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Spin symmetric solutions of Dirac equation with Pöschl–Teller potential*

Ferhat Taşkın and Gökhan Koçak†

Department of Physics, Erciyes University, Kayseri 38039, *Turkey*

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The approximate analytical solutions of the Dirac equation with the Pöschl–Teller potential is presented for arbitrary spin-orbit quantum number κ within the framework of the spin symmetry concept. The energy eigenvalues and the corresponding two Dirac spinors are obtained approximately in closed forms. The limiting cases of the energy eigenvalues and the two Dirac spinors are briefly discussed.

Keywords: Dirac equation, Pöschl–Teller potential, spin symmetry

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1. Introduction

In the last decades, a large amount of research work has been devoted to the study of the Dirac equation with the spin or the pseudospin symmetry. In the Dirac equation, the spin symmetry arises when the magnitudes of the attractive scalar potential $S(r)$ and the repulsive vector potential $V(r)$ are nearly equal, i.e., $\Delta(r) = V(r) - S(r) =$ constant. On the other hand, the pseudospin symmetry occurs if $S(r)$ and $-V(r)$ are nearly equal, i.e., $\Sigma(r) = V(r) + S(r) =$ constant.[1−3] The spin and the pseudospin symmetries have many application in the nuclear physics and the related areas.[2−5] In nuclear physics, the spin and the pseudospin symmetries are important for explaining aspects of the deformed nuclei in the hadrons and only recently become one of the topics of theoretical investigations.[2,6−8] Spin symmetry occurs in the spectrum of a meson with one heavy quark and one antinucleon bounded in a nucleus.[2] The pseudospin symmetry occurs in the spectrum of certain nuclei.[4] By using relativistic theories with scalar and vector potentials, Ginocchio showed that these symmetries could explain degeneracies in meson spectra or in single-particle energy levels.[3] The symmetries have also been used to elucidate the properties of the deformed nuclei[9] and of the super deformation,[10] and have been used to establish an effective nuclear shell-model scheme.[11] The spin symmetry in nuclear theory refers to a quasi-degeneracy of the single-nucleon doublets and can be characterized with the non-relativistic quantum numbers $(n, l, j = l + 1/2)$ and $(n, l, j = l - 1/2)$, where n, l and j are the single-nucleon radial, orbital and total angular momentum quantum numbers for a single particle, respectively. The pseudospin symmetry means that $(n, l, j = l + 1/2)$ and $(n - 1, l + 2, j = l + 3/2)$ states are degenerate. Some authors have applied the spin and the pseudospin symmetries to several physical potentials, including the harmonic oscillator,[12−15] the Morse,[16,17] the Hulthén,[18] the Eckart[19−21] and the Manning–Rosen potentials.[22]

Recently, the bound state solutions for the Pöschl–Teller (PT) potential have been investigated by using a proper approximation for the spin-orbit coupling term.[23] The modified PT and the deformed generalized PT potentials for the Dirac equation with the scalar and vector have been discussed by using the algebraic method and the s-wave bound states.[24,25] The pseudospin symmetry properties for the generalized PT potential have been studied by using the supersymmetry quantum mechanics (SUSYQM) and the shape invariance methods.[26] The bound and the scattering states for the generalized PT potential have been analyzed in the nonrelativistic case.[27,28] Grosche[29] has also studied the deformed generalized PT potential by using the path integral method. In this paper, we will study the Dirac equation with the PT potential and the spin-orbit coupling term by using the approximation given in Refs. [27] and [30]. The

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†Corresponding author. E-mail: gkocak@erciyes.edu.tr

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rest of the paper is organized as follows. In Section 2, the approximate closed forms for the energy eigenvalues and the corresponding two Dirac spinors are obtained within the framework of the spin symmetry concept. A numerical example is presented in Table 1. The last section gives our conclusion.

