Landau damping of longitudinal oscillation in ultra-relativistic plasmas with nonextensive distribution*

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(Received 3 November 2010; revised manuscript received 25 November 2010)

The generalized dispersion equation for longitudinal oscillation in an unmagnetized, collisionless, isotropic and relativistic plasma is derived in the context of nonextensive q-distribution. An analytical expression for the Landau damping is obtained in an ultra-relativistic regime, which is related to q-parameter. In the limit $q \to 1$, the result based on the relativistic Maxwellian distribution is recovered. It is shown that the interactions between the wave and particles are stronger and the waves are more strongly damped for lower values of q-parameter. The results are explained by the increased number of superthermal particles or low velocity particles contained in the plasma with the nonextensive distribution.

Keywords: relativistic plasma, Landau damping, nonextensive distribution, longitudinal oscillation

PACS: 52.20.–j, 52.25.Dg, 52.27.Ny, 52.35.–g **DOI:** 10.1088/1674-1056/20/6/065201

1. Introduction

Over the last few years, a great deal of attention has been paid to nonextensive statistical mechanics (NSM), which is described by a nonextensive parameter q, based on the derivations of Boltzmann–Gibbs (B–G) entropic measure. NSM can be considered as the more appropriate basis of a theoretical framework to describe the complex systems whose properties cannot be exactly described by B–G statistical mechanics.[1] For $q \neq 1$, it gives power-law distribution functions and only when the parameter $q \to 1$, is Maxwellian distribution recovered.[2] NSM has been successfully applied to stellar polytropes,[3] two-dimensional Euler and drift turbulence in a pure electron plasma column,[4] as well as to the peculiar velocity function of galaxies clusters.[5] In particular, there is a growing interest to high energy physics applications of nonextensive statistics. Several authors outline the possibility that experimental observations in relativistic heavy-ion collisions can reflect nonextensive features during the early stage of the collisions and the thermalization evolution of the system.[6–9]

Landau damping is important for studies of wave-particle interactions, emission, absorption and other kinetic processes which require the damping rates as input. Many authors have discussed Landau damping from different aspects, since it is impractical to mention all of them, therefore, few will be cited here.[10–15] Recently, there has been a great deal of interest in studying the Landau damping in plasmas in the context of the nonextensive statistics. Lima *et al.*[16] have studied the dispersion relation and Landau damping of longitudinal oscillation based on q-distribution in a nonrelativistic plasma and the result showed that nonextensive formalism presents a good fit to the experimental data, while the standard Maxwellian distribution only provides a crude description. Valentini[17] studied the nonlinear Landau damping of electrostatic wave with nonextensive distribution in a nonrelativistic plasma. Liu and Du[18] have investigated the Landau damping of ion acoustic wave when the plasma is described by the nonextensive distribution. However, relativistic plasmas are present in many physical systems, like pulsar magnetospheres,[19,20] AGN jets,[21,22] and experiments in laser plasma interactions.[23] Thus, it is of interest to study such relativistic systems in the q-nonextensive context. In this paper, the Landau damping for longitudinal oscillation in an unmagnetized, collisionless, isotropic and relativistic plasma is discussed in the context of nonextensive q-distribution. The analytical expression is obtained in ultra-relativistic regime, which is discussed under the different constraints on the value of q-parameter.

*Project supported by the National Natural Science Foundation of China (Grant No. 10963002), the International S & T Cooperation Program of China and Jiangxi Province (Grant No. 2009DFA02320), the Program for Innovative Research Team of Nanchang University, and the National Basic Research Program of China (Grant No. 2010CB635112).

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The paper is organized as follows. In Section 2 we briefly introduce the relativistic nonextensive distribution function. The generalized Landau damping for longitudinal oscillations is obtained in Section 3. In Section 4, the Landau damping is discussed. Summary and conclusion are given in Section 5.

