Complete leaky mode coupling in dual-core photonic crystal fibre based on the coupled-mode theory*

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(Received 10 May 2010; revised manuscript received 5 January 2011)

This paper theoretically investigates the dependence of leaky mode coupling between inner core fundamental mode and outer core defect mode on phase and loss matching in pure silica dual-core photonic crystal fibres with the multi-pole method. The complete mode coupling can take place when both the phase and loss matching conditions are satisfied at the avoided anti-crossing wavelength. It shows the influences of cladding structure parameters including the diameters of cladding air holes d_1, diameters of outer core holes d_2 and hole to hole pitch Λ on the characteristics of leaky modes coupling. The coupled-mode theory is used to analyse the mode transition characteristics and the complete coupling can be clearly indicated by comparing the real and imaginary parts of propagation constant of the leaky modes.

Keywords: dual-core photonic crystal fibres, complete leaky mode coupling, multi-pole method, coupled-mode theory

PACS: 41.20.Jb, 42.81.–I, 42.81.Dp **DOI:** 10.1088/1674-1056/20/6/064101

1. Introduction

Dispersion compensating fibres are widely used for compensating the dispersion accumulated over a transmission distance. The photonic crystal fibres (PCFs)[1,2] with air holes embedded in the silica matrix have attracted lots of attention because of their remarkable optical properties such as large mode area and controllable dispersion,[3,4] and so on. The designing degrees of freedom in PCFs facilitate a flexible control on its dispersion properties such as ultra-flattened and large negative dispersion. Several designs of dispersion compensating PCFs have been reported in Refs. [5]–[19] and some designs are based on dual-core refractive index profile.[16−19] In the dual-core PCFs (DCPCFs), two modes including the central core and ring core mode are supported and the optical fields transits to the ring core from the central core after the phase-matching wavelength through coupling effect.

Because of modes in PCFs being leaky, large negative dispersion in DCPCFs is caused by leaky mode coupling. Engeness *et al.* investigated the dispersion compensation based on modal interactions in OmniGuide fibres.[4] Roberts *et al.*[20] showed mode coupling between the inner core fundamental mode of PCF and outer core defect mode to obtain large negative dispersion at the avoided anti-crossing point. Saitoh *et al.* designed single mode large hollow-core fibre based on the index-matching mechanism of air-core modes with outer core defect modes.[21] Renversez *et al.*[22] explained a core mode transition induced by avoided crossing between a core leaky mode and a higher-index cylinder leaky mode in anti-resonant PCFs. Zhang *et al.*[23] investigated the dependence of leaky mode coupling on loss in PCF with

*Project supported by the National Key Basic Research Special Foundation of China (Grant No. 2010CB327605), the National High-Technology Research and Development Program of China (Grant No. 2009AA01Z220), the Key Grant of the Chinese Ministry of Education (Grant No. 109015), the Discipline Co-construction Project of Beijing Municipal Commission of Education (Grant No. YB20081001301), the Open Fund of Key Laboratory of Information Photonics and Optical Communications (Beijing University of Posts and Telecommunications), Ministry of Education, and the Specialized Research Fund for the Doctoral Program of Beijing University of Posts and Telecommunications (Grant No. CX201023).

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hybrid cladding and showed that the imaginary part of propagation constant was also special for a leaky mode like the real part.

In this paper, the dependence of leaky mode coupling between inner core fundamental mode and outer core defect mode on phase and loss matching in DCPCFs is theoretically investigated based on the multi-pole method (MPM).[24,25] In such pure silica DCPCFs, the phase and loss matching of leaky modes can be adjusted by changing the diameters of cladding air holes d_1, diameters of outer core holes d_2 and hole to hole pitch Λ and the coupled-mode theory is used to analyse the characteristics of leaky mode coupling.

2. Numerical simulation

The designed DCPCF consists of a silica core surrounded by five rings of large air holes and one ring of small air holes in the third ring in a hexagonal lattice, as shown in Fig. 1. The diameters of large and small air holes are fixed at $d_1 = 1.1$ μm and $d_2 = 0.51$ μm and the hole to hole pitch is set to $\Lambda = 1.2$ μm. The indices of inner core region and cladding background are 1.45 and the index of the air holes is 1.0.

