A meshless method for the compound KdV-Burgers equation*

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The element-free Galerkin (EFG) method for numerically solving the compound Korteweg–de Vries–Burgers (KdVB) equation is discussed in this paper. The Galerkin weak form is used to obtain the discrete equation and the essential boundary conditions are enforced by the penalty method. The effectiveness of the EFG method of solving the compound Korteweg–de Vries–Burgers (KdVB) equation is illustrated by three numerical examples.

Keywords: element-free Galerkin (EFG) method, meshless method, compound Korteweg–de Vries–Burgers (KdVB) equation, solitary wave

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1. Introduction

The compound Korteweg–de Vries–Burgers (KdVB) equation is a nonlinear partial differential equation which was initially formulated by Su and Gardner.[1] This model arises from many physical applications such as the propagation of undular bores in shallow water waves,[2] propagation of waves in elastic tube filled with a viscous fluid,[3] and weakly nonlinear plasma waves with certain dissipative effects.[4] The KdVB equation is the combination of the Burgers' equation and Korteweg–de Vries (KdV) equation. Burgers' equation[5] was first used by Burger for the study of turbulence in 1939, whereas the KdV equation[6] was first suggested by Korteweg and de Vries who used this as a nonlinear model to study the change in shape of long waves moving in a rectangular channel.

Many researchers focus on the KdVB equation and a number of theoretical issues and numerical techniques have been obtained. The traveling wave solution in particular has been studied extensively. Demiray,[7] Antar and Demiray[8] derived the KdVB equation as the governing evolution equation for wave propagation in fluid-filled elastic or viscoelastic tubes in which the effects of dispersion, dissipation and nonlinearity were present. Zaki[9] used the quintic B spline finite elements method and Talaat and Danaf[10] used septic B-spline to obtain numerical methods of solving the KdVB equation. The Adomian decomposition method and the variational iteration method were used to obtain the exact solution of KdVB equation.[11,12] Hon and Mao,[13] and Sirajul and Siraj[14] used the radial basis function method to solve the KdVB equation.

In recent years, meshless techniques have attracted the attention of researchers. In a meshless (mesh-free) method, a set of scattered nodes is used instead of meshing the domain of the problem. The meshless method is a new and an interesting numerical technique by which many engineering problems not suited to conventional numerical methods with a minimum of meshing or no meshing at all,[15−29] can be solved. The element-free Galerkin (EFG) method is greatly developed and widely used in various fields. To the best knowledge of the authors, this method has not been applied to the KdVB equation.

In this paper, the EFG method is used to obtain the numerical solution of the KdVB equation. In this method, the function in the solution domain needs only a set of nodes, it does not require element connectivity. The Galerkin weak form is used to obtain the corresponding discrete equations and the essential boundary conditions are enforced by the penalty

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method. The numerical results which are in good agreement with the exact solutions are presented.

2. Moving least-square approximation

In the moving least-square approximation,[15] Lancaster and Salkauskas defines a local approximation by

$$u^h(\boldsymbol{x},\overline{\boldsymbol{x}}) = \sum_{j=1}^{m} p_j(\overline{\boldsymbol{x}})a_j(\boldsymbol{x}) \equiv \boldsymbol{p}^{\mathrm{T}}(\overline{\boldsymbol{x}})\boldsymbol{a}(\boldsymbol{x}), \quad (1)$$

where m is the number of terms in the basis, $p_j(\boldsymbol{x})$ are monomial basis functions and $\boldsymbol{a}_j(\boldsymbol{x})$ are their coefficients, which as indicated are the functions of the spatial coordinates $\boldsymbol{x}$. This defines the quadratic form as

$$J = \sum_{i=1}^{n} w(\boldsymbol{x}-\boldsymbol{x}_i)\left[u^h(\boldsymbol{x},\boldsymbol{x}_i) - u(\boldsymbol{x}_i)\right]^2, \quad (2)$$