2. Bound state solutions

The Dirac equation with both a scalar potential $S(r)$ and a vector potential $V(r)$ is ($\hbar = c = 1$)

$$[\boldsymbol{\alpha}\cdot\boldsymbol{p} + \boldsymbol{\beta}\,(M + S(r))]\,\boldsymbol{\Psi}(r) = [E - V(r)]\,\Psi(r), \quad (1)$$

where E is the relativistic energy, M is the mass of a single particle. The $\boldsymbol{p}$ is the momentum operator, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the 4×4 Dirac matrices:

$$\boldsymbol{p} = -\,\mathrm{i}\,\boldsymbol{\nabla}, \quad \boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma}_i \\ \boldsymbol{\sigma}_i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} \boldsymbol{I} & 0 \\ 0 & -\boldsymbol{I} \end{pmatrix},$$

where $\boldsymbol{I}$ is the unit matrix and $\boldsymbol{\sigma}_i$ is the Pauli matrix. For a particle in a central field, the total angular momentum $\boldsymbol{J}$ and $\hat{\boldsymbol{K}} = -\boldsymbol{\beta}(\boldsymbol{\alpha}\cdot\boldsymbol{L}+1)$ commute with the Dirac Hamiltonian, where $\boldsymbol{L}$ is the orbital angular momentum. The eigenvalues of the $\hat{\boldsymbol{K}}$ are $\kappa = \pm(j+1/2)$, where "$-$" for aligned spin and "$+$" for unaligned spin to given a total angular momentum j. The wavefunctions can be classified according to the angular momentum j and the spin-orbit quantum number κ as follows:

$$\boldsymbol{\Psi}_{n\kappa}(r,\theta,\phi) = \frac{1}{r}\begin{pmatrix} F_{n\kappa}(r)\mathrm{Y}^{l}_{jm}(\theta,\phi) \\ \mathrm{i}\,G_{n\kappa}(r)\mathrm{Y}^{\tilde{l}}_{jm}(\theta,\phi) \end{pmatrix}, \quad (2)$$

where $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ are the upper and the lower components, $\mathrm{Y}^{l}_{jm}(\theta,\phi)$ and $\mathrm{Y}^{\tilde{l}}_{jm}(\theta,\phi)$ are the spherical harmonic functions, n is the radial quantum number and m is the projection of the angular momentum on the z axis. The orbital angular momentum quantum numbers l and $\tilde{l}$ represent the spin and the pseudospin quantum numbers, respectively. Substituting Eq. (2) into Eq. (1), we obtain two couple equations for the radial part of the Dirac equation as follows:

$$\left(\frac{\mathrm{d}}{\mathrm{d}r} + \frac{\kappa}{r}\right) F_{n\kappa}(r) = [M + E_{nk} - \Delta(r)]\,G_{n\kappa}(r), \quad (3)$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}r} - \frac{\kappa}{r}\right) G_{n\kappa}(r) = [M - E_{nk} + \Sigma(r)]\,F_{n\kappa}(r), \quad (4)$$

where $\Delta(r) = V(r) - S(r)$ and $\Sigma(r) = V(r) + S(r)$. By eliminating $G_{n\kappa}(r)$ in Eq. (3) and $F_{n\kappa}(r)$ in Eq. (4), we obtain two second-order differential equations for the upper and the lower spinor components as follows:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{\kappa(\kappa+1)}{r^2} - (M + E_{n\kappa} - \Delta)(M - E_{n\kappa} + \Sigma) + \frac{\frac{\mathrm{d}\Delta}{\mathrm{d}r}\left(\frac{\mathrm{d}}{\mathrm{d}r} + \frac{\kappa}{r}\right)}{M + E_{n\kappa} - \Delta}\right] F_{n\kappa}(r) = 0, \quad (5)$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{\kappa(\kappa-1)}{r^2} - (M + E_{n\kappa} - \Delta)(M - E_{n\kappa} + \Sigma) - \frac{\frac{\mathrm{d}\Sigma}{\mathrm{d}r}\left(\frac{\mathrm{d}}{\mathrm{d}r} - \frac{\kappa}{r}\right)}{M - E_{n\kappa} + \Sigma}\right] G_{n\kappa}(r) = 0. \quad (6)$$