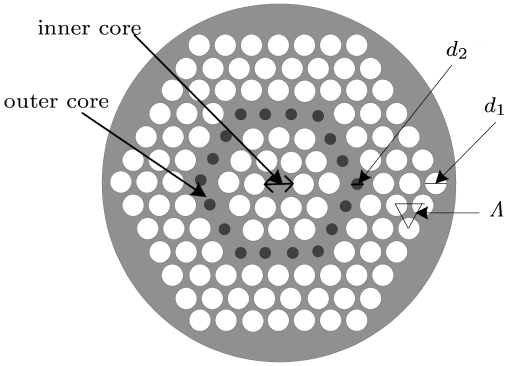


Fig. 1. The cross section of DCPCF along with the effective index profile.

The inner core fundamental mode and outer core defect mode can be supported in the DCPCF. The complete leaky mode coupling between the central core and outer ring core at avoided crossing wavelength can happen while the phase and loss matching conditions are satisfied. As presented in Figs. 2(a) and 2(b), the real and imaginary parts of mode effective index change rapidly around the phase and loss matching wavelength of 1.538 μm and the mode transitions happen to the phases and losses which are in proportion to the real and imaginary parts of effective index. In Fig. 2(c), the dispersion curves of two leaky modes with peak value of about ±23500 ps·nm·km around 1.538 μm are shown. As shown in inserts of Fig. 2(c), the mode field of inner core fundamental mode mainly resides in the inner core region at shorter wavelength, leaks into the outer core region at longer wavelength, and a critical mode state emerges at the phase and loss matching wavelength. However, an inverse transiting trend is appearing for the outer core defect mode. Subsequently the influences of d_1, d_2 and Λ on the characteristics of leaky modes coupling will be analysed.

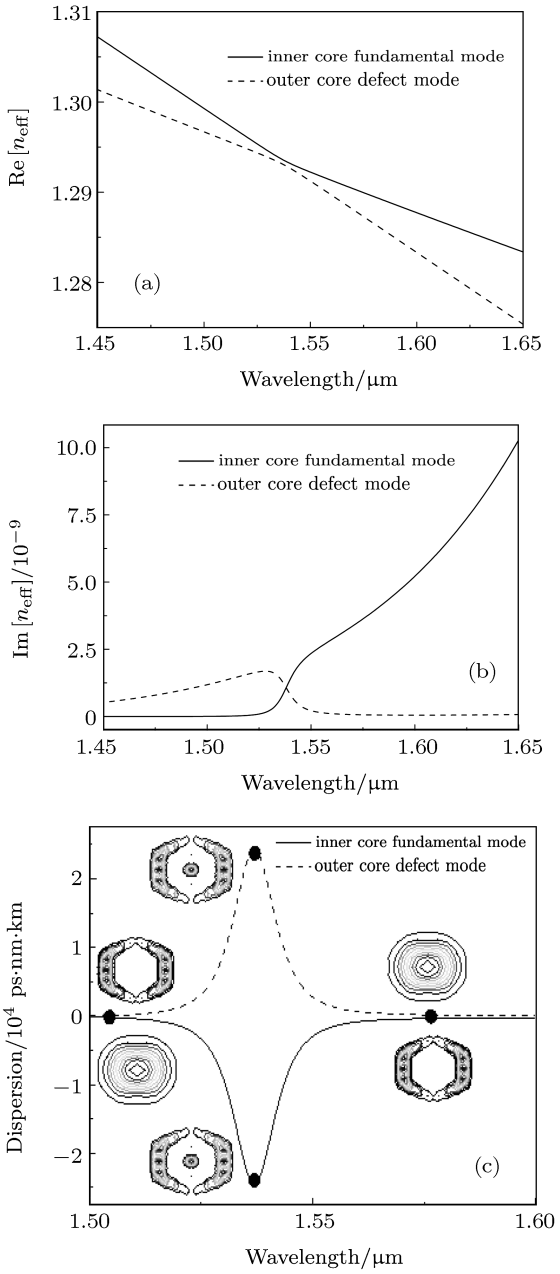


Fig. 2. Real part (a) and imaginary part (b) of the effective index of inner core fundamental mode and outer core defect mode as a function of wavelength when the complete coupling happens. (c) The dispersions of two modes, the inserts showing the intensity distribution of mode fields at given wavelengths.