where $w(\boldsymbol{x}-\boldsymbol{x}_i)$ are the weight functions with compact support and $\boldsymbol{x}_i$ are the nodes in the compacted supported domain of $\boldsymbol{x}$. Equation (2) can be rewritten in a vector form as

$$\boldsymbol{J} = (\boldsymbol{p}\boldsymbol{a}-\boldsymbol{u})^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x})(\boldsymbol{p}\boldsymbol{a}-\boldsymbol{u}), \quad (3)$$

where

$$\boldsymbol{u}^{\mathrm{T}} = (u_1, u_2, ..., u_n), \quad (4)$$

$$\boldsymbol{p} = \begin{bmatrix} p_1(\boldsymbol{x}_1) & p_2(\boldsymbol{x}_1) & \cdots & p_m(\boldsymbol{x}_1) \\ p_1(\boldsymbol{x}_2) & p_2(\boldsymbol{x}_2) & \cdots & p_m(\boldsymbol{x}_2) \\ \vdots & \vdots & \ddots & \vdots \\ p_1(\boldsymbol{x}_n) & p_2(\boldsymbol{x}_n) & \cdots & p_m(\boldsymbol{x}_n) \end{bmatrix}, \quad (5)$$

and

$$\boldsymbol{W}(\boldsymbol{x}) = \begin{bmatrix} w(\boldsymbol{x}-\boldsymbol{x}_1) & 0 & \cdots & 0 \\ 0 & w(\boldsymbol{x}-\boldsymbol{x}_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w(\boldsymbol{x}-\boldsymbol{x}_n) \end{bmatrix}. \quad (6)$$

To find the coefficients $\boldsymbol{a}(\boldsymbol{x})$, we obtain the extremum of $\boldsymbol{J}$ by

$$\frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}} = \boldsymbol{A}(\boldsymbol{x})\boldsymbol{a}(\boldsymbol{x}) - \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u} = 0, \quad (7)$$

where $\boldsymbol{A}$ is called the moment matrix and is given by

$$\boldsymbol{A} = \boldsymbol{p}^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x})\boldsymbol{p}, \quad (8)$$

$$\boldsymbol{B} = \boldsymbol{p}^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x}). \quad (9)$$

Then we have

$$\boldsymbol{a}(\boldsymbol{x}) = \boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}. \quad (10)$$

The approximation $u^h(\boldsymbol{x})$ can be expressed as

$$u(\boldsymbol{x}) \approx u^h(\boldsymbol{x}) = \sum_{i=1}^{n} \Phi_i^k(\boldsymbol{x})\, u_i = \boldsymbol{\Phi}^k(\boldsymbol{x})\boldsymbol{u}, \quad (11)$$

where the shape functions are given by

$$\boldsymbol{\Phi}^k(\boldsymbol{x}) = [\Phi_1^k(\boldsymbol{x}), \cdots, \Phi_n^k(\boldsymbol{x})] = \boldsymbol{p}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x}), \quad (12)$$

where the superscript k is the order of the polynomial basis. The functions corresponding to $k=0$ are called the Shepard functions and are given by

$$\Phi_i(\boldsymbol{x}) = \frac{w(\boldsymbol{x}-\boldsymbol{x}_i)}{\sum_{j=1}^{n} w(\boldsymbol{x}-\boldsymbol{x}_j)}. \quad (13)$$

The derivatives of the unknown u are approximated by

$$D^\alpha u^h(\boldsymbol{x}) = \sum_{i=1}^{n} D^\alpha \Phi_i^k(\boldsymbol{x})u_i = D^\alpha \boldsymbol{\Phi}^k(\boldsymbol{x})\boldsymbol{u}. \quad (14)$$

3. EFG method for the compound KdVB equation

The compound KdVB equation is given by

$$u_t + \varepsilon u u_x + \lambda u^2 u_x + \gamma u_{xx} - \mu u_{xxx} = 0, \quad a < x < b. \quad (15a)$$

The boundary conditions are

$$u(a,t) = \alpha_1,\ u(b,t) = \beta_1. \quad (15b)$$

The initial condition is

$$u(x,0) = f(x), \quad (15c)$$

where μ, ε, γ and λ are parameters and subscripts x and t denote differentiation.

