A new approach to obtaining positive-definite Wigner operator for two entangled particles with different masses*

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Based on our previously proposed Wigner operator in entangled form, we introduce the generalized Wigner operator for two entangled particles with different masses, which is expected to be positive-definite. This approach is able to convert the generalized Wigner operator into a pure state so that the positivity can be ensured. The technique of integration within an ordered product of operators is used in the discussion.

Keywords: positive-definite Wigner operator, entangled form, marginal distribution

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1. Introduction

In quantum mechanics theory, due to Heisenberg's uncertainty principle, one cannot measure precisely a particle's position and momentum simultaneously,[1] instead, Wigner proposed a distribution function $W(x,p)$ for the quantum state $|\psi\rangle$ in phase space, whose two marginal distributions $\int_{-\infty}^{\infty}\mathrm{d}xW(x,p)$ and $\int_{-\infty}^{\infty}\mathrm{d}pW(x,p)$ lead to $|\langle p\,|\psi\rangle|^2$ and $|\langle x\,|\psi\rangle|^2$ respectively.[2] Nowadays, the Wigner function theory has become one of the major topics in both quantum statistics and quantum optics.[3] Though the Wigner distribution function helps to demonstrate some properties of quantum states, it is not a probability distribution because it is not always positive.[4] To overcome this inconvenience, Husimi[5] smoothed out the Wigner function by averaging over a coarse graining function,

$$W_{\mathrm{h}}(x,p;\kappa) \equiv 2\int \mathrm{d}x'\mathrm{d}p'W(x',p')\,\mathrm{e}^{-\kappa\left(x'-x\right)^2-\left(p'-p\right)^2/\kappa}, \quad (1)$$

where positive κ is the Gaussian spatial width parameter, which is free to be chosen and determines the relative resolution in p-space versus x-space. On the other hand, in recent years, much attention has been paid to quantum entanglement and entangled states.[6−13] Therefore, for entangled states the physical meaning of Wigner function's marginal distributions should not be focused on single particle's variables, but lies in the measurement probability for the combined observables $(X_1+X_2),(P_1-P_2)$ or $(X_1-X_2),(P_1+P_2)$.[14−17] Hence examining the Wigner operator from the point of view of entanglement is of necessity. In Ref. [18], Fan has introduced the two-mode Wigner operator in entangled form (entangled Wigner operator)

$$\begin{aligned}&\Delta(\sigma,\gamma)\\ &\equiv \frac{1}{\pi^2}\colon \exp\left[-\left(\sigma_1-\frac{X_1-X_2}{\sqrt{2}}\right)^2\right.\\ &\quad-\left(\sigma_2-\frac{P_1+P_2}{\sqrt{2}}\right)^2\\ &\quad\left.-\left(\gamma_1-\frac{X_1+X_2}{\sqrt{2}}\right)^2-\left(\gamma_2-\frac{P_1-P_2}{\sqrt{2}}\right)^2\right]\colon\\ &=\frac{1}{\pi^2}\colon \exp\left[-(\sigma-a_1+a_2^\dagger)(\sigma^*-a_1^\dagger+a_2)\right.\\ &\quad\left.-(\gamma-a_1-a_2^\dagger)(\gamma^*-a_1^\dagger-a_2)\right]\colon \end{aligned} \quad (2)$$

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where $\sigma = \sigma_1 + \mathrm{i}\sigma_2$, $\gamma = \gamma_1 + \mathrm{i}\gamma_2$, the symbol : : denotes normal ordering. Its marginal distribution is

$$\int \mathrm{d}^2\sigma \Delta(\sigma,\gamma) = \frac{1}{\pi} : \mathrm{e}^{-(\gamma - a_2^\dagger - a_1)(\gamma^* - a_1^\dagger - a_2)} : = \frac{1}{\pi} |\gamma\rangle \langle\gamma| , \tag{3}$$

where

$$|\gamma\rangle = \mathrm{e}^{-|\gamma|^2/2 + \gamma a_1^\dagger + \gamma^* a_2^\dagger - a_1^\dagger a_2^\dagger} |00\rangle \tag{4}$$