As shown in Fig. 3(a) where d_1 changes from 1.1 μm to 1.085 μm and d_2 and Λ keep constant, the real part of effective index of inner core fundamental mode increases with decreasing d_1 due to larger cladding effective index and the same case also happens to the outer core defect mode around the phase-matching wavelength of 1.538 μm. In Fig. 3(b), for different d_1, although the losses between the two leaky modes are not equal, the loss matching condition is satisfied around the loss matching wavelength of 1.538 μm. With increasing d_1, the imaginary part of the effective index of outer core defect mode gradually increases while the one of inner core fundamental mode decreases first and increases subsequently and the interaction strength of the leaky mode becomes stronger as d_1 decreases due to a smaller air filling fraction. It is also found that the losses of the two leaky modes gradually increase as d_1 decreases. Moreover, the loss curves of coupled modes intersect at the phase matching point and it is also easy to distinguish whether the mode coupling happens from it or not. Because of similar

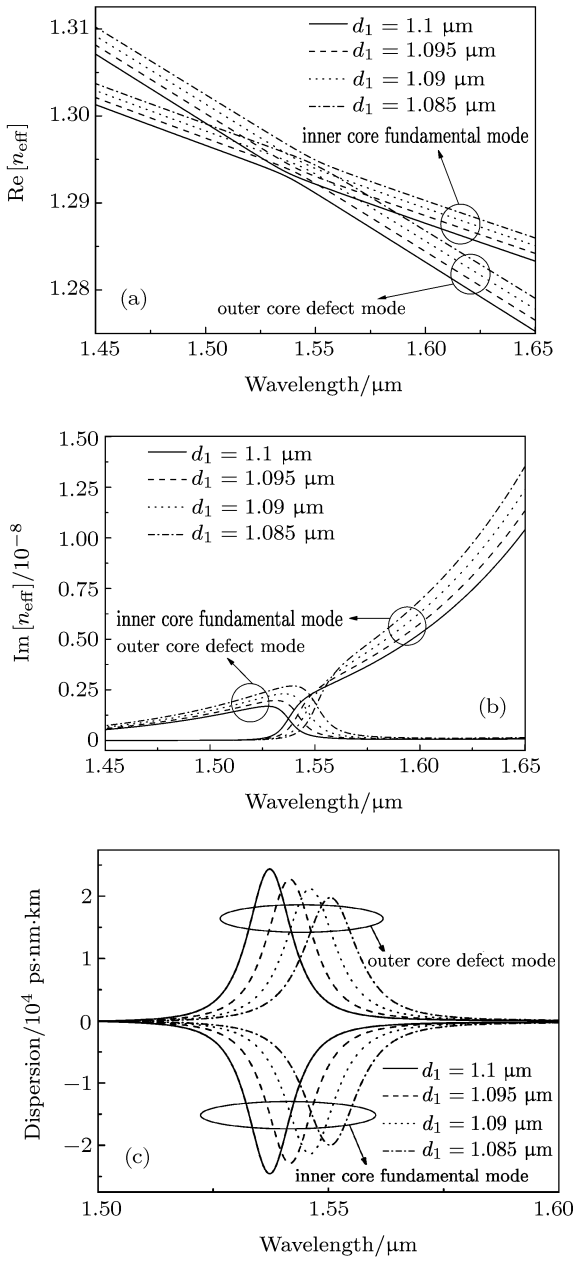


Fig. 3. Real part (a) and imaginary part (b) of the effective index of inner core fundamental mode and outer core defect mode as a function of wavelength when d_1 changes. (c) The dispersions of two modes as a function of wavelength when d_1 changes.