The weak form of Eq. (15) is obtained as

$$\int_\Gamma \delta u(u_t + \varepsilon u u_x + \lambda u^2 u_x + \Gamma u_{xx} - \mu u_{xxx})\mathrm{d}\Gamma + \alpha\delta u(u-\overline{u})|_{\Gamma_u} = 0, \quad (16)$$

where Γ denotes the domain of x, Γ_u denotes the boundaries, $\overline{u}$ are the values of x in the boundaries and α is a penalty parameter.

By the subsection integration, we can have

$$\int_\Gamma \delta u u_t \mathrm{d}\Gamma + \varepsilon\int_\Gamma \delta u u u_x \mathrm{d}\Gamma + \lambda\int_\Gamma \delta u u^2 u_x \mathrm{d}\Gamma - \gamma\int_\Gamma \delta u_x u_x \mathrm{d}\Gamma + \mu\int_\Gamma \delta u_x u_{xx}\mathrm{d}\Gamma + \alpha\delta u(u-\overline{u})|_{\Gamma_u} = 0. \quad (17)$$

Using Eq. (11), we can obtain the approximation of the test function as

$$u^h(x,t) = \sum_{I=1}^{n} \Phi_I(x) \cdot u_I(t) = \boldsymbol{\Phi}(x) \cdot \boldsymbol{U}(t), \tag{18}$$

$$\frac{\partial u^h(x,t)}{\partial t} = \frac{\partial}{\partial t} \sum_{I=1}^{n} \Phi_I(x) \cdot u_I(t) = \sum_{I=1}^{n} \Phi_I(x) \cdot \frac{\partial U_I(t)}{\partial t} = \boldsymbol{\Phi}(x) \cdot \dot{\boldsymbol{U}}(t), \tag{19}$$

$$\frac{\partial u^h(x,t)}{\partial x} = \frac{\partial}{\partial x} \sum_{I=1}^{n} \Phi_I(x) \cdot u_I(t) = \sum_{I=1}^{n} \Phi_{I,x}(x) \cdot u_I(t) = \boldsymbol{\Phi}_x(x) \cdot \boldsymbol{U}(t), \tag{20}$$

$$\frac{\partial^2 u^h(x,t)}{\partial x^2} = \frac{\partial^2}{\partial x \partial x} \sum_{I=1}^{n} \Phi_I(x) \cdot u_I(t) = \sum_{I=1}^{n} \Phi_{I,xx}(x) \cdot u_I(t) = \boldsymbol{\Phi}_{xx}(x) \cdot \boldsymbol{U}(t), \tag{21}$$

where

$$\boldsymbol{\Phi}(x) = (\Phi_1(x), \Phi_2(x), \cdots, \Phi_n(x)), \tag{22}$$

$$\boldsymbol{U}(t) = (u_1(t), u_2(t), \cdots, u_n(t))^{\mathrm{T}}, \tag{23}$$

$$\dot{\boldsymbol{U}}(t) = \left(\frac{\partial u_1(t)}{\partial t}, \frac{\partial u_2(t)}{\partial t}, \cdots, \frac{\partial u_n(t)}{\partial t}\right)^{\mathrm{T}}, \tag{24}$$

$$\boldsymbol{\Phi}_x(x) = (\Phi_{1,x}(x), \Phi_{2,x}(x), \cdots, \Phi_{n,x}(x)), \tag{25}$$

$$\boldsymbol{\Phi}_{xx}(x) = (\Phi_{1,xx}(x), \Phi_{2,xx}(x), \cdots, \Phi_{n,xx}(x)). \tag{26}$$