is (an entangled state) the common eigenvector of two particles' centre-of-mass coordinate $(X_1 + X_2)/2$ and relative momentum $(P_1 - P_2)$ with eigenvalues $\gamma_1/\sqrt{2}$ and $\sqrt{2}\gamma_2$, respectively. On the other hand,

$$\int \mathrm{d}^2\gamma \Delta(\sigma,\gamma) = \frac{1}{\pi} : \mathrm{e}^{-(\sigma + a_2^\dagger - a_1)(\sigma^* - a_1^\dagger + a_2)} : = \frac{1}{\pi} |\sigma\rangle \langle\sigma| , \tag{5}$$

where

$$|\sigma\rangle = \mathrm{e}^{-|\sigma|^2/2 + \sigma a_1^\dagger - \sigma^* a_2^\dagger + a_1^\dagger a_2^\dagger} |00\rangle \tag{6}$$

is the eigenvector of two particles' relative position and total momentum, i.e.

$$\begin{aligned} (X_1 - X_2)|\sigma\rangle &= \sqrt{2}\sigma_1 |\sigma\rangle , \\ (P_1 + P_2)|\sigma\rangle &= \sqrt{2}\sigma_2 |\sigma\rangle . \end{aligned} \tag{7}$$

Since

$$\begin{aligned} \left[\frac{1}{2}(X_1 + X_2), (P_1 + P_2)\right] &= \mathrm{i}, \\ \left[\frac{1}{2}(X_1 - X_2), (P_1 - P_2)\right] &= \mathrm{i}, \end{aligned} \tag{8}$$

$|\sigma\rangle$ and $|\gamma\rangle$ are conjugate to each other. It then follows the corresponding marginal distributions of Wigner function of $|\psi\rangle$,

$$\int \mathrm{d}^2\sigma \langle\psi| \Delta(\sigma,\gamma) |\psi\rangle = \frac{1}{\pi} |\psi(\gamma)|^2 , \tag{9}$$

and

$$\int \mathrm{d}^2\gamma \langle\psi| \Delta(\sigma,\gamma) |\psi\rangle = \frac{1}{\pi} |\psi(\sigma)|^2 , \tag{10}$$

where $|\psi(\gamma)|^2$ denotes the probability of measuring two particles' centre-of-mass coordinate with eigenvalue $\gamma_1/\sqrt{2}$ and simultaneously relative momentum with eigenvalue $\sqrt{2}\gamma_2$, while $|\psi(\sigma)|^2$ denotes the probability of measuring total momentum with eigenvalue $\sqrt{2}\sigma_2$ and simultaneously relative position with eigenvalue $\sqrt{2}\sigma_1$.

The Wigner function $\langle\psi| \Delta(\sigma,\gamma) |\psi\rangle$ is not always positive either. The aim of this work is to present a convenient and direct approach for introducing the generalized Wigner operator in entangled state form (including the case when two particles have different masses) which is expected to be positive-definite. Our approach is trying to convert the generalized Wigner operator into a pure state so that the positivity can be ensured.

2. Positive-definite generalized Wigner operator

Let us begin with asingle-mode Wigner operator in normal product form[19,20]

$$\Delta(x,p) = \frac{1}{\pi} : \mathrm{e}^{-(x-X)^2 - (p-P)^2} : . \tag{11}$$

We reform it by multiplying the X-term by $\kappa/(1+\kappa)$, while multiplying the P-term by $(1/\kappa)/(1+1/\kappa)$e (noting X is conjugate to P),

$$\begin{aligned} &\Delta\left(x,p;\sqrt{\kappa}\right) \\ &\equiv \frac{\sqrt{\kappa}}{\pi(1+\kappa)} : \mathrm{e}^{-(\kappa/(1+\kappa))(x-X)^2 - (1/\kappa)/(1+1/\kappa)(p-P)^2} : \\ &= |x,p;\kappa\rangle \langle x,p;\kappa| , \end{aligned} \tag{12}$$