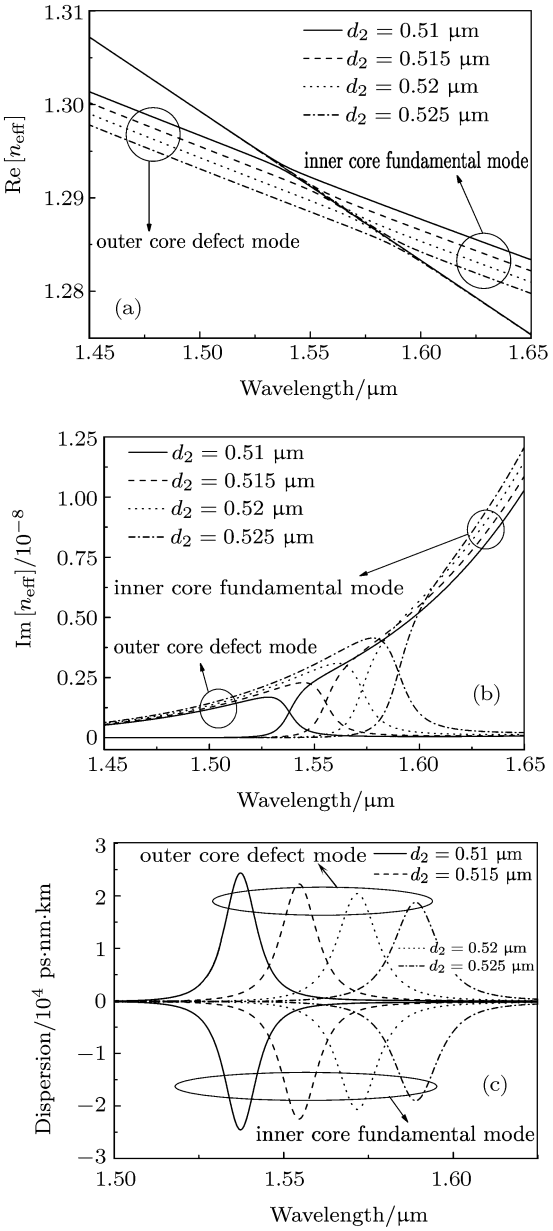


Fig. 4. Real part (a) and imaginary part (b) of the effective index of inner core fundamental mode and outer core defect mode as a function of wavelength when d_2 changes. (c) The dispersions of two modes as a function of wavelength with d_2 changing.

changing trend on mode loss, the intersections of loss curves correspond to higher loss values as d_1 decreases. Considering real and imaginary parts of the effective index of leaky modes, we find that a stronger coupling can happen within a wider wavelength range through adjusting d_1 and the propagation characteristics of two leaky modes appear alternately. As presented in Fig. 3(c), for two leaky modes, the dispersion curves shift to longer wavelengths and the values decrease as d_1 becomes smaller and the wavelengths of peak values correspond to the phase and loss matching wavelength.

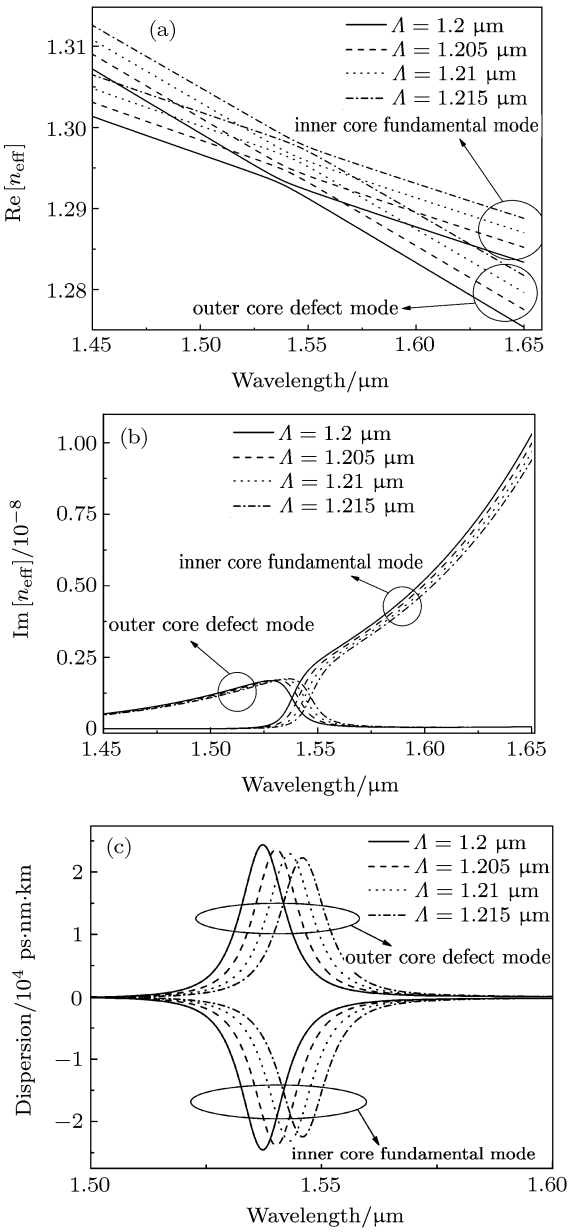


Fig. 5. Real part (a) and imaginary part (b) of the effective index of inner core fundamental mode and outer core defect mode as a function of wavelength when Λ changes. (c) The dispersions of two modes as a function of wavelength when Λ changes.