2. Relativistic nonextensive distribution function

First let us recall some basic facts about Tsallis statistics. In Tsallis statistics, the entropy has the form[2] of

$$S_q = k_{\rm B}\frac{1-\sum_i p_i^q}{q-1}, \tag{1}$$

where $k_{\rm B}$ is the Boltzmann constant, q is a parameter quantifying the degree of nonextensivity, p_i is the probability of the i-th microstate. The B–G entropy is recovered in the limit $q \to 1$. The basic property of Tsallis entropy is the nonadditivity or nonextensivity for $q \neq 1$. For example, for two systems A and B, the rule of composition[2] reads

$$S_q({\rm A}+{\rm B}) = S_q({\rm A}) + S_q({\rm B}) + (1-q)S_q({\rm A})S_q({\rm B}). \tag{2}$$

In the nonextensive description, the equilibrium distribution function for a relativistic plasma can be written as[24]

$$f_q(\boldsymbol{p}) = A_q\left[1-(q-1)\frac{\varepsilon}{k_{\rm B}T}\right]^{1/(q-1)}, \tag{3}$$

the normalization constant is

$$A_q = \frac{n_0}{4\pi(mc)^3\lambda}, \quad 2/3 < q \leq 1+1/\alpha, \tag{4}$$

in which

$$\lambda = \int_0^{u_{\max}}\left[1-(q-1)\alpha\sqrt{1+u^2}\right]^{1/(q-1)} u^2\,{\rm d}u, \tag{5}$$

where $u = p/mc$ is the reduced velocity, the temperature parameter $\alpha = mc^2/k_{\rm B}T$, $\varepsilon = mc^2\sqrt{1+u^2}$ is the energy; $\boldsymbol{p}$, c, m, $k_{\rm B}$, T, and n_0 denote the momentum of particles, speed of light in vacuum, rest mass of particles, Boltzmann constant, temperature of particles and particle number density, respectively. As one may check, for $q < 2/3$, the q-distribution is unnormalizable. For $2/3 < q \leq 1$, the momentum of the particles can take any value ($p_{\max} = \infty$, i.e., $u_{\max} = \infty$). For $q \geq 1$, the distribution function Eq. (3) exhibits a cutoff on the maximum value allowed for the momentum of the particles, which is given by

$$p_{\max} = mc\sqrt{[1/\alpha(q-1)]^2 - 1}, \tag{6}$$

namely,

$$u_{\max} = \sqrt{[1/\alpha(q-1)]^2 - 1}, \tag{7}$$

thus, a real value for the cutoff in Eq. (7) exists only if

$$q \leq 1 + 1/\alpha. \tag{8}$$

Finally, then, unlike the nonrelativistic case,[16] where $1/3 < q < \infty$, in the relativistic regime $2/3 < q \leq 1+1/\alpha$. We see that in the limit $q \to 1$, $p_{\max}$ goes to infinity and equation (3) reduces to the Boltzmann distribution function

$$f_{q=1}(\boldsymbol{p}) = A_1\exp(-\varepsilon/k_{\rm B}T). \tag{9}$$

In order to illustrate the effects of q-parameter on system energy in relativistic plasmas, the curve of the average energy of particle as a function of the q-parameter is plotted in Fig. 1, where temperature parameter $\alpha = 0.1$. From Fig. 1, we see that the average energy of particle decreases with the increase of q-parameter. In other words, the number of the superthermal particles in the system increases as the value of q decreases.

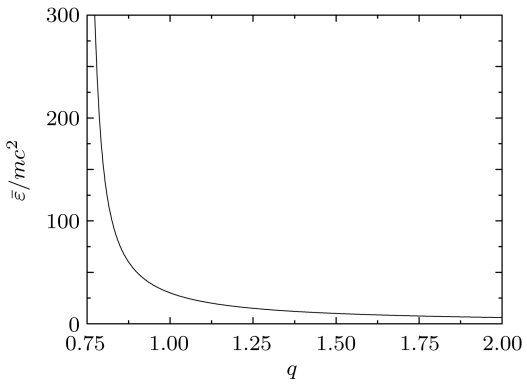


Fig. 1. The average energy of particles as a function of q-parameter at $\alpha = 0.1$

