Donor-bound electron states in a two-dimensional quantum ring under uniform magnetic field*

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The electron states in a two-dimensional GaAs/AlGaAs quantum ring are theoretically studied in effective mass approximation. On-centre donor impurity and uniform magnetic field perpendicular to the ring plane are taken into account. The energy spectrum with different angular momentum changes dramatically with the geometry of the ring. The donor impurity reduces the energies with an almost fixed value; however, the magnetic field alters energies in a more complex way. For example, energy levels under magnetic field will cross each other when increasing the inner radius and outer radius of the ring, leading to the fact that the arrangement of energy levels is distinct in certain geometry of the ring. Moreover, energy levels with negative angular momentum exhibit the non-monotonous dependence on the increasing magnetic field.

Keywords: quantum ring, electron state, donor impurity, uniform magnetic field

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1. Introduction

As the promising nanostructure, quantum rings have been the focus in condensed matter and optoelectronics for more than two decades. Nowadays, advanced techniques for epitaxial growth make it possible to form various novel-shaped quantum rings based on III–V and IV–IV materials.[1−6] Not only the fundamental research on growth mechanism, but also the application of quantum rings is attractive for physicists. Possessing the three-dimensional confinement for the carriers, quantum rings are suitable in fabricating lasers,[7] photo-detector,[8] and even the single photon sources.[9] However, as the precondition of exploring the application in optoelectronic devices, the electronic structures of quantum rings are investigated experimentally and theoretically by many groups.[10−18]

Moreover, the electron states of quantum rings placed in the applied electric or magnetic field are of great interest. The external potential has strong competition with the natural geometric confinement of the heterostructure. Therefore, modified energy levels and changed localization of the electronic wavefunctions can be obtained.[19−23] For example, the magnetic field gives rise to a parabolic potential for the quantum rings, which is comparable to the geometric confinement. Besides that, through trapping the magnetic flux in their cylindrical interior, quantum rings are the best candidate to observe the Aharonov–Bohm oscillation among the diverse nanostructures.[24,25] Another interesting point is the quantum ring with an on-centre impurity. The energy levels and absorption spectrum of this system has been studied recently.[26,27]

In this paper, by solving the single-electron effective mass Schrödinger equation, we calculate the electron states in a two-dimensional GaAs/AlGaAs quantum ring containing an on-centre donor impurity. The uniform magnetic field is applied perpendicular to the quantum ring plane. The discontinuity of the electron effective mass, together with confinements originating from both the heterostructure and the magnetic field are taken into account. We theoretically study the dependence of electron energy levels on geometric parameters of the ring and on the magnetic induction. It is found that in the presence and absence of the magnetic field, the energy levels have distinct prop-

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erties. With the magnetic field and negative angular momentum, the energy crossing appears and the Aharonov–Bohm energy oscillation is obtained.

2. Theory

In the x–y plane we consider a two-dimensional quantum ring with a finite width, which is positioned in a uniform magnetic field along the z axis. The material of the heterostructure is GaAs/$Al_{0.3}Ga_{0.7}$As and a donor impurity is located in the ring centre. All material parameters throughout our calculation are taken from Ref. [28].

In the cylindrical coordinate system, considering the magnetic field and the donor impurity, the single-electron effective mass Schrödinger equation for this system is expressed as[22]

$$-\frac{\hbar^2}{2m^*}\left(\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\varphi^2}\right) - \frac{\mathrm{i}\hbar\omega_{\mathrm{C}}}{2}\frac{\partial\Psi}{\partial\varphi}+\frac{1}{8}m^*\omega_{\mathrm{C}}^2r^2\Psi+V\Psi=E\Psi, \quad (1)$$

where $\omega_{\mathrm{C}} = eB/m^*$ is the electron–cyclotron frequency and $V = V_{\mathrm{h}} + V_{\mathrm{c}}$ indicates the heterostructure confinement (i.e. the geometric confinement) and the Coulomb interaction between the donor impurity and the electron within the ring. Here we set the heterostructure confinement V_{h} to be 70% of the bandgap difference of GaAs and $Al_{0.3}Ga_{0.7}$As. The Coulomb interaction is $V_{\mathrm{c}} = -e^2/4\pi\varepsilon_0\varepsilon_{\mathrm{r}}r$. Both V_{h} and V_{c} have nothing to do with the polar angle for the cylindrical symmetry of the ring. By writing the wavefunction as