Figures 4(a) and 4(b) show the real and imaginary parts of the effective index of the inner core fundamental mode and outer core defect mode as a function of wavelength when d_2 respectively increases from 0.51 μm to 0.525 μm. As shown in Fig. 4(a), for two leaky modes, the real part of inner core fundamental mode decreases with increasing d_2 and the similar cases happen to the outer core defect mode. In Fig. 4(b), the losses of two leaky modes rapidly increase and decrease near the loss matching wavelengths and the mode coupling transitions are achieved in a wider wavelength range compared to Fig. 3(b). In Fig. 4(c), the dispersion curves of two leaky modes show similar changing trend to that in Fig. 3(c). The dispersion curves shift to longer wavelengths and the values decrease with increasing d_2.

The real and imaginary parts of effective indices of inner core fundamental mode and outer core defect mode as a function of wavelength when Λ changes are presented in Figs. 5(a) and 5(b) respectively. With Λ increasing from 1.2 μm to 1.215 μm, for two leaky modes, the real parts of effective indices increase with increasing Λ, as shown in Fig. 5(a). In Fig. 5(b), for different Λ, the contrary changing trend takes place on the imaginary parts of effective indices of two leaky modes due to the relative changing of modal and cladding effective indices. Considering the phase and loss matching conditions, the complete mode coupling is achieved within a narrower wavelength range. With increasing Λ and the phase and loss matching wavelength shifting to longer wavelengths, the dispersion curves gradually shift to longer wavelengths and the values decrease, as shown in Fig. 5(c).

3. Theoretical analysis

The coupled-mode theory can be used to explain the leaky mode coupling between the inner core fundamental mode and outer core defect mode. For two leaky modes, the coupled-mode equations can be written as

$$\frac{\mathrm{d}E_1}{\mathrm{d}z} = \mathrm{i}\,\beta_1 E_1 + \mathrm{i}\,\gamma_1 E_2, \tag{1}$$

$$\frac{\mathrm{d}E_2}{\mathrm{d}z} = \mathrm{i}\,\beta_2 E_2 + \mathrm{i}\,\gamma_2 E_1, \tag{2}$$

where E_1 and E_2 are the electromagnetic intensities of two leaky modes, respectively, β_1 and β_2 are the alterative propagation constants caused by changing waveguide geometry, γ_1 and γ_2 correspond to the coupling coefficients between the leaky modes. Assuming $E_1 = E_{01}\mathrm{e}^{-\mathrm{i}\,\omega_1 t}\mathrm{e}^{\mathrm{i}\,k_1 z} = E_{m1}\mathrm{e}^{\mathrm{i}\,k_1 z}$ and $E_2 = E_{02}\mathrm{e}^{-\mathrm{i}\,\omega_2 t}\mathrm{e}^{\mathrm{i}\,k_2 z} = E_{m2}\mathrm{e}^{\mathrm{i}\,k_2 z}$, considering $\gamma_1 = \gamma_2 = \gamma$,

$k_1 = k_2 = k$ and substituting them into Eqs. (1) and (2), finally we obtain

$$\begin{bmatrix} k-\beta_1 & -\gamma \\ -\gamma & k-\beta_2 \end{bmatrix} \begin{bmatrix} E_{m1} \\ E_{m2} \end{bmatrix} = 0. \quad (3)$$

Considering $\det \begin{vmatrix} k-\beta_1 & -\gamma \\ -\gamma & k-\beta_2 \end{vmatrix} = 0$, we can obtain the propagation constants of leaky modes k as follows:

$$k = \frac{\beta_1+\beta_2}{2} \pm \sqrt{\left(\frac{\beta_1-\beta_2}{2}\right)^2 + \gamma^2}. \quad (4)$$