3. Generalized Landau damping

For the longitudinal wave propagating in an unmagnetized, collisionless, isotropic and relativistic ion–electron plasma, it is justified to consider the ion as immobile uniform background that simply maintains the charge neutrality. Therefore, we neglect the ion terms and only consider the electron terms. According to kinetic theory, the longitudinal dielectric function of electron can be written as[25]

$$\varepsilon_k^l = 1 + \frac{4\pi e^2}{\omega k^2}\int {\rm d}\boldsymbol{p}\frac{\partial f_q}{\partial\varepsilon}\frac{(\boldsymbol{k}\cdot\boldsymbol{v})^2}{\omega - \boldsymbol{k}\cdot\boldsymbol{v} + {\rm i}\delta}. \tag{10}$$

We now choose the z axis along the direction of the wave vector $\boldsymbol{k}$, $\boldsymbol{k} = k\boldsymbol{z}$, so that equation (10) becomes

$$\varepsilon_k^l = 1 + \frac{4\pi e^2}{\omega}\int \mathrm{d}\boldsymbol{p}\frac{\partial f_q}{\partial \varepsilon}\frac{v_z^2}{\omega - kv_z + \mathrm{i}\delta}, \tag{11}$$

where e is electronic charge, $\mathrm{i}\delta$ comes from Landau rules.[26] Introducing the reduced velocity, we have $u = \boldsymbol{p}/m_e c = \gamma \boldsymbol{v}/c$, so $\gamma = (1+u^2)^{1/2}$. Here $(\boldsymbol{u}, \theta, \varphi)$ represent the spherical coordinate system, so equation (11) becomes

$$\begin{aligned}\varepsilon_k^l \;=\; & 1 - \frac{8\pi^2 e^2 c}{\omega k}(m_e c)^3 \int \frac{\partial f_q}{\partial \varepsilon}\frac{u^3}{\gamma}\mathrm{d}u \\ & \times \int_{-1}^{1}\frac{x^2\,\mathrm{d}x}{x - \gamma\bar{v}_p/u - \mathrm{i}\delta},\end{aligned} \tag{12}$$

in which $\bar{v}_p = \omega/kc$.

Using the Plemelj formula[25]

$$\frac{1}{z \pm \mathrm{i}0} = \wp\frac{1}{z} \mp \mathrm{i}\pi\delta(z), \tag{13}$$

where $\wp$ denotes the principal value, the x integral in Eq. (13) becomes[25]

$$\begin{aligned}&\int_{-1}^{1}\frac{x^2}{x - \gamma\bar{v}_p/u - \mathrm{i}\delta}\mathrm{d}x \\ &= 2\frac{\gamma\bar{v}_p}{u}\left[1 - \frac{\gamma\bar{v}_p}{4u}\ln\left(\frac{\bar{v}_p + u/\gamma}{\bar{v}_p - u/\gamma}\right)^2\right. \\ &\quad \left. + \frac{\mathrm{i}\pi}{2}\frac{\gamma\bar{v}_p}{u}\theta\left(1 - \frac{\gamma\bar{v}_p}{u}\right)\right],\end{aligned} \tag{14}$$

where $\theta(\xi)$ is the step function, indicating that $\theta(\xi) = 1$ at $\xi > 0$ and $\theta(\xi) = 0$ at $\xi < 0$.