Substituting Eqs. (18)–(21) into Eq. (17), we have

$$\begin{aligned} &\delta \boldsymbol{U}^{\mathrm{T}}(t) K \dot{\boldsymbol{U}}(t) + \delta \boldsymbol{U}^{\mathrm{T}}(t) \int_{\Gamma} \varepsilon \boldsymbol{\Phi}(u u_x) \mathrm{d}\Gamma \\ &+ \delta \boldsymbol{U}^{\mathrm{T}}(t) \int_{\Gamma} \lambda \boldsymbol{\Phi}(u^2 u_x) \mathrm{d}\Gamma + \delta \boldsymbol{U}^{\mathrm{T}}(t) G \boldsymbol{U}(t) \\ &- \delta \boldsymbol{U}^{\mathrm{T}}(t) N \boldsymbol{U}(t) + \alpha \delta \boldsymbol{U}^{\mathrm{T}}(t) H \boldsymbol{U}(t) - \alpha \delta \boldsymbol{U}^{\mathrm{T}}(t) Q = 0. \end{aligned} \tag{27}$$

Since $\delta \boldsymbol{U}^{\mathrm{T}}(t)$ are arbitrary in Eq. (27), the following equations can be obtained:

$$\begin{aligned} &\boldsymbol{K} \dot{\boldsymbol{U}}(t) + (\boldsymbol{H} - \boldsymbol{N} + \boldsymbol{G}) \boldsymbol{U}(t) \\ &\quad + \int_{\Gamma} \varepsilon \boldsymbol{\Phi}(u u_x) \mathrm{d}\Gamma + \int_{\Gamma} \lambda \boldsymbol{\Phi}(u^2 u_x) \mathrm{d}\Gamma = \boldsymbol{Q}(t), \end{aligned} \tag{28}$$

where

$$K_{IJ} = \int_{\Gamma} \Phi_I(x) \Phi_J(x) \mathrm{d}\Gamma, \tag{29}$$

$$N_{IJ} = \gamma \int_{\Gamma} \Phi_{I,x}(x) \Phi_{J,x}(x) \mathrm{d}\Gamma, \tag{30}$$

$$G_{IJ} = \mu \int_{\Gamma} \Phi_{I,x}(x) \Phi_{J,xx}(x) \mathrm{d}\Gamma, \tag{31}$$

$$H_{IJ} = \alpha(\Phi_I \Phi_J|_{x=a} + \Phi_I \Phi_J|_{x=b}), \tag{32}$$

$$Q_I = \alpha(\Phi_I \bar{u}|_{x=a} + \Phi_I \bar{u}|_{x=b}). \tag{33}$$

Forming the time discretization of Eq. (28) with the central difference method, we obtain

$$\begin{aligned} &\boldsymbol{K} \frac{\boldsymbol{U}^{n+1} - \boldsymbol{U}^n}{\Delta t} + (\boldsymbol{H} - \boldsymbol{N} + \boldsymbol{G}) \frac{\boldsymbol{U}^{n+1} + \boldsymbol{U}^n}{2} \\ &\quad + \int_{\Gamma} \varepsilon \boldsymbol{\Phi} \frac{(u u_x)^{n+1} + (u u_x)^n}{2} \mathrm{d}\Gamma \\ &\quad + \int_{\Gamma} \lambda \boldsymbol{\Phi} \frac{(u^2 u_x)^{n+1} + (u^2 u_x)^n}{2} \mathrm{d}\Gamma \\ &\quad = \frac{\boldsymbol{Q}^{n+1} + \boldsymbol{Q}^n}{2}, \end{aligned} \tag{34}$$

where Δt is the time of step and

$$\boldsymbol{U}^n = \boldsymbol{U}(n\Delta t) = (u_1(n\Delta t), u_2(n\Delta t), \cdots, u_n(n\Delta t)). \tag{35}$$

The nonlinear terms $(u u_x)^{n+1}$ and $(u^2 u_x)^{n+1}$ in the above Eq. (34) are linearized by using the following term:[30]

$$(uv)^{n+1} = u^{n+1} v^n + u^n v^{n+1} - u^n v^n, \tag{36}$$

then we have

$$(u u_x)^{n+1} = u^{n+1} u_x^n + u_x^n u^{n+1} - u^n u_x^n, \tag{37}$$

$$(u^2 u_x)^{n+1} = 2u^{n+1} u^n u_x^n + u_x^{n+1} u^n u^n - 2u^n u^n u_x^n. \tag{38}$$