where

$$\begin{aligned} &|x,p;\kappa\rangle \\ &= \frac{\kappa^{1/4}}{\sqrt{\pi(1+\kappa)}} \exp\left\{-\frac{1}{1+\kappa}\left[\frac{\kappa x^2}{2} + \frac{p^2}{2}\right.\right. \\ &\quad \left.\left. - \left(\sqrt{2}\kappa x + \mathrm{i}\sqrt{2}p\right) a_1^\dagger - \frac{\kappa - 1}{2} a_1^{\dagger 2}\right]\right\} |0\rangle , \end{aligned} \tag{13}$$

which is a squeezed-coherent state. Clearly, the generalized Wigner function $\langle\phi| \Delta(x,p;\sqrt{\kappa}) |\phi\rangle = |\langle x,p;\kappa|\ \phi\rangle|^2$ is positive-definite. Enlightened by this idea and based on Eq. (2) we introduce

$$\begin{aligned} &\Delta(\sigma,\gamma;\kappa) \\ &\equiv \frac{\kappa}{\pi^2(1+\kappa)^2} : \exp\left[-\frac{\kappa}{1+\kappa}\left(\sigma - a_1 + a_2^\dagger\right)\right. \\ &\quad \times \left(\sigma^* - a_1^\dagger + a_2\right) - \frac{1/\kappa}{1+1/\kappa}\left(\gamma - a_1 - a_2^\dagger\right) \\ &\quad \left. \times \left(\gamma^* - a_1^\dagger - a_2\right)\right] : , \end{aligned} \tag{14}$$

from which one can see that our approach lies in multiplying the σ-term by $\kappa/(1+\kappa)$, while multiplying the γ-term by $(1/\kappa)(1+1/\kappa)$, this motivation arises from the observation that $|\sigma\rangle$ is conjugate to $|\gamma\rangle$. Still using $: \mathrm{e}^{-a_1^\dagger a_1 - a_2^\dagger a_2} : = |00\rangle \langle 00|$, we can see

$$\Delta(\sigma,\gamma;\kappa) = |\sigma,\gamma;\kappa\rangle \langle\sigma,\gamma;\kappa| , \tag{15}$$

which is a pure state, where

$$\begin{aligned}&|\sigma,\gamma;\kappa\rangle\\ &=\frac{\sqrt{\kappa}}{\pi\left(1+\kappa\right)}\exp\left\{-\frac{1}{1+\kappa}\left[\frac{\kappa\left|\sigma\right|^{2}}{2}+\frac{\left|\gamma\right|^{2}}{2}\right.\right.\\ &-\left(\kappa\sigma+\gamma\right)a_{1}^{\dagger}+\left(\kappa\sigma^{*}-\gamma^{*}\right)a_{2}^{\dagger}\\ &\left.\left.-\left(\kappa-1\right)a_{1}^{\dagger}a_{2}^{\dagger}\right]\right\}\left|00\right\rangle\end{aligned}\tag{16}$$

is a two-mode coherent squeezed state. It then follows the positivity

$$\left\langle\psi\right|\Delta\left(\sigma,\gamma;\kappa\right)\left|\psi\right\rangle=\left|\left\langle\sigma,\gamma;\kappa\right|\left.\psi\right\rangle\right|^{2}\geq0.\tag{17}$$

3. Positive definite Wigner operator for two entangled particles with different masses

In Ref. [21] a normally ordered entangled Wigner operator for two particles with different masses is introduced, i.e.,

$$\begin{aligned}\Delta\left(\sigma,\gamma\right)_{g}=&\frac{1}{\pi^{2}}\colon\exp\{-[\gamma-f(a_{1}+a_{2}^{\dagger})-g(a_{1}^{\dagger}-a_{2})]\\ &\times[\gamma^{*}-f(a_{1}^{\dagger}+a_{2})-g(a_{1}-a_{2}^{\dagger})]\\ &-[\sigma-f(a_{1}^{\dagger}-a_{2})+g(a_{1}+a_{2}^{\dagger})]\\ &\times[\sigma^{*}-f(a_{1}-a_{2}^{\dagger})+g(a_{1}^{\dagger}+a_{2})]\}\colon,\end{aligned}\tag{18}$$

where f, g are two real parameters, obeying $f^{2}+g^{2}=1$, the subscript g means g-dependence of the entangled Wigner operator. Its marginal distribution is

$$\begin{aligned}&\int\frac{\mathrm{d}^{2}\sigma}{\pi}\Delta\left(\sigma,\gamma\right)_{g}\\ &=\frac{1}{\pi}\colon\exp\{-[\gamma-f(a_{1}+a_{2}^{\dagger})-g(a_{1}^{\dagger}-a_{2})]\\ &\times[\gamma^{*}-f(a_{1}^{\dagger}+a_{2})-g(a_{1}-a_{2}^{\dagger})]\}\colon.\end{aligned}\tag{19}$$