According to the longitudinal dispersion relation we obtain

$$\varepsilon_k^l(\omega, k) = 0. \tag{15}$$

Substituting Eqs. (3) and (14) into Eq. (12), we obtain the generalized dispersion equation of longitudinal oscillation

$$\begin{aligned}&\varepsilon_k^l \\ &= 1 + \frac{\alpha\omega_{pe}^2}{\lambda k^2 c^2}\int_0^{u_{\max}}\left[1 - (q-1)\gamma\alpha\right]^{1/(q-1)-1} G u^2\,\mathrm{d}u \\ &= 0,\end{aligned} \tag{16}$$

where $\omega_{pe} = \sqrt{4\pi n_0 e^2/m_e}$ is the plasma frequency, $u_{\max}$ is the maximum allowed by Eq. (7) if $1 \le q \le 1 + 1/\alpha$, or ∞ if $2/3 < q \le 1$ and

$$\begin{aligned}G \;=\; & 1 - \frac{\gamma\bar{v}_p}{4u}\ln\left(\frac{\bar{v}_p + u/\gamma}{\bar{v}_p - u/\gamma}\right)^2 \\ & + \frac{\mathrm{i}\pi}{2}\frac{\gamma\bar{v}_p}{u}\theta\left(1 - \frac{\gamma\bar{v}_p}{u}\right).\end{aligned} \tag{17}$$

When $\bar{v}_p > 1$, the oscillations are superluminous ($\omega/k > c$) in this range of wave numbers, $\theta(1 - \gamma\bar{v}_p/u) = 0$, the solution of Eq. (16) is of real number, so that there is no Landau damping in this case. when $\bar{v}_p < 1$, the oscillation becomes subluminous ($\omega/k < c$), $\theta(1 - \gamma\bar{v}_p/u) = 1$, then the oscillation frequency is found to be complex. Therefore, it is necessary to take into account the resonant interactions between the particles and such oscillations (Landau damping).

In the case of an ultrarelativistic plasma, namely, $\alpha \ll 1$, the characteristic values of u and γ are large, $u \sim \gamma \sim 1/\alpha$ and

$$1 - \frac{u}{\gamma} \approx \frac{1}{2u^2} \approx \alpha^2. \tag{18}$$

Due to this fact, equation (17) for G is simplified and takes the form

$$\begin{aligned}G \;=\; & 1 - \frac{\bar{v}_p}{4}\ln\left(\frac{\bar{v}_p + 1}{\bar{v}_p - 1 + 1/2u^2}\right)^2 \\ & + \frac{\mathrm{i}\pi}{2}\bar{v}_p\theta\left(1 - \frac{\gamma\bar{v}_p}{u}\right).\end{aligned} \tag{19}$$

Then the longitudinal dispersion equation (16) can be written in terms of the real and imaginary parts as

$$\begin{aligned}&\mathrm{Re}\,\varepsilon_k^l \\ &= 1 + \frac{\alpha\omega_{pe}^2}{\lambda k^2 c^2}\int_0^{u_{\max}}\left[1 - \frac{\bar{v}_p}{4}\ln\left(\frac{\bar{v}_p + 1}{\bar{v}_p - 1 + 1/2u^2}\right)^2\right] \\ &\quad \times\left[1 - (q-1)\gamma\alpha\right]^{1/(q-1)-1} u^2\,\mathrm{d}u \\ &= 0,\end{aligned} \tag{20}$$

$$\begin{aligned}&\mathrm{Im}\,\varepsilon_k^l \\ &= \frac{\pi}{2}\frac{\alpha\omega_{pe}^2}{\lambda k^2 c^2}\int_{u_0}^{u_{\max}}\bar{v}_p\left[1 - (q-1)\gamma\alpha\right]^{1/(q-1)-1} u^2\,\mathrm{d}u,\end{aligned} \tag{21}$$

where

$$u_0 = 1/\sqrt{2(1 - \bar{v}_p)}, \tag{22}$$

Note that equation (20) has the exact solution $\bar{v}_p = 1$, i.e., $\omega = ck$, if $k = k_0$; where k_0 satisfies

$$k_0 = \frac{\omega_{pe}}{c}\left[(3q-2)\,\alpha\ln\frac{1}{\alpha}\right]^{1/2}, \tag{23}$$

which is the critical wave number. Namely, there is no Landau damping for $k < k_0$. In this case

$$\omega_l = \omega_0 = \omega_{pe}\left[(3q-2)\,\alpha\ln\frac{1}{\alpha}\right]^{1/2}. \tag{24}$$