$$\Psi = \frac{\mathrm{e}^{\mathrm{i}\,l\varphi}}{\sqrt{2\pi}}R(r)$$

and substituting it into Eq. (1), we obtain the following equation only about the radial wavefunction $R(r)$,[26]

$$\left[-\frac{\hbar^2}{2m^*}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}}{\mathrm{d}r}\right)+\frac{\hbar^2}{2m^*}\frac{l^2}{r^2}+\frac{m^*}{8}\omega_{\mathrm{C}}^2r^2 + \frac{l}{2}\hbar\omega_{\mathrm{C}}+V(r)\right]R(r)=ER(r), \quad (2)$$

where l is the angular momentum quantum number. We use the finite element method to solve Eq. (2). The electron states influenced by both the magnetic field and the donor are studied.

3. Results and discussion

First, only considering the heterostructure confinement, we study the lowest electron energy level corresponding to different angular momentum l. Figures 1(a) and 1(b) show the energy levels changing with the inner and outer radius of the ring. The outer radius in Fig. 1(a) is $R_2 = 40$ nm and the inner radius in Fig. 1(b) is $R_1 = 10$ nm. Dependence of the energy levels on the quantum ring geometry is clearly obtained. Since there is no magnetic field applied, according to Eq. (2), energy levels with negative angular momentum l are identical to those of corresponding positive values. The rise or drop of energy levels in Fig. 1 originates from the geometric confinement. Besides that, we notice that the lines in Fig. 1(a) are closer to each other, which is more evident when R_1 is larger. However, the lines in Fig. 1(b) have larger and changeless spacing. Energy spacing between lines of l=0 and 1 is smaller than that between lines of l=1 and 2 in both Figs. 1(a) and 1(b).

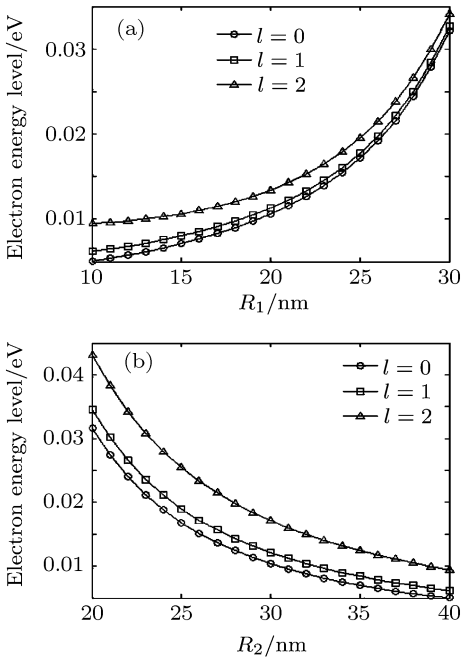


Fig. 1. Lowest energy levels with different angular momenta as a function of (a) inner radius R_1 (with R_2 = 40 nm) and (b) outer radius R_2 (with $R_1 = 10$ nm) of the quantum ring, with neither the on-centre donor impurity nor the magnetic field.

Figure 2 is the electron states in the on-centre donor impurity quantum ring. Figures 2(a) and 2(b) show energy levels versus inner radius R_1 and outer radius R_2, respectively. The trend of the lines is similar to that in Fig. 1 while the energy values decrease about 5 meV. However, when the ring is relatively small, i.e. in the left ends of Figs. 1(b) and 2(b), the

energies drop rapidly. The reason for that is the potential induced by the donor impurity obeys the inverse square law, inducing more effect near the donor.

