite mixed state.

There is considerable research on the dynamics of entanglement for the Jaynes–Cummings model (JCM)[25] with a mixed state. The entanglement for the model consisting of a two-level atom interacting with a single mode field through a two-photon process was studied.[26] In Ref. [27], the authors studied the effects of phase damping on the entanglement evolution, where the two-level atom initially in a mixed state interacts with a multi-quanta single mode quantized cavity field including the cavity loss. Akhtarshenas and Farsi[28] studied the entanglement of the JCM with the atom initially in a mixed state and the field prepared in a Fock state. Very recently, the entanglement for the JCM with a two-level atom in a lossy cavity was manipulated by a classical driving field,[29] which was a novel scheme to control the behaviour of the entanglement. Xu H S and Xu J B[30] studied the

*Project supported by the National Natural Science Foundation of China (Grant No. 10704031) and the Fundamental Research Funds for the Central Universities of China (Grant No. lzujbky-2010-75).

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entanglement dynamics of a three-level atom in $\Xi-$ configuration and they found that the maximal value of the entanglement decreases with the temperature of thermal field and can be controlled by the detuning. Zhang and Xu[31] considered the entanglement of JCM interacting with two mode quantized fields for two different initial situations. Yonäc *et al.*[32] studied the entanglement dynamics of two separate Jaynes–Cummings Hamiltonians which consist of two identical two-level atoms inside two separate cavities, respectively. They investigated how the two sets of pure initial states affected the entanglement dynamics in the resonant case. In Ref. [33] it is shown that the pure entangled states are very difficult to obtain due to the interaction of uncontrollable environment and thus the atomic initial state prepared in mixed states is more practical and general. In this paper, we will focus on the problem of a moving two-level atom coupled with a quantized field in a multi-photon transition process. The effects of detuning, transition photon number, atomic motion, field-mode structure and temperatures on the entanglement are all discussed. In fact, the model including all these factors is extremely difficult to deal with using the conventional methods. Here we adopt the algebraic dynamical approach[34,35] to study the nonlinear system, and the key idea is to introduce a canonical transformation that transforms the Hamiltonian into a liner function in terms of a set of Lie algebraic generators. The nonlinear system is thus integrable and solvable, then the evolution of density operator with time can be obtained easily.

The rest of the present paper is organized as follows. In Section 2, we present the basic model and derive the time evolution expression of the density matrix using the algebraic dynamical approach. In Section 3, we study the dynamics evolution of the entanglement between the two subsystems for different cases with the atom initially in a pure state and a mixed state. Finally, a brief conclusion is given in Section 4.

2. Multi-photon JCM

We investigate a moving two-level atom interacting with a single mode quantized radiation field in a multi-photon transition process. Within the rotating wave approximation, the Hamiltonian of the bipartite system can be described as

$$H = \omega_0 S_z + \omega a^\dagger a + g[f(z)]^l (a^{\dagger l} S_- + a^l S_+) \ (\hbar = 1), \tag{1}$$

where ω_0 is the atomic transition frequency, ω is the field frequency, $a^\dagger(a)$ denotes the creation (annihilation) operator of the radiation field and S_+ (S_-) represents the atomic raising (lowering) operator, S_z is the atomic inversion operator, g is the atom–field coupling constant and $f(z)$ is the shape function of the cavity field mode.[36] The atomic motion can be incorporated as

$$f(z) \to f(\upsilon t), \tag{2}$$

where υ is the atomic motion velocity. A specific mode TEM_{mnp} is defined as

$$f(z) = \sin(\frac{p\pi\upsilon t}{L}), \tag{3}$$

where p represents the number of half-wavelengths of the field mode inside a cavity with the length L. When the atomic motion velocity is considered to be $\upsilon = gL/\pi$, the evolution function $\theta(t)$ can be obtained as

$$\theta(t) = \int_0^t [f(\upsilon t')]^l \mathrm{d}t'. \tag{4}$$

We restrict the motion of the atom along the z-axis. There exists one conservative quantity for the system as follows:

$$N = a^\dagger a + l(S_z + \frac{1}{2}), \tag{5}$$

which commutes with Hamiltonian (1). To linearize the Hamiltonian by the algebraic dynamical approach, a canonical transformation operator is introduced as