Here, assuming $(\beta_1+\beta_2)/2 = \bar{\beta}$ and $(\beta_1-\beta_2)/2 = \delta$, we can describe Eqs. (4) as

$$k = \bar{\beta} \pm \sqrt{\delta^2+\gamma^2}. \quad (5)$$

For the leaky modes, $\beta_1 = \beta_{1\mathrm{R}} + \mathrm{i}\,\beta_{1\mathrm{I}}$ and $\beta_2 = \beta_{2\mathrm{R}} + \mathrm{i}\,\beta_{2\mathrm{I}}$, so $\delta = \delta_{\mathrm{R}} + \mathrm{i}\,\delta_{\mathrm{I}}$ where $\delta_{\mathrm{R}} = (\beta_{1\mathrm{R}} - \beta_{2\mathrm{R}})/2$ and $\delta_{\mathrm{I}} = (\beta_{1\mathrm{I}} - \beta_{2\mathrm{I}})/2$. Considering the phase-matching condition, we find that the real parts of propagation constant of two leaky modes are equal, $\delta_{\mathrm{R}} = 0$. Moreover, $k_+ = \bar{\beta} + \sqrt{-{\delta_{\mathrm{I}}}^2+\gamma^2}$ and $k_- = \bar{\beta} - \sqrt{-{\delta_{\mathrm{I}}}^2+\gamma^2}$, when $\delta_{\mathrm{I}} < \gamma$, $\sqrt{-{\delta_{\mathrm{I}}}^2+\gamma^2}$ is real,

$$k_+ = \frac{\beta_{1\mathrm{R}}+\beta_{2\mathrm{R}}}{2} + \sqrt{-{\delta_{\mathrm{I}}}^2+\gamma^2} + \mathrm{i}\,\frac{\beta_{1\mathrm{I}}+\beta_{2\mathrm{I}}}{2}, \quad (6)$$

$$k_- = \frac{\beta_{1\mathrm{R}}+\beta_{2\mathrm{R}}}{2} - \sqrt{-{\delta_{\mathrm{I}}}^2+\gamma^2} + \mathrm{i}\,\frac{\beta_{1\mathrm{I}}+\beta_{2\mathrm{I}}}{2}. \quad (7)$$

Comparing the real and imaginary parts of Eq. (6) with those of Eq. (7), we find that the real parts are not equal but the imaginary parts are equal, under this condition the complete coupling (avoided anti-crossing) can happen.

When $\delta_{\mathrm{I}} > \gamma$, $\sqrt{-{\delta_{\mathrm{I}}}^2+\gamma^2}$ is imaginary,

$$k_+ = \frac{\beta_{1\mathrm{R}}+\beta_{2\mathrm{R}}}{2} + \mathrm{i}\left[\frac{\beta_{1\mathrm{I}}+\beta_{2\mathrm{I}}}{2} + \sqrt{{\delta_{\mathrm{I}}}^2-\gamma^2}\right], \quad (8)$$

$$k_- = \frac{\beta_{1\mathrm{R}}+\beta_{2\mathrm{R}}}{2} + \mathrm{i}\left[\frac{\beta_{1\mathrm{I}}+\beta_{2\mathrm{I}}}{2} - \sqrt{{\delta_{\mathrm{I}}}^2-\gamma^2}\right]. \quad (9)$$

In Eqs. (8) and (9), the real parts are equal but the imaginary parts are not equal, so the complete mode coupling can not happen.

Here, the mode coupling mainly refers to leaky modes not bound modes. In general, the complete coupling of two leaky modes in DCPCFs takes place only when the phase and loss matching conditions are satisfied at the same time. The propagation characteristics of inner and outer core modes start to exchange within the wavelength range of mode field diverting near the mode coupling wavelengths. In such pure silica DCPCFs, the characteristics of phase and loss matching can be evidently changed by adjusting the structural parameters. Based on the above theoretical analysis, whether a complete coupling being or not being happened can be clearly indicated through comparing the real and imaginary parts of propagation constant of the leaky modes and the propagation characteristics of leaky modes can also be well predicted.

4. Conclusion

In summary, the complete leaky mode coupling in such pure silica DCPCFs is deeply investigated. When both phase and loss matching conditions are satisfied at the same time, the larger negative dispersion will emerge and the propagation characteristics of inner and outer core modes will happen to exchange completely. The influences of structural parameters on the leaky modes coupling are shown. The coupled-mode theory is used to analyse and predict the characteristics of complete leaky mode coupling.

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