Using $:\mathrm{e}^{-a_{1}^{\dagger}a_{1}-a_{2}^{\dagger}a_{2}}:\;=\left|00\right\rangle\left\langle00\right|$, we can reform Eq. (19) as

$$\int\mathrm{d}^{2}\sigma\Delta\left(\sigma,\gamma\right)_{g}=\frac{1}{\pi}\left|\gamma\right\rangle_{gg}\left\langle\gamma\right|,\tag{20}$$

where $\left|\gamma\right\rangle_{g}$ is a new entangled state,

$$\begin{aligned}&\left|\gamma\right\rangle_{g}\\ &=\exp\left[-\frac{1}{2}\left|\gamma\right|^{2}+\left(f\gamma+g\gamma^{*}\right)a_{1}^{\dagger}+\left(f\gamma^{*}-g\gamma\right)a_{2}^{\dagger}\right.\\ &\left.-fg(a_{1}^{\dagger2}-a_{2}^{\dagger2})-(f^{2}-g^{2})a_{1}^{\dagger}a_{2}^{\dagger}\right]\left|00\right\rangle.\end{aligned}\tag{21}$$

The $\left|\gamma\right\rangle_{g}$ obeys the eigenvector equations

$$\begin{aligned}&[f(a_{1}+a_{2}^{\dagger})+g(a_{1}^{\dagger}-a_{2})]\left|\gamma\right\rangle_{g}=\gamma\left|\gamma\right\rangle_{g},\\ &[f(a_{1}^{\dagger}+a_{2})+g(a_{1}-a_{2}^{\dagger})]\left|\gamma\right\rangle_{g}=\gamma^{*}\left|\gamma\right\rangle_{g},\end{aligned}\tag{22}$$

or

$$\begin{aligned}&[(f+g)(a_{1}+a_{1}^{\dagger})+(f-g)(a_{2}+a_{2}^{\dagger})]|\gamma\rangle_{g}=2\gamma_{1}|\gamma\rangle_{g},\\ &[(f-g)(a_{1}-a_{1}^{\dagger})+(f+g)(a_{2}^{\dagger}-a_{2})]|\gamma\rangle_{g}=2\mathrm{i}\gamma_{2}|\gamma\rangle_{g}.\end{aligned}\tag{23}$$

On the other hand, Eq. (18) also yields

$$\begin{aligned}&\int\mathrm{d}^{2}\gamma\Delta\left(\sigma,\gamma\right)_{g}\\ &=\frac{1}{\pi}\colon\exp\Big\{-[\sigma-f(a_{1}^{\dagger}-a_{2})+g(a_{1}+a_{2}^{\dagger})]\\ &\times[\sigma^{*}-f(a_{1}-a_{2}^{\dagger})+g(a_{1}^{\dagger}+a_{2})]\Big\}\colon,\end{aligned}\tag{24}$$

which can be further decomposed as

$$\int\mathrm{d}^{2}\gamma\Delta\left(\sigma,\gamma\right)_{g}=\frac{1}{\pi}\left|\sigma\right\rangle_{gg}\left\langle\sigma\right|,\tag{25}$$

where

$$\begin{aligned}&\left|\sigma\right\rangle_{g}\\ &=\exp\left[-\frac{1}{2}\left|\sigma\right|^{2}+\left(f\sigma-g\sigma^{*}\right)a_{1}^{\dagger}-\left(f\sigma^{*}+g\sigma\right)a_{2}^{\dagger}\right.\\ &\left.+fg(a_{1}^{\dagger2}-a_{2}^{\dagger2})+(f^{2}-g^{2})a_{1}^{\dagger}a_{2}^{\dagger}\right]\left|00\right\rangle\end{aligned}\tag{26}$$

is another new entangled state which obeys the eigenvector equations

$$\begin{aligned}&[f(a_{1}^{\dagger}-a_{2})-g(a_{1}+a_{2}^{\dagger})]\left|\sigma\right\rangle_{g}=\sigma\left|\sigma\right\rangle_{g},\\ &[f(a_{1}-a_{2}^{\dagger})-g(a_{1}^{\dagger}+a_{2})]\left|\sigma\right\rangle_{g}=\sigma^{*}\left|\sigma\right\rangle_{g}.\end{aligned}\tag{27}$$