If we consider exact spin symmetry, (i.e., $\mathrm{d}\Delta/\mathrm{d}r = 0$ or $\Delta = C =$ constant), Eq. (5) can be reduced to the following form:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{\kappa(\kappa+1)}{r^2} - (M + E_{n\kappa} - C) \times (M - E_{n\kappa} + \Sigma(r))\right] F_{n\kappa}(r) = 0, \quad (7)$$

where $\kappa = l$ for $\kappa < 0$ and $\kappa = -(1+l)$ for $\kappa > 0$. Equation (7) shows that the energy eigenvalues is a function of n and κ. In Eq. (7), we assume that $\Sigma(r)$ is the PT potential[31] with

$$V(r) = \frac{V_1 - V_2\cosh(\alpha r)}{\sinh^2(\alpha r)}, \quad (8)$$

where parameters V_1 and V_2 describe the property of the potential well, $V_1 > V_2$ and the screening parameter α determines the range of potential. Substituting Eq. (8) into Eq. (7), the upper spinor component takes the following form:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{\kappa(\kappa+1)}{r^2} + \widehat{E} - \frac{\widehat{V}_1 - \widehat{V}_2\cosh(\alpha r)}{\sinh^2(\alpha r)}\right] \times F_{n\kappa}(r) = 0, \quad (9)$$

where

$$\widehat{E} = E^2_{n\kappa} - M^2 + C(M - E_{n\kappa}),$$
$$\widehat{V}_1 = (M + E_{n\kappa} - C)V_1,$$
$$\widehat{V}_2 = (M + E_{n\kappa} - C)V_2. \quad (10)$$

The Dirac equation with the PT potential has an exact solution only for the $\kappa = -1$ case. For $\kappa \neq -1$ states, we have to use an approximation to deal with the spin-orbit coupling term. In the literature, many authors have applied a proper approximation to the spin-orbit coupling term, i.e. $1/r^2 \approx \alpha^2\,\mathrm{e}^{-\alpha r}/(1 - \mathrm{e}^{-\alpha r})^2$ and have obtained approximate analytical solutions to the κ state of the Dirac equation with several physical potentials.[18–23] Recently, Dong *et al.*[32] proposed a new approximation to the centrifugal potential

$$\frac{1}{r^2} \approx \gamma_0 + \frac{\gamma_1}{\sinh^2(\alpha r)} + \frac{\gamma_2\cosh(\alpha r)}{\sinh^2(\alpha r)}, \quad (11)$$

where

$$\gamma_0 = \frac{\alpha^2}{2d^4}\left(6 - 3d\sqrt{\frac{2b}{a}} + 2d^2\right), \quad \gamma_1 = \frac{\alpha^2}{V_2^2 d^4}\left[-6V_1 b + \sqrt{\frac{2b}{a}}d(2V_2^2 + V_1 b)\right],$$
$$\gamma_2 = \frac{\alpha^2 V_2}{d^3\sqrt{2a^2 - 2aV_1 + V_2^2}}\left\{\frac{\sqrt{2}\left[(8V_1^2 - V_2^2)(aV_1 + V_1^2 - V_2^2) - 4aV_1V_2^2\right]}{2V_1 b - V_2^2} - \frac{6}{d}\right\}, \tag{12}$$
$$a = \sqrt{V_1^2 - V_2^2}, \quad b = a + V_1, \quad d = \operatorname{arctanh}(\sqrt{2a/b}).$$

Using Eq. (11) in Eq. (9), a second order differential equation is obtained

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}r^2} + [\widehat{E} - \gamma_0\kappa(\kappa+1)] - \frac{\widehat{V}_1 + \gamma_1\kappa(\kappa+1) - [\widehat{V}_2 - \gamma_2\kappa(\kappa+1)]\cosh(\alpha\ r)}{\sinh^2(\alpha r)}\right\} F_{n\kappa}(r) = 0. \tag{13}$$

If we introduce a new variable of the form $z = (1 - \cosh(\alpha r))/2$ into Eq. (13), we obtain

$$z(1-z)\frac{\mathrm{d}^2}{\mathrm{d}z^2}F_{n\kappa}(z) + \left(\frac{1}{2} - z\right)\frac{\mathrm{d}}{\mathrm{d}z}F_{n\kappa}(z) + \left[-\widetilde{E} - \frac{\widetilde{V}_1}{4z(1-z)} + \frac{\widetilde{V}_2\ (1-2z)}{4z(1-z)}\right]F_{n\kappa}(z) = 0, \tag{14}$$

where

$$\widetilde{E} = \frac{1}{\alpha^2}[\widehat{E} - \gamma_0\kappa(\kappa+1)],$$
$$\widetilde{V}_1 = \frac{1}{\alpha^2}[\widehat{V}_1 + \gamma_1\kappa(\kappa+1)],$$
$$\widetilde{V}_2 = \frac{1}{\alpha^2}[\widehat{V}_2 - \gamma_2\kappa(\kappa+1)]. \tag{15}$$