$$U_g = \exp\Big[\frac{\alpha}{F^{1/2}}(a^l S_+ - a^{\dagger l} S_-)\Big], \tag{6}$$

where

$$\alpha = -\arctan[(\sqrt{\Delta^2/4 + \tilde{g}^2 F} - \Delta/2)/\tilde{g}F^{1/2}],$$

$$\Delta = \omega_0 - l\omega,$$

$$\tilde{g} = g\theta(t)/t,$$

and

$$F = N!/(N-l)!.$$

Based on the algebraic dynamical method, SU(2) algebra generators $\{J_0,\ J_+,\ J_-\}$ are introduced, with $J_0 = S_z$, $J_+ = F^{-1/2} a^l S_+$, $J_- = F^{-1/2} a^{\dagger l} S_-$, which satisfy the following commutation relations:

$$[J_0, J_+] = J_+,\ [J_0, J_-] = -J_-,\ [J_+, J_-] = 2J_0. \tag{7}$$

Then the canonical transformation operator is reduced to

$$U_g = \exp[\alpha(J_+ - J_-)]. \tag{8}$$

After the canonical transformation, the dressed Hamiltonian can be written in terms of SU(2) algebra generators as

$$H' = U_g^{-1}\Big(\int H\mathrm{d}t\Big)U_g = [E(N) + \lambda J_0]t, \quad (9)$$

where

$$\lambda = \sqrt{\Delta^2 + 4\tilde{g}^2 F},$$

and

$$E(N) = \omega(N - l/2).$$

Then the time evolution operator can be expressed as

$$\begin{aligned} U(t) &= \mathrm{e}^{-\mathrm{i}\int H\mathrm{d}t} \\ &= U_g \exp\big[-\mathrm{i}(E(N) + \lambda J_0)t\big]U_g^{-1} \\ &= \mathrm{e}^{-\mathrm{i}E(N)t}\Big[\cos\frac{\lambda t}{2} - 2\mathrm{i}J_0 \sin\frac{\lambda t}{2}\cos 2\alpha \\ &\quad + \mathrm{i}(J_+ + J_-)\sin\frac{\lambda t}{2}\sin 2\alpha\Big]. \end{aligned} \quad (10)$$

Assuming that the cavity field is initially in the single-mode thermal state

$$\rho_f(0) = \sum_{n=0}^{\infty} P_n|n\rangle\langle n|, \quad P_n = \frac{m^n}{(m+1)^{n+1}}, \quad (11)$$

with $m = 1/[\exp(\omega/T) - 1]$, which is the mean photon number of the cavity field for thermal equilibrium at a certain temperature T, and the atom is initially in a statistical mixed state

$$\rho_a(0) = C_e|e\rangle\langle e| + C_g|g\rangle\langle g|, \quad (12)$$

where $C_e + C_g = 1$, $C_g = 0$ ($C_g = 1$) means that the atom is initially prepared in the excited state (the ground state) and $0 < C_g < 1$ represents that the initial state of the atom is in the mixed state. Since there is no system that can be decoupled perfectly from environmental influence, mixing is unavoidable. Mixed states are more realistic and meaningful than pure states in experimental situation.[37] Accordingly, the initial state of the total system can be given as

$$\begin{aligned} \rho_{af}(0) &= \rho_f(0) \otimes \rho_a(0) \\ &= \sum_n P_n C_g|n, g\rangle\langle n, g| + \sum_n P_n C_e|n, e\rangle\langle n, e|. \end{aligned} \quad (13)$$

Using Eqs. (10) and (13), the density operator with the time evolution can be written as

$$\begin{aligned} \rho_{af}(t) &= U(t)\rho_{af}(0)U^+(t) \\ &= \sum_n P_n C_g \rho_g(t) + \sum_n P_n C_e \rho_e(t), \end{aligned} \quad (14)$$