Substituting Eqs. (37) and (38) into Eq. (34), we have

$$\begin{aligned} &\boldsymbol{K} \frac{\boldsymbol{U}^{n+1} - \boldsymbol{U}^n}{\Delta t} + (\boldsymbol{H} - \boldsymbol{N} + \boldsymbol{G}) \frac{\boldsymbol{U}^{n+1} + \boldsymbol{U}^n}{2} \\ &+ \left(\int_{\Gamma} \varepsilon \boldsymbol{\Phi} \frac{\boldsymbol{U}^n (\boldsymbol{\Phi}_x \boldsymbol{\Phi}^{\mathrm{T}} + \boldsymbol{\Phi} \boldsymbol{\Phi}_x^{\mathrm{T}})}{2} \mathrm{d}\Gamma \right. \\ &+ \left. \int_{\Gamma} \lambda \boldsymbol{\Phi} \boldsymbol{\Phi}^{\mathrm{T}} \frac{(\boldsymbol{U}^n)^2 (2\boldsymbol{\Phi}_x \boldsymbol{\Phi}^{\mathrm{T}} + \boldsymbol{\Phi} \boldsymbol{\Phi}_x^{\mathrm{T}})}{2} \mathrm{d}\Gamma \right) \cdot \boldsymbol{U}^{n+1} \\ &= \frac{\boldsymbol{Q}^{n+1} + \boldsymbol{Q}^n}{2}. \end{aligned} \tag{39}$$

By solving the above iteration equation, we can obtain the numerical solution of compound KdVB equation.

4. Numerical examples

The application of the EFG method of numerically solving compound KdVB equation is demonstrated in this section by three examples, which are

chosen because their exact solutions are known. A regular arrangement of nodes and the background mesh of cells are used for numerical integrations to calculate the system equation. The linear basis and the cubic spline weight function are used in the EFG method. The cubic spline weight function used in the present analysis can be written as a function of normalized radius r as follows:

$$w_i = w(x - x_i) \equiv w(r) = \begin{cases} \dfrac{2}{3} - 4r^2 + 4r^3, & r \le \dfrac{1}{2}, \\ \dfrac{4}{3} - 4r + 4r^2 - \dfrac{4}{3}r^3, & \dfrac{1}{2} < r \le 1, \\ 0, & r > 1, \end{cases} \tag{40}$$

where

$$r = d_i/d_{m_i}, \quad d_i = \|\boldsymbol{x} - \boldsymbol{x}_i\|, \quad d_{m_i} = d_{\max} c_i, \tag{41}$$

with $d_{\max}$ being a scaling parameter and the distances c_i are chosen such that the moment matrix $\boldsymbol{A}$ is not singular.

The spatial derivatives of weight function are obtained as

$$\frac{\mathrm{d}w_i}{\mathrm{d}x} = \frac{\mathrm{d}w_i}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}x} = \begin{cases} (-8r + 12r^2)\dfrac{\mathrm{sign}(x - x_i)}{d_{m_i}}, & r \le \dfrac{1}{2}, \\ (-4 + 8r - 4r^2)\dfrac{\mathrm{sign}(x - x_i)}{d_{m_i}}, & \dfrac{1}{2} < r \le 1, \\ 0, & r > 1. \end{cases} \tag{42}$$