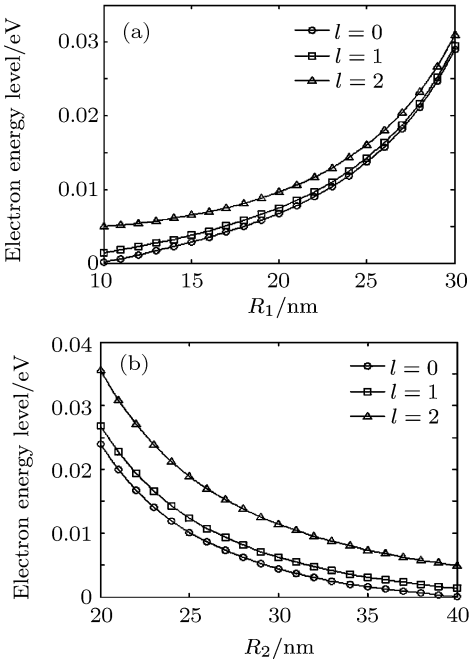


Fig. 2. The lowest energy levels with different angular momenta as a function of (a) inner radius R_1 (with $R_2 = 40$ nm) and (b) outer radius R_2 (with $R_1 = 10$ nm) of the quantum ring, with the on-centre donor impurity but without the magnetic field.

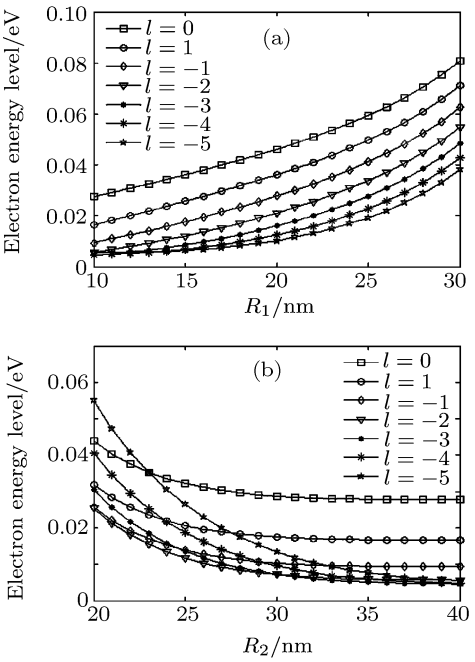


Fig. 3. The lowest energy levels with different angular momentum as a function of (a) inner radius R_1 (with $R_2 = 40$ nm) and (b) outer radius R_2 (with $R_1 = 10$ nm) of the quantum ring, with the on-centre donor impurity and the magnetic induction of 10 T.

Next we apply an applied magnetic induction of 10 T. Figure 3 shows the lowest energy level as a function of inner and outer radii of the ring containing the donor impurity. The energy levels with negative angular momentum values are paid more attention since they change more with the geometry. Lines in Fig. 3(b) exhibit more disorder. When R_2 is smaller than 27 nm, the level with $l = -5$ is higher than that with $l = 0$; and when R_2 is smaller than 23 nm, it is even higher than that with $l = 1$. The level with $l = -4$ crosses that with $l = 0$ at $R_2 = 24$ nm and gets close to that with $l = 1$ at $R_2 = 20$ nm, where the level with $l = -3$ approximates that with l=0. If the ring is relatively wider (the left end of Fig. 3(a) and the right part of Fig. 3(b)), the energy levels with l=2, 3, 4, and 5 are nearly the same.

The electron states versus the width d and the average radius R_0 of the ring are depicted in Figs. 4 and 5. The results without and with the magnetic field are compared. We keep the average radius R_0 equal to 25 nm in Fig. 4 and the width d 10 nm in Fig. 5.

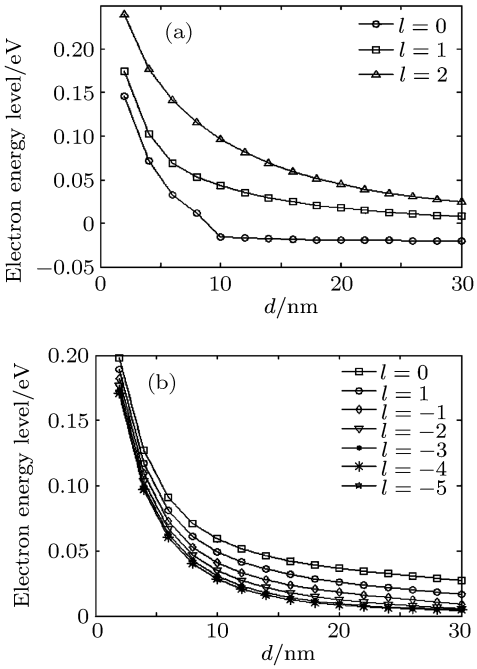


Fig. 4. The lowest energy levels with different angular momenta as a function of width d of the quantum ring (with $R_0 = 25$ nm), with the on-centre donor impurity and the magnetic induction of (a) 0 T and (b) 10 T.