where

$$\begin{aligned} \rho_g(t) = &\Big[\cos^2\Big(\frac{\lambda_n t}{2}\Big) + \sin^2\Big(\frac{\lambda_n t}{2}\Big)\cos^2(2\alpha_n)\Big]|n, g\rangle\langle n, g| \\ &- \mathrm{i}\sin(\frac{\lambda_n t}{2})\sin(2\alpha_n)\Big[\cos\frac{\lambda_n t}{2} + \mathrm{i}\sin\frac{\lambda_n t}{2}\cos(2\alpha_n)\Big]|n, g\rangle\langle n - l, e| \\ &+ \mathrm{i}\sin\Big(\frac{\lambda_n t}{2}\Big)\sin(2\alpha_n)\Big[\cos\frac{\lambda_n t}{2} - i\sin\frac{\lambda_n t}{2}\cos(2\alpha_n)\Big]|n - l, e\rangle\langle n, g| \\ &+ \sin^2\Big(\frac{\lambda_n t}{2}\Big)\sin^2(2\alpha_n)|n - l, e\rangle\langle n - l, e|, \end{aligned} \quad (15)$$

$$\begin{aligned} \rho_e(t) = &\Big[\cos^2\Big(\frac{\lambda_{n+l} t}{2}\Big) + \sin^2\Big(\frac{\lambda_{n+l} t}{2}\Big)\cos^2(2\alpha_{n+l})\Big]|n, e\rangle\langle n, e| \\ &- \mathrm{i}\sin\Big(\frac{\lambda_{n+l} t}{2}\Big)\sin(2\alpha_{n+l})\Big[\cos\frac{\lambda_{n+l} t}{2} - \mathrm{i}\sin\frac{\lambda_{n+l} t}{2}\cos(2\alpha_{n+l})\Big]|n, e\rangle\langle n + l, g| \\ &+ \mathrm{i}\sin\Big(\frac{\lambda_{n+l} t}{2}\Big)\sin(2\alpha_{n+l})\Big[\cos\frac{\lambda_{n+l} t}{2} + \mathrm{i}\sin\frac{\lambda_{n+l} t}{2}\cos(2\alpha_{n+l})\Big]|n + l, g\rangle\langle n, e| \\ &+ \sin^2\Big(\frac{\lambda_{n+l} t}{2}\Big)\sin^2(2\alpha_{n+l})|n + l, g\rangle\langle n + l, g|, \end{aligned} \quad (16)$$

where

$$\lambda_n = \sqrt{\Delta^2 + 4\tilde{g}^2 F(n)},$$

$$\alpha_n = -\arctan[(\sqrt{\Delta^2/4 + \tilde{g}^2 F(n)} - \Delta/2)/(\tilde{g}F^{1/2}(n))],$$

and

$$F(n) = n!/(n - l)!.$$

3. Entanglement evolution for bipartite system

In this section the negativity is employed to investigate the entanglement for the coupled atom-field system described by Eq. (14). The entanglement de-

gree is quantified into[23]

$$\mathcal{N}(\boldsymbol{\rho}) \equiv \frac{\|\boldsymbol{\rho}^{\mathrm{T}}\| - 1}{2}, \tag{17}$$

where $\boldsymbol{\rho}^{\mathrm{T}}$ is the partial transpose of $\boldsymbol{\rho}$ and $\|\boldsymbol{\rho}^{\mathrm{T}}\|$ denotes the trace norm of $\boldsymbol{\rho}^{\mathrm{T}}$ which is equal to the sum of the absolute values for all eigenvalues of $\boldsymbol{\rho}^{\mathrm{T}}$. Therefore, $\mathcal{N}(\boldsymbol{\rho})$ can also be expressed as the absolute value of the sum of the negative eigenvalues of $\boldsymbol{\rho}^{\mathrm{T}}$, which has been proposed as a quantitative entanglement measure. It is a good measure of entanglement because the negativity is an entanglement monotone, it does not increase under local operations and classical communication (LOCC) and can also be easily treated mathematically. From Eqs. (14) and (17), the amount of entanglement can be obtained from the following result:

$$\begin{aligned}\|\boldsymbol{\rho}^{\mathrm{T}}(t)\| = &\sum_{n=0}^{l-1}|\xi_n| + \frac{1}{2}\sum_{n=0}^{\infty}\Big[|\mu_n + \xi_{n+l} \\ &+ \sqrt{(\mu_n - \xi_{n+l})^2 + 4\phi_n\chi_n}| + |\mu_n + \xi_{n+l} \\ &- \sqrt{(\mu_n - \xi_{n+l})^2 + 4\phi_n\chi_n}|\Big], \end{aligned}\tag{18}$$