In order to find the variation of $\omega_l = \omega(k)$ near $k = k_0$, assume $k - k_0$ to be sufficiently small; we shall solve the dispersion relation by expansion into series with respect to small parameter $(k - k_0)/k$. Taking $\bar{v}_p = 1 + \Delta\bar{v}_p$, $k = k_0 + \Delta k$ in the first approximation with respect to Δk, we obtain

$$\left(\partial\varepsilon^l/\partial k\right)_{\bar{v}_p} \Delta k + \left(\partial\varepsilon^l/\partial\bar{v}_p\right)_k \Delta\bar{v}_p = 0, \tag{25}$$

where the derivatives ε_k^l as functions of k and $\bar{v}_p$ are taken for $k = k_0$, $\bar{v}_p = 1$. The value $\Delta\omega_l = \omega_l - ck_0$ we are interested in is found by taking into account $\Delta\omega_l = c(\Delta k + k_0\Delta\bar{v}_p)$ so we obtain

$$\Delta\omega_l = c\Delta k\left[1 - k_0\left(\partial\varepsilon^l/\partial k\right)_{\bar{v}_p}/\left(\partial\varepsilon^l/\partial\bar{v}_p\right)_k\right]. \tag{26}$$

According to Eq. (20), the quantity $\left(\partial\varepsilon^l/\partial k\right)_{\bar{v}_p}$ has a universal form not depending on plasma parameters,

$$\left(\partial\varepsilon^l/\partial k\right)_{\bar{v}_p} = \frac{2}{k_0}. \tag{27}$$

Using Eqs. (20) and (23) we obtain

$$\left(\partial\varepsilon^l/\partial\bar{v}_p\right)_k \approx \frac{1}{(3q-2)(4q-3)}\frac{12}{\alpha^2\ln(1/\alpha)}. \tag{28}$$

Substituting Eqs. (27) and (28) into Eq. (26), we obtain

$$\Delta\omega_l = c\Delta k\left[1 - (3q-2)(4q-3)\alpha^2\ln\frac{1}{\alpha}/6\right]. \tag{29}$$

Finally, taking into account $\omega_l = ck_0 + \Delta\omega_l$, we determine the variation of the dispersion relation near the critical wave number k_0

$$\omega_l = c\left[k - (3q-2)(4q-3)(k-k_0)\alpha^2\ln(1/\alpha)/6\right]. \tag{30}$$

As expected, the dispersion relation near the critical wave number k_0 in the framework of B–G statistics, $\omega_l = c\left[k - (k-k_0)\alpha^2\ln(1/\alpha)/6\right]$, is recovered in the limiting case $(q \to 1)$.[27] Calculate the difference $1 - \bar{v}_p$ entering into Eq. (22) for u_0 by using Eq. (30), then equation (22) becomes

$$u_0 = \frac{1}{\alpha}\sqrt{\frac{3k_0}{(3q-2)(4q-3)(k-k_0)\ln(1/\alpha)}}. \tag{31}$$

Considering $(\partial\mathrm{Re}\,\varepsilon_k^l/\partial\omega)|_{\omega=\omega_l} = ((\partial\mathrm{Re}\,\varepsilon_k^l/\partial\bar{v}_p)(\partial\bar{v}_p/\partial\omega))|_{\omega=\omega_l}$ and using Eq. (28), we find

$$\left(\partial\mathrm{Re}\,\varepsilon_k^l/\partial\omega\right)\big|_{\omega=\omega_l} = \frac{1}{(3q-2)(4q-3)}\frac{12}{\alpha^2k_0c\ln(1/\alpha)}. \tag{32}$$

Then the Landau damping rate can be written as

$$\gamma_k^l = -\frac{\mathrm{Im}\,\varepsilon_k^l}{\dfrac{\partial}{\partial\omega}\,\mathrm{Re}\,\varepsilon_k^l\big|_{\omega=\omega_l}}. \tag{33}$$