One can see that $\left|\gamma\right\rangle_{g}$ and $\left|\sigma\right\rangle_{g}$ are mutually conjugate.

Now for two entangled particles having different masses m_j, $\mu_j=mj/(m_1+m_2)$, $j=1,2$, we can identify

$$\begin{aligned}&\sqrt{\frac{\mu_{1}}{2}}+\sqrt{\frac{\mu_{2}}{2}}=f,\qquad\sqrt{\frac{\mu_{1}}{2}}-\sqrt{\frac{\mu_{2}}{2}}=g,\\ &f^{2}+g^{2}=\mu_{1}+\mu_{2}=1.\end{aligned}\tag{28}$$

When we set $\hbar=1$, $\omega_j=1$, $M=m_1+m_2=2$, then coordinate operator $X_j=\sqrt{\hbar/2m_j\omega_j}(a_j+a_j^{\dagger})$ and momentum operator $P_j=-\mathrm{i}\sqrt{m_j\omega_j\hbar/2}(a_j-a_j^{\dagger})$ take the form

$$X_{j}=\frac{a_{j}+a_{j}^{\dagger}}{2\sqrt{\mu_{j}}},\quad P_{j}=-\mathrm{i}\sqrt{\mu_{j}}(a_{j}-a_{j}^{\dagger}),\ j=1,2.\tag{29}$$

Thus, the centre-of-mass coordinate takes the form

$$X_{\mathrm{cm}}\equiv\mu_{1}X_{1}+\mu_{2}X_{2}$$

$$= \sqrt{\mu_1}(a_1 + a_1^\dagger) + \sqrt{\mu_2}(a_2 + a_2^\dagger)$$
$$= \frac{f+g}{\sqrt{2}}(a_1 + a_1^\dagger) + \frac{f-g}{\sqrt{2}}(a_2 + a_2^\dagger), \quad (30)$$

and the mass-weighted relative momentum

$$\begin{aligned} P_{\rm r} &= \mu_2 P_1 - \mu_1 P_2 \\ &= -{\rm i}\mu_2\mu_1\left(\frac{a_1 - a_1^\dagger}{\sqrt{\mu_1}} - \frac{a_2 - a_2^\dagger}{\sqrt{\mu_2}}\right) \\ &= -{\rm i}\sqrt{\frac{\mu_2\mu_1}{2}}\Big[(f-g)\left(a_1 - a_1^\dagger\right) \\ &\quad - (f+g)\left(a_2 - a_2^\dagger\right)\Big]. \end{aligned} \quad (31)$$

Comparing Eqs. (30) and (31) with Eq. (22), we can see

$$\begin{aligned} X_{\rm cm}\left|\gamma\right\rangle_g &= \frac{1}{\sqrt{2}}\gamma_1\left|\gamma\right\rangle_g, \\ P_r\left|\gamma\right\rangle_g &= \frac{1}{\sqrt{2}}(f^2 - g^2)\gamma_2\left|\gamma\right\rangle_g, \end{aligned} \quad (32)$$

where $\gamma = \gamma_1 + {\rm i}\gamma_2$. So $\left|\gamma\right\rangle_g$ is the common eigenvector of two particles' centre-of-mass coordinate and the mass-weighted relative momentum. On the other hand, from

$$X_r = X_1 - X_2 = \frac{a_1 + a_1^\dagger}{2\sqrt{\mu_1}} - \frac{a_2 + a_2^\dagger}{2\sqrt{\mu_2}}, \quad (33)$$