where

$$\begin{aligned}\mu_n =& C_g P_n\Big[\cos^2\Big(\frac{\lambda_n t}{2}\Big) + \sin^2\Big(\frac{\lambda_n t}{2}\Big)\cos^2(2\alpha_n)\Big] \\ &+ C_e P_{n-l}\sin^2\Big(\frac{\lambda_n t}{2}\Big)\sin^2(2\alpha_n), \\ \xi_n =& C_e P_n\Big[\cos^2\Big(\frac{\lambda_{n+l} t}{2}\Big) + \sin^2\Big(\frac{\lambda_{n+l} t}{2}\Big)\cos^2(2\alpha_{n+l})\Big] \\ &+ C_g P_{n+l}\sin^2\Big(\frac{\lambda_{n+l} t}{2}\Big)\sin^2(2\alpha_{n+l}), \\ \phi_n =& \chi_n^* \\ =& \mathrm{i}(C_g P_{n+l} - C_e P_n)\sin\Big(\frac{\lambda_{n+l} t}{2}\Big)\sin(2\alpha_{n+l}) \\ &\times\Big[\cos\frac{\lambda_{n+l} t}{2} - \mathrm{i}\sin\frac{\lambda_{n+l} t}{2}\cos(2\alpha_{n+l})\Big]. \end{aligned}\tag{19}$$

In what follows, we present some interesting numerical results for different parameters in order to demonstrate their effects on the entanglement degree. In Fig. 1, the negativity is shown as a function of C_g and scaled time gt/π. We find that different atomic initial state have a great influence on the entanglement. The larger the probability of the atom initially in the ground state is, the smaller the value of the entanglement is. When the value of C_g exceeds 0.5, the entanglement approaches zero. Therefore, we choose $C_g = 0.2$ in the following figures to clearly demonstrate the effects of the other parameters on the entanglement. Figure 2 displays the relationships between the entanglement and the atomic motion and the field-mode structure. Here, the atomic motion is considered to be constant: $\upsilon = gL/\pi$ and it is neglected by performing $\theta(t) \to t$. By comparing Fig. 2(a) and Fig. 2(b), it is found that the atomic motion leads to the regular periodic evolution of the negativity. Furthermore, the increasing of p shortens the period of the entanglement and results in a slight decrease of the amplitude, as depicted in Figs. 2(b) and 2(c). In fact, when the field-mode structure parameter p varies in a small region, the amplitude of the entanglement does not change a lot. Interestingly, when the atomic motion is considered, the negativity can go to zero abruptly and then revives after an interval of time, which is called entanglement sudden death (ESD).[38–40]

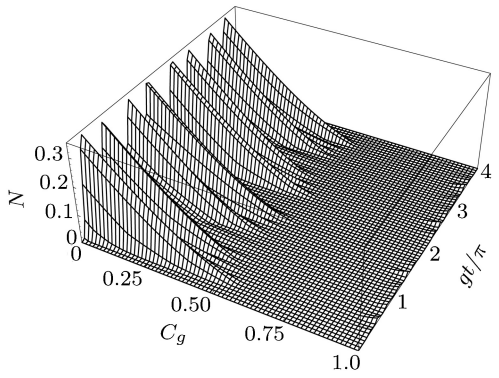


Fig. 1. The evolution of the negativity as functions of C_g and the scaled time gt/π. The atomic motion is considered, other parameters include: $m = 1$, $\Delta = 0$, $p = 1$, $l = 2$.

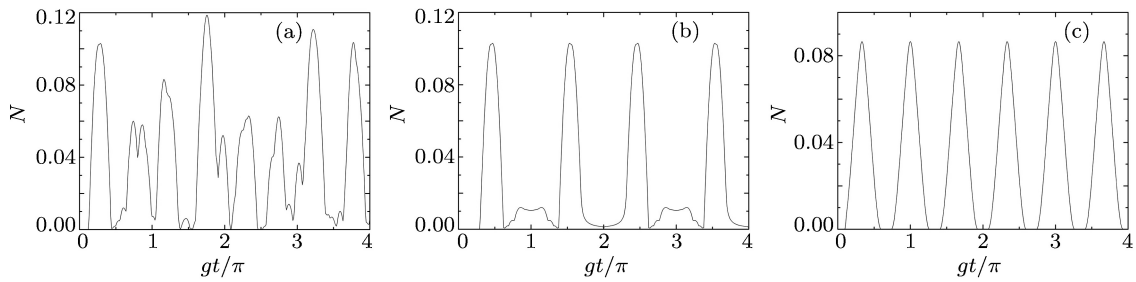


Fig. 2. The evolution of the negativity as a function of the scaled time gt/π for different parameters p. Other parameters include: $m = 1$, $\Delta = 0$, $l = 1$, $C_g = 0.2$. (a) The atomic motion is neglected, (b) $p = 1$, (c) $p = 3$.