4.1. Example 1

Consider the following problem:

$$u_t + \varepsilon u u_x + \lambda u^2 u_x + \gamma u_{xx} - \mu u_{xxx} = 0, \ a < x < b, \tag{43}$$

with initial condition

$$u(x, 0) = -\frac{3\varepsilon\mu + \lambda\beta\gamma}{6\lambda\mu}(1 + \tanh(kx)), \tag{44}$$

and boundary conditions

$$u(a, t) = -\frac{3\varepsilon\mu + \lambda\beta\gamma}{6\lambda\mu}(1 + \tanh(k(a - vt))), \tag{45}$$

$$u(b, t) = -\frac{3\varepsilon\mu + \lambda\beta\gamma}{6\lambda\mu}(1 + \tanh(k(b - vt))). \tag{46}$$

The exact solution is[31]

$$u(x, t) = -\frac{3\varepsilon\mu + \lambda\beta\gamma}{6\lambda\mu}(1 + \tanh(k(x - vt))), \tag{47}$$

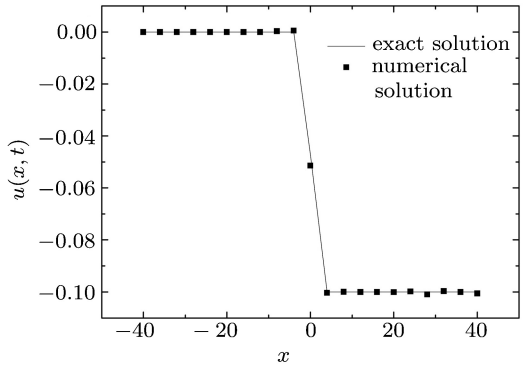


Fig. 1. Numerical solution and exact solution of $u(x, t)$ at $t = 0.5$.

where $\beta = \sqrt{\dfrac{6\mu}{\lambda}}$, $k = -\dfrac{3\varepsilon\mu + 2\lambda\beta\gamma}{6\lambda\beta\mu}$ and

$$v = \frac{(2\lambda\beta\lambda - 3\varepsilon\mu)(\lambda\beta\lambda - 3\varepsilon\mu)}{54b\mu^2}.$$

The EFG method is applied to the above compound KdVB equation with $a = -40$, $b = 40$, $\varepsilon = 0.05$, $\lambda = -0.15$, $\gamma = 0.5$, $\mu = -1$, $\Delta t = 0.01$ and the penalty factor $\alpha = 10^5$. In Fig. 1, the numerical solution and the exact solution are plotted at time $t = 0.5$.

4.2. Example 2

If we assume parameter $\lambda = 0$, the compound KdVB equation (15) is reduced to the following KdVB equation:

$$u_t + \varepsilon u u_x - \gamma u_{xx} + \mu u_{xxx} = 0, \ -20 < x < 20, \tag{48}$$

with initial condition

$$u(x, 0) = -\frac{6\gamma^2}{25\mu^2}\left[1 + \tanh(\frac{\gamma x}{10\mu}) + \frac{1}{2}\,\mathrm{sech}^2(\frac{\gamma x}{10\mu})\right]. \tag{49}$$

The exact solution is

$$u(x, t) = -\frac{6\gamma^2}{25\mu^2}\left[1 + \tanh(y) + \frac{1}{2}\,\mathrm{sech}^2(y)\right], \tag{50}$$

$$y = \frac{\gamma}{10\mu}\left(x + \frac{6\gamma^2}{25\mu}t\right). \tag{51}$$

The EFG method is used to solve the above KdVB equation with $\Delta t = 0.01$ and the penalty factor $\alpha = 10^5$. In Fig. 2, the numerical solution and the exact solution are plotted at time $t = 1$ for different values of viscosity $\gamma = 0.001$, 0.1 and 1.