$$P_{\rm c} = P_1 + P_2 = -{\rm i}\sqrt{\mu_1}(a_1 - a_1^\dagger) + {\rm i}\sqrt{\mu_2}(a_2 - a_2^\dagger), \quad (34)$$

and in reference to Eq. (27), we see that $\left|\sigma\right\rangle_g$ is the common eigenvector of two particles' relative coordinate and the total momentum,

$$X_{\rm r}\left|\sigma\right\rangle_g = \frac{\sqrt{2}\sigma_1}{f^2 - g^2}\left|\sigma\right\rangle_g, \quad P_{\rm c}\left|\sigma\right\rangle_f = \sqrt{2}\sigma_2\left|\sigma\right\rangle_g. \quad (35)$$

Thus, we can see that Eq. (18) is the correct Wigner operator for two entangled particles with different masses. It turns out that in order to guarantee its marginal distributions to be physically meaningful, the mass dependent Wigner operator should be introduced.

Now by multiplying the σ-term by $\kappa/(1+\kappa)$, while multiplying the γ-term by $(1/\kappa)(1+1/\kappa)$, we introduce

$$\begin{aligned} \Delta\left(\sigma,\gamma;\kappa\right)_g = \frac{\kappa}{\pi^2\left(1+\kappa\right)^2} :\, \exp\Bigg\{ &-\frac{\kappa}{1+\kappa}\left[\left(\gamma - fa_1 - fa_2^\dagger - ga_1^\dagger + ga_2\right)\times\left(\gamma^* - fa_1^\dagger - fa_2 - ga_1 + ga_2^\dagger\right)\right] \\ &- \frac{1/\kappa}{1/\kappa+1}\left[\left(\sigma - fa_1^\dagger + fa_2 + ga_1 + ga_2^\dagger\right)\times\left(\sigma^* - fa_1 + fa_2^\dagger + ga_1^\dagger + ga_2\right)\right]\Bigg\} : \end{aligned} \quad (36)$$

Still using $: {\rm e}^{-a_1^\dagger a_1 - a_2^\dagger a_2} : \; = \left|00\right\rangle\left\langle 00\right|$, we find the positive-definite Wigner operator for two entangled particles with different masses

$$\Delta\left(\sigma,\gamma;\kappa\right)_g = \left|\sigma,\gamma;\kappa\right\rangle_{gg}\left\langle\sigma,\gamma;\kappa\right|, \quad (37)$$

where

$$\begin{aligned} &\left|\sigma,\gamma;\kappa\right\rangle_g \\ &= \frac{\sqrt{\kappa}}{\pi\left(1+\kappa\right)}\exp\Bigg\{-\frac{1}{1+\kappa}\Bigg[\frac{\kappa\left|\gamma\right|^2}{2} + \frac{\left|\sigma\right|^2}{2} \\ &\quad - \left(\kappa\gamma f + \kappa g\gamma^* - \sigma g + f\sigma^*\right)a_1^\dagger \\ &\quad - \left(\kappa f\gamma^* - \kappa\gamma g - \sigma f - g\sigma^*\right)a_2^\dagger \\ &\quad + \left(\kappa - 1\right)\left(f^2 - g^2\right)a_1^\dagger a_2^\dagger + fg\left(\kappa - 1\right)a_1^{\dagger 2} \\ &\quad - fg\left(\kappa - 1\right)a_2^{\dagger 2}\Bigg]\Bigg\}\left|00\right\rangle. \end{aligned} \quad (38)$$

It then follows the positivity

$$\left\langle\psi\right|\Delta\left(\sigma,\gamma;\kappa\right)_g\left|\psi\right\rangle = \left|{}_g\left\langle\sigma,\gamma;\kappa\right|\,\psi\right\rangle\right|^2 \geq 0. \quad (39)$$

4. Conclusion

In summary, we have proposed a convenient approach for obtaining the generalized Wigner operator, which is positive-definite, i.e., based on the Wigner operator in anormally ordered form (see Eq. (36)), multiplying the two mutually-conjugate-variable terms by $\kappa/(1+\kappa)$ and by $(1/\kappa)(1+1/\kappa)$, respectively, we find that the resulting generalized Wigner operator is a pure state to guarantee its positivity.

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