Now, we propose a reasonable wavefunction of the form $F_{n\kappa}(z) = (1-z)^\delta z^\lambda f_{n\kappa}(z)$. The upper spinor component $F_{n\kappa}(r)/r$ has to satisfy the boundary conditions, i.e. $F_{n\kappa}(0) \to 0$ and $F_{n\kappa}(\infty) \to 0$. For the bound states, the regularity conditions are $\lambda > 0$, $\delta < 0$ and $\lambda < -\delta$. Substituting this wavefunction into Eq. (14), we obtain a second-order differential equation

$$(1-z)z\frac{\mathrm{d}^2}{\mathrm{d}z^2}f_{n\kappa}(z) + \left[2\lambda + \frac{1}{2} - (2\lambda + 2\delta + 1)z\right]\frac{\mathrm{d}}{\mathrm{d}z}f_{n\kappa}(z) + \left[-\widetilde{E} + (\lambda+\delta)^2\right]f_{n\kappa}(z) = 0, \tag{16}$$

where

$$\lambda = \frac{1}{4}\left(1 + \sqrt{1 + 4\widetilde{V}_1 - 4\widetilde{V}_2}\right),$$
$$\delta = \frac{1}{4}\left(1 - \sqrt{1 + 4\widetilde{V}_1 + 4\widetilde{V}_2}\right). \tag{17}$$

The solution of Eq. (16) is the hypergeometric function

$$f_{n\kappa}(z) = {}_2\mathrm{F}_1(a, b, c; z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{k=0}^{\infty}\frac{\Gamma(a+k)\Gamma(b+k)}{\Gamma(c+k)}\frac{z^k}{k!}, \tag{18}$$

where

$$a = \lambda + \delta + \sqrt{-\widetilde{E}}, \quad b = \lambda + \delta - \sqrt{-\widetilde{E}},$$
$$c = 2\lambda + 1/2. \tag{19}$$

From the properties of the hypergeometric functions, the series given by Eq. (18) approaches infinity unless either a or b equals to a negative integer. Therefore, the wavefunction $F_{n\kappa}(z)$ will not be finite everywhere unless

$$a = \lambda + \delta + \sqrt{-\widetilde{E}} = -n, \quad n = 1, 2, 3, \ldots. \tag{20}$$

Using Eqs. (10), (15) and (17), we obtain the explicit expression of the energy eigenvalues of the Dirac equation with the PT potential for the arbitrary κ state,

$$E_{n\kappa}^2 - M^2 + C\,(M - E_{n\kappa}) = \gamma_0\kappa(\kappa+1) - \alpha^2\left\{n + \frac{1}{2} + \frac{1}{4}\left[\sqrt{1 + 4(\widetilde{V}_1 - \widetilde{V}_2)} - \sqrt{1 + 4(\widetilde{V}_1 + \widetilde{V}_2)}\right]\right\}^2. \tag{21}$$

Equation (21) leads to quadratic energy eigenvalues for the spin symmetric limit. Therefore, the solution of this equation consists of positive and negative energy eigenvalues for different n and κ. It has been shown that there are only bound states to the positive energy eigenvalues in the spin symmetric limit,[3] otherwise the lower spinor component $G_{n\kappa}(r)$ will diverge if $E_{n,\kappa} = M$ and $C = 0$. We can write the upper component as

$$F_{n\kappa}(r) = N(1-z)^\delta z^\lambda \times {}_2\mathrm{F}_1\left(-n, n + 2(\lambda+\delta), 2\lambda + 1/2; z\right), \tag{22}$$

where N is a normalization constant. By using Eqs. (3) and (22), we obtain the corresponding lower spinor component $G_{n\kappa}(r)$ as

$$G_{n\kappa}(r) = \frac{N}{M+E_{n\kappa}-C}\left[\frac{\kappa}{r}+\frac{\alpha\sinh(\alpha\ r)}{2}\left(\frac{\delta}{1-z}-\frac{\lambda}{z}\right)\right](1-z)^{\delta}z^{\lambda}$$
$$\times\ {}_2\mathrm{F}_1\left(-n, n+2(\lambda+\delta), 2\lambda+1/2; z\right) - \frac{Nz^{\lambda}(1-z)^{\delta}n(n+2\delta+2\lambda)\alpha\sinh(\alpha\ r)}{(M+E_{n\kappa}-C)(4\lambda+1)}$$
$$\times\ {}_2\mathrm{F}_1\left(1-n, n+1+2(\lambda+\delta), 2\lambda+3/2; z\right). \quad (23)$$