From Fig. 4, it is clear that the energies decrease with d no matter whether the magnetic field exists or not. In Fig. 4(b), levels with $l = 2$, 3, 4, and 5 almost superpose especially in large width, which is also discussed in the above paragraph. The line of the ground

state in Fig. 4(a) is not smooth; moreover, it is flat when d exceeds 10 nm. The best example of the influence of the magnetic field on electron states is shown in Fig. 5. If we increase the average radius while fixing the width, the geometric confinement will not have much effect on the energy levels, so the slight change in Fig. 5(a) mainly reflects the donor-induced Coulomb interaction. However, the quantum ring with a larger average radius lies in stronger magnetic-field-induced potential, which brings a rising dependence of the energy levels on the average radius.

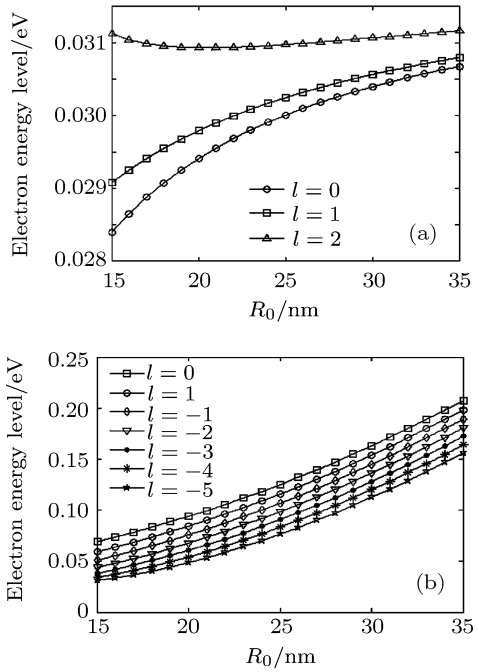


Fig. 5. The lowest energy levels with different angular momenta as a function of average radius R_0 of the quantum ring (with d = 10 nm), with the on-centre donor impurity and the magnetic induction of (a) 0 T and (b) 10 T.

Now we focus on the Aharonov–Bohm energy oscillation. Figure 6 depicts the oscillation as a function of the magnetic field. As described above, the magnetic field changes the behaviour of the electron with negative angular momentum markedly. This leads to that in Fig. 6 lines with l = 0 and 1 are changed monotonously while those with negative l are not and have their respective minima, which means that some energy levels will degenerate at certain magnetic induction. The minimum for each line only appears once because what we consider is an ideal single ring. It is definitely to observe the competition between the heterostructure and magnetic confinement in Fig. 6. Therefore, we can tailor the energy levels by adjusting the size of the quantum ring and applying magnetic field with certain magnetic induction.

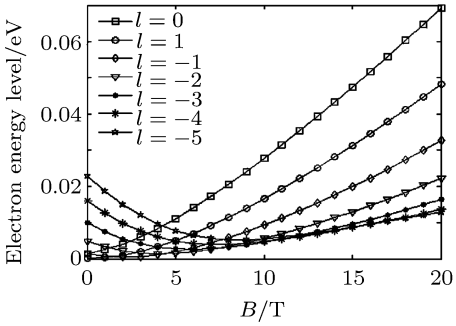


Fig. 6. The lowest energy levels with different angular momenta as a function of the magnetic induction B, with R_1 = 10 nm and R_2 = 40 nm and the on-centre donor impurity.

4. Conclusion

To sum up, we theoretically calculated the electron states in a two-dimensional $GaAs/Al_{0.3}Ga_{0.7}As$ quantum ring with an on-centre donor impurity, analysed the energy levels as functions of the geometry and the magnetic induction and discussed the Aharonov–Bohm energy oscillation. Although the energy levels possess common properties in quantum ring without the magnetic field, they exhibit the novelty under the applied magnetic field. The levels intersect each other when changing the geometry of the ring, so one certain energy level may be higher or lower than others for different ring geometries. The results give instructions to further investigate the device applications of magnetic quantum ring.

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