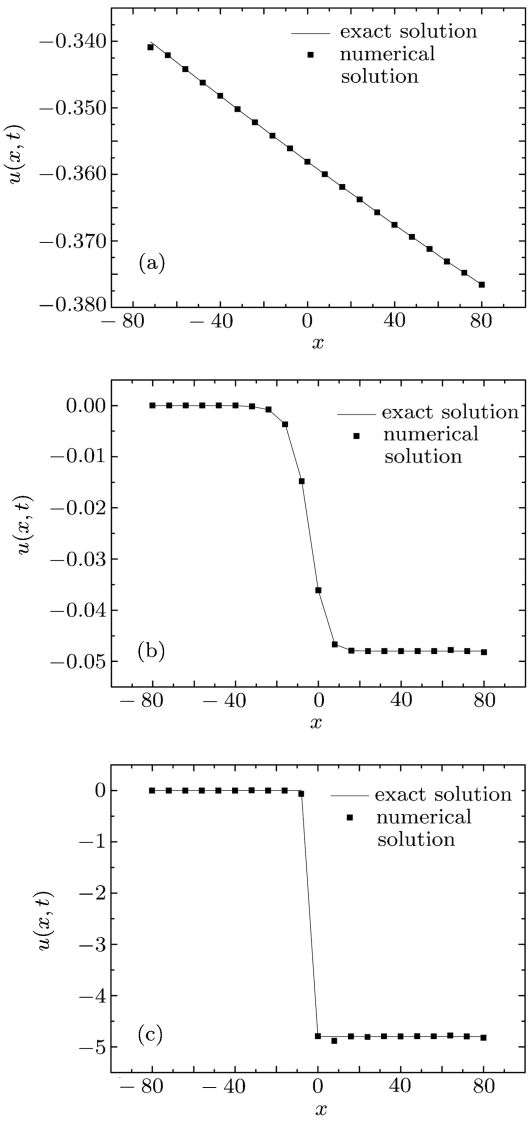


Fig. 2. Numerical solutions and exact solutions of $u(x,t)$ (a) at $t=1$ and $\gamma=0.001$, (b) at $t=1$ and $\gamma=0.1$, (c) at $t=1$ and $gamma=1$.

4.3. Example 3

If we assume parameters $\lambda=\gamma=0$, the compound KdVB equation (15) is reduced to the following KdV equation:

$$u_t+\varepsilon uu_x+\mu u_{xxx}=0,\ 0<x<2, \tag{52}$$

with the following initial condition and the boundary conditions:

$$u(x,0)=3c\cdot\operatorname{sech}^2(Ax+D), \tag{53}$$

$$u(0,t)=u(2,t)=0,\quad t>0. \tag{54}$$

In this case, the KdV equation has the analytical solution as follows:

$$u(x,t)=3c\cdot\operatorname{sech}^2(Ax-Bt+D), \tag{55}$$

where $A=\frac{1}{2}\sqrt{\varepsilon c/\mu}$, $B=\varepsilon cA$.

The EFG method is used to solve the above KdV equation with $\varepsilon=1$, $\mu=4.84\times10^{-4}$, $c=0.3$, $D=-6$ and $\Delta t=0.0001$. In Fig. 3, the numerical solution and the exact solution are plotted at time $t=0.5$.

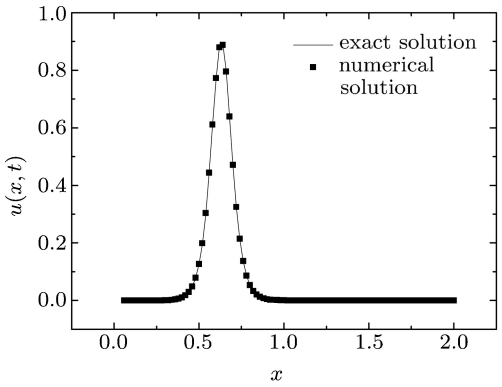


Fig. 3. Numerical solution and exact solution of $u(x,t)$ at $t=0.5$.

From these figures, it is shown that numerical results using EFG to solve the compound KdVB equation is in good agreement with the exact solutions.

5. Conclusion

The EFG method is used to solve the compound KdVB equation in this paper. The numerical results, which are in good agreement with the exact solutions, show that the technique is accurate and efficient. The method shows advantages over traditional mesh-dependent techniques, such as the finite difference method, the finite element method and the boundary element method, in solving problems with non-regular geometry. The EFG method should be extended to solving the problems of nonlinear partial differential equations which arise from the theory of solitons and other areas.

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