It can be easily seen from Eqs. (22) and (23) that when $\lambda > 0$, $\delta < 0$ and $\lambda < -\delta$, the upper spinor component $F_{n\kappa}(r)$ and the lower spinor component $G_{n\kappa}(r)$ satisfy the regularity conditions for the bound state. In order to show the procedure of determining the bound state energy eigenvalues with Eq. (21), we take the following set of physical parameters, i.e., $\alpha = 0.2$, $V_1 = 5$, $V_2 = 3$, $C = 2$ and $M = 10$, to give a numerical example. For $n = 1$ and $\kappa = -1$, Eq. (21) yields $E_{1,-1} = -7.996092626$, 9.596090284. If we choose $E_{1,-1} = 9.596090284$ as the solution of Eq. (21), we find that the values of λ and δ are 15.08285638 and -29.41255238, respectively. If we take $E_{1,-1} = -7.996092626$ as the solution of Eq. (21), then $\lambda = 0.5836797492$ and $\delta = -0.2578077392$, which do not satisfy the regularity condition $\lambda < -\delta$. As a result, we can only take the positive energy eigenvalue $E_{1,-1} = 9.596090284$ as the solution of Eq. (21). The numerical solutions of Eq. (21) for the other values of n and κ are presented in Table 1 with the same values of α, V_1, V_2, C and M.

Table 1. The bound state energy eigenvalues of the Dirac equation with the PT potential for several values of n and κ. The other parameters are $V_1 = 5$, $V_2 = 3$, $C = 2$ and $M = 10$.

	$E_{n,\kappa<0}$					
$n,\ \kappa$	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.01$	$\alpha = 0.001$	$\alpha = 0.0001$	$\alpha = 0.00001$
1, –1	9.596090284	9.549372681	9.505057517	9.500506958	9.500050708	9.500005071
1, –2	9.676276884	9.571850151	9.505298247	9.500509378	9.500050732	9.500005071
1, –3	9.804409955	9.614428518	9.505779448	9.500514217	9.500050781	9.500005072
1, –4	9.931518195	9.672791720	9.506500605	9.500521476	9.500050853	9.500005072
2, –1	9.654189021	9.580782289	9.508413991	9.500844779	9.500084512	9.500008452
2, –2	9.727785266	9.602399880	9.508653836	9.500847197	9.500084536	9.500008452
2, –3	9.843732786	9.643302087	9.509133269	9.500852035	9.500084584	9.500008452
2, –4	9.954028513	9.699245930	9.509851775	9.500859291	9.500084657	9.500008453

At the limit of α becomes zero, the energy eigenvalues and the eigenfunctions go to

$$\lim_{\alpha\to 0} E^{\pm}_{n,\kappa} = M - \frac{V_1}{2} \pm \frac{1}{2}\sqrt{V_1^2 - V_2^2}, \quad (24)$$
$$\lim_{\alpha\to 0} F_{n\kappa}(r) = 0, \quad (25)$$
$$\lim_{\alpha\to 0} G_{n\kappa}(r) = 0. \quad (26)$$

From Eqs. (25) and (26), we conclude that the upper spinor component $F_{n\kappa}(r)$ and the lower spinor component $G_{n\kappa}(r)$ become unbound when α approaches to zero. From Eq. (24), we conclude that the energy eigenvalues approach a constant when α goes to zero. Since $E^{-}_{n\kappa}$ does not give the limit values, the physically acceptable limit energy value is $E^{+}_{n\kappa}$.

3. Conclusion

In this study, we have investigated an approximate analytical solution of the Dirac equation with the PT potential and the spin symmetry for the arbitrary spin-orbit quantum number κ. We have obtained the energy eigenvalues and the associated two Dirac spinors in a closed form by employing a new approximation to the spin-orbit coupling term. We have found that there are only positive energy states for bound states in the case of spin symmetry based on the strong regularity restriction condition $\lambda < -\delta$ for the wave functions. The numerical results have shown that energy levels approach a constant when α goes to zero.

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