Using Eqs. (21) and (32), the generalized Landau damping can be obtained from Eq. (33) as

$$\gamma_k^l = -\frac{\pi}{24}(3q-2)(4q-3)\alpha^3k_0c\ln\left(\frac{1}{\alpha}\right)\frac{\omega_{pe}^2}{\lambda k_0^2c^2}\int_{u_0}^{u_{\max}}\bar{v}_p\left[1-(q-1)\gamma\alpha\right]^{1/(q-1)-1}u^2\,\mathrm{d}u. \tag{34}$$

When $2/3 < q \le 1$, $u_{\max} = \infty$, equation (34) becomes

$$\gamma_k^l = -\frac{\pi}{48}\alpha^4k_0cu_0^2\tau_2\mathrm{F}_1\left(\frac{1}{1-q}+1, \frac{1}{1-q}-2; \frac{1}{1-q}-1; -\frac{1}{(1-q)\alpha u_0}\right), \tag{35}$$

where

$$\tau = \frac{4q-3}{2q-1}\frac{(1-q)^3\,\Gamma\left[1/(1-q)\right]}{\Gamma\left[1/(1-q)-3\right]\left[(1-q)\alpha u_0\right]^{1/(1-q)}}, \tag{36}$$

and ${}_2\mathrm{F}_1$ is the hypergeometric function, Γ is the Gamma function.

When $1 \le q \le 1 + 1/\alpha$, $u_{\max}$ is the maximum allowed by Eq. (7), equation (34) becomes

$$\gamma_k^l = -\frac{\pi}{48}\alpha^2k_0c(3q-2)(4q-3)\left\{2 - \mu_2\mathrm{F}_1\left(1-\frac{1}{q-1}, 3; 4; (q-1)\alpha u_0\right)\right\}, \tag{37}$$

where

$$\mu = \frac{(q-1)^3\,\Gamma\left[1/(q-1)+3\right]}{3\Gamma\left[1/(q-1)\right]}\alpha^3u_0^3. \tag{38}$$

If the usual B–G statistics is used, the Landau damping for ultra-relativistic plasma is[27]

$$\gamma_k^l = -\frac{\pi}{48}\alpha^4k_0cw_0^2\,\mathrm{e}^{-\alpha w_0}, \tag{39}$$

where

$$w_0 = \frac{1}{\alpha}\sqrt{\frac{3k_0}{(k-k_0)\ln(1/\alpha)}}. \tag{40}$$

In Fig. 2 we have plotted, for the value of $q \to 1$, the ratio γ_k^l/ω_{pe} as a function of the dimensionless parameter kc/ω_{pe} according to Eqs. (35), (37) and (39). As expected, the agreement is very well. Confirming that the Landau damping based on the relativistic Maxwellian distribution is recovered in our analytical result for $q \to 1$. It should be noted that the damping rate is correct in a sufficiently wide vicinity near critical wave number k_0 (i.e., $|k-k_0|/k \ll 1$). When the inequality is broken, these waves become strongly damped, then the wave may no longer exist.

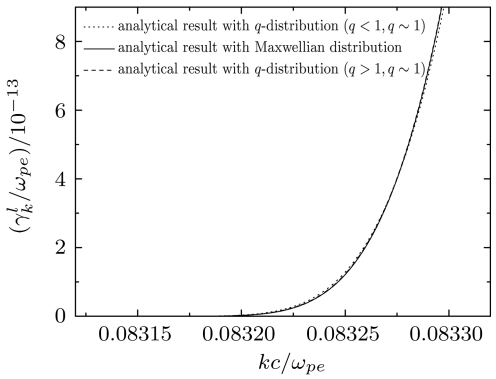


Fig. 2. The curves of γ_k^l/ω_{pe} versus kc/ω_{pe}.

4. The Landau damping and q-parameter

In this section, we will explore the Landau damping under the different constraints for some selected values of q-parameter. In Fig. 3, we plot the curves of γ_k^l/ω_{pe} versus the dimensionless parameter kc/ω_{pe} for some selected values of the nonextensive parameter q in the case of $\alpha = 0.001$. The selected values of the q-parameter are around 0.9, 1.0 and 1.1, respectively, in Figs. 3(a), 3(b), and 3(c).