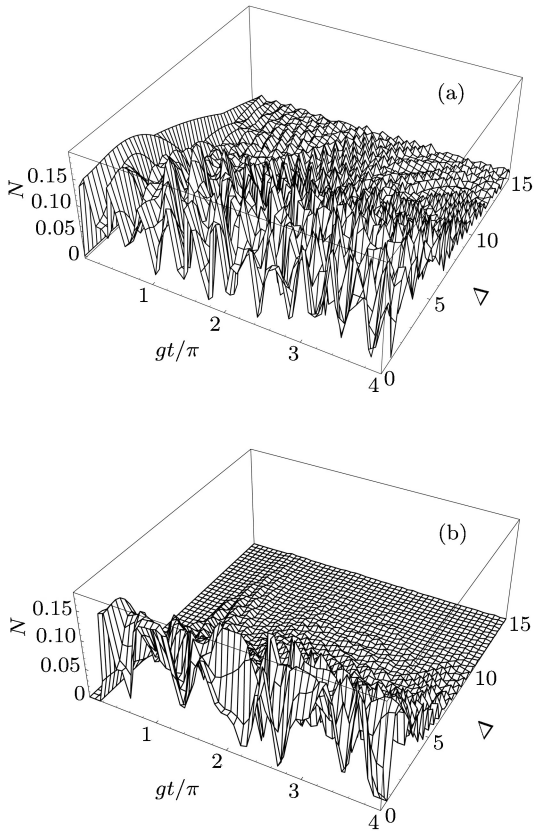


Fig. 3. The evolution of the negativity as functions of the scaled time gt/π and Δ. Other parameters include: $m = 1$, $l = 2$, $C_g = 0.2$. (a) The atomic motion is neglected, (b) the atomic motion is considered ($p = 1$).

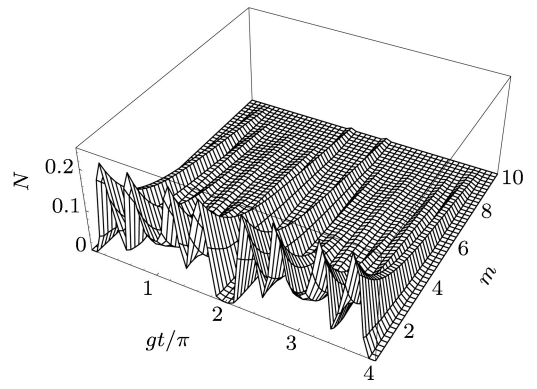


Fig. 4. The evolution of the negativity as functions of the scaled time gt/π and the average photon number m. The atomic motion is considered, other parameters include: $\Delta = 0$, $p = 1$, $l = 2$, $C_g = 0.2$.

Now we turn our attention to the effects of the detuning. What is worth noting is that in most previous researches only the case of the resonance is considered to avoid the complexity. Without loss of generality, we consider that the detuning is necessary and meaningful. In Fig. 3, it clearly shows that when the detuning increases the entanglement decreases and oscillates fast,[41] and it is interesting to note that when the detuning and the atomic motion are all considered, a damping evolution of the entanglement arises. The former is attributed to the fact that when dutuning is large, the atomic system is weakly coupled to the radiation field and the latter may be referred to as a quantum decoherence[42−44] when the interaction between the internal and the external atomic dynamics is considered (see Fig. 3(b)). In addition, it is shown that the atomic motion can lead to a damping in the correlation function of non classical behaviours between the field and internal atomic variables, which induces the separability of the atom and the field.[43]

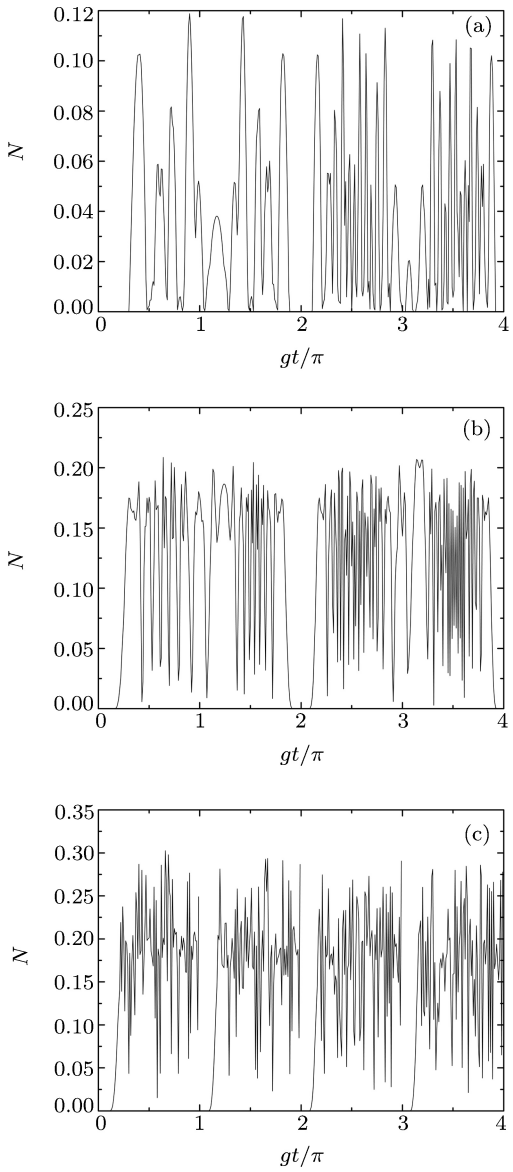


Fig. 5. The evolution of the negativity as a function of the scaled time gt/π for different parameters l. The atomic motion is considered, other parameters include: $m = 1$, $\Delta = 0$, $p = 1$, $C_g = 0.2$. (a) $l = 1$, (b) $l = 3$, (c) $l = 8$.

Figure 4 shows how the average photon number m influences the entanglement evolution. One can see that with the increase of the average photon number the negativity decreases, but the evolution periodicity of the entanglement does not change. These results show that the maximum value of the entanglement depends on the mean photon number of the field (or the thermal equilibrium temperature T), but the period does not, which accords well with the results in Ref. [45]. As a matter of fact, for the thermal field, when the average photon number increases, the weight mixture of number states denoted by P_n becomes small, so the entanglement feature is washed out greatly. In addition, photon transition process can also affect the entanglement as shown in Fig. 5. The entanglement increases as the photon number l increases, meanwhile the oscillation of the negativity becomes faster and faster.[46]

Experimentally, the realization of the JCM in this paper is not difficult. We can inject a highly excited Rydberg atom along the axis of the cavity which moves fast to eliminate reflection effect from the cavity, then we can assume that it has a constant velocity along the cavity.[47] The transition between the upper and the lower levels of the atom may involve l photons by adjusting the energy separation between the levels close to the energy of l quanta of the electromagnetic field. Photons acting as the fastest information carriers have only very weak coupling to the environment and are thus the most successful physical system for the observation of multi-partite entanglement and proof-of-principle demonstrations of a diversity of quantum information applications so far. Multi-photon entanglement has become a hot topic recently and we believe that the new approach in this paper will promote the relevant development.

4. Conclusion

We have used negativity to study the entanglement for the system of a moving two-level atom interacting with a quantized single mode field in the mixed state with a multi-photon process. With the algebraic dynamical method, its solution is obtained. In our work, the atomic motion, the field-mode structure, the detuning, the transition photon number and the average photon number of the field are all considered. Our studies show that the increase of the detuning or the average photon number can lead to the decrease of the negativity. For lager detuning, the entanglement oscillates fast. Atomic motion and the field-mode structure also give rise to many interesting effects on the entanglement. The ESD appears and an increase of the parameter p can shorten the evolution periodicity of the entanglement. Importantly, a damping evolution of the entanglement happens when the detuning and the atomic motion are considered together. Furthermore, we demonstrate that a lager photon number l makes the oscillation of the negativity become quick and leads its amplitude to go up slightly. Our researches might shed light on the entanglement of the JCM and provide a simple way to study the dynamics of the entanglement.

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