From Fig. 3, we note that the curve of damping, which is strongly dependent on the q-parameter, shifts to the right with increasing q-parameter. Namely, the critical wave number k_0 increases with the increases in the value of q-parameter. For a fixed value of the dimensionless parameter kc/ω_{pe}, if q increases the Landau damping decreases. Now we discuss the reason for this phenomenon. As one knows, Landau damping stems from the resonant interactions between the wave and particles moving with the speed close to the phase velocity, as a result, the damping conditions are determined by the number of resonant particles. It is evident from Fig. 1 that the average energy of particle decreases with increasing q-parameter, namely the number of the superthermal particles in the system increases as the values of q decrease. Therefore, there are more resonant particles in the system with lower values of q-parameter. Consequently, the interactions between the wave and particles are stronger and the waves get more damped at lower values of q-parameter.

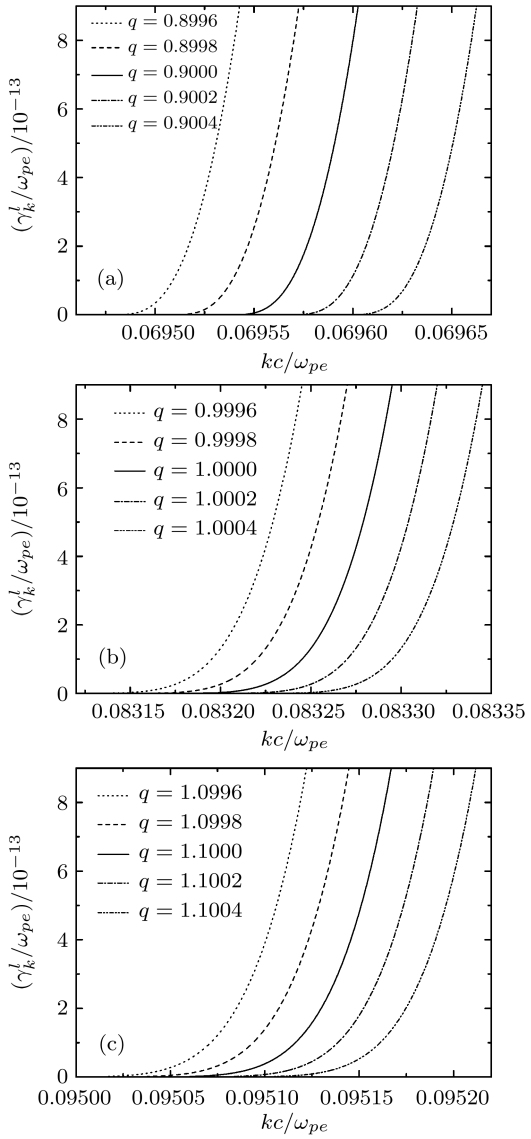


Fig. 3. The curves of γ_k^l/ω_{pe} versus kc/ω_{pe} for some selected values of q-parameter.

5. Summary and conclusion

In this paper, we have discussed the Landau damping for longitudinal oscillation in an unmagnetized, collisionless, isotropic and relativistic plasma in

the context of nonextensive distribution. The analytical damping rate is obtained in the ultra-relativistic regime. In the limiting case ($q \to 1$), the result based on the relativistic Maxwellian distribution is recovered. It is shown that the curve of damping, which is strongly dependent on the q-parameter, shifts to the right with increasing q-parameter. Namely, the critical wave number k_0 increases for the increase in the value of q-parameter. For a fixed value of the dimensionless parameter kc/ω_{pe}, the Landau damping increases if the value of q decreases, namely, the interactions between the wave and particles are stronger and the waves get more damped at lower values of q-parameter. The results are explained by the increased number of superthermal particles or low-velocity particles contained in the plasma with the nonextensive